Random number generator: testing and whitening

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ESCAPE / LIRMM

invitation to an internship (2020)

What is a random sequence of digits?

- `0000000000000000000000000000000000000000`

- 0000000000000000000000000000000000000000
- 0101010101010101010101010101010101010101

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- `0101010101010101010101010101010101010101`
- `1101111011100001101101010110001111101010`

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- 1100100100001111110110101010001000100001

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  (btw, the last sequence is a binary expansion of π)

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but some look more random than others.

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$x_1 \dots x_n$ is random means $C(x_1 \dots x_n) \approx n$
$x_1 \dots x_n$ is non-random means $C(x_1 \dots x_n) \ll n$

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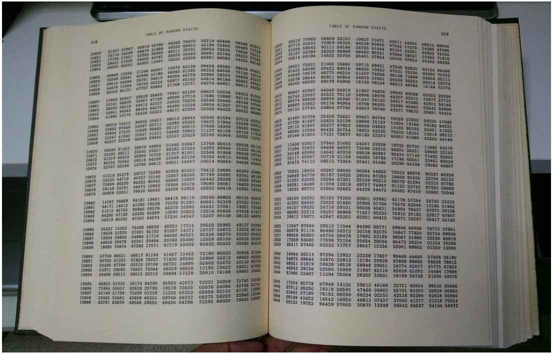
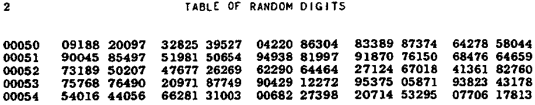
Big practical question: where do we get truly random bits?



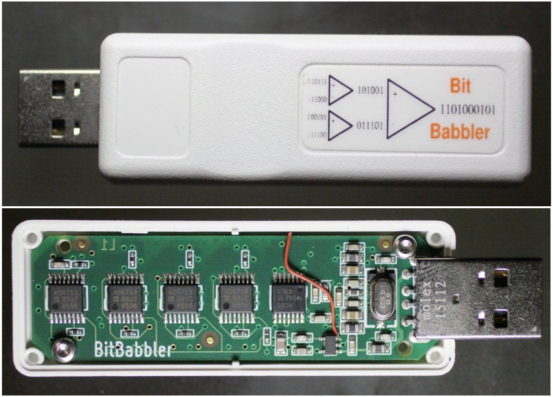


2 TABLE OF RANDOM DIGITS

00050	09188	20097	32825	39527	04220	86304	83389	87374	64278	58044
00051	90045	85497	51981	50654	94938	81997	91870	76150	68476	64659
00052	73189	50207	47677	26269	62290	64464	27124	67018	41361	82760
00053	75768	76490	20971	87749	90429	12272	95375	05871	93823	43178
00054	54016	44056	66281	31003	00682	27398	20714	53295	07706	17813

Rand Corporation, *A Million Random Digits with 100,000 Normal Deviates* (1955)

[random digits kindly generated for us in 1955]



[random digits from a noise in an electric circuit]

[random bits from quantum phenomena]

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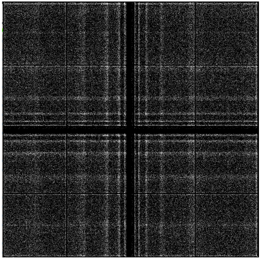
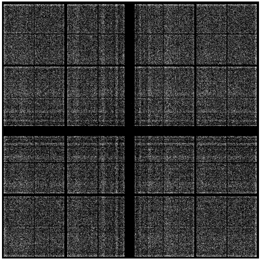
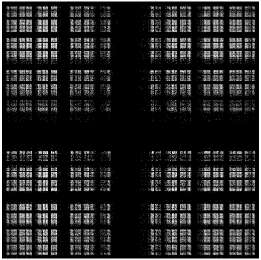


100 kHz



default rate 2.5 MHz



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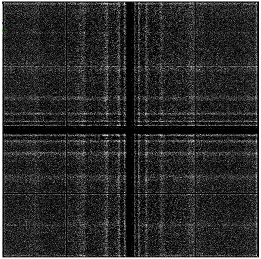

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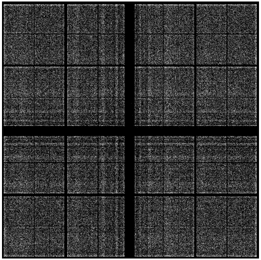

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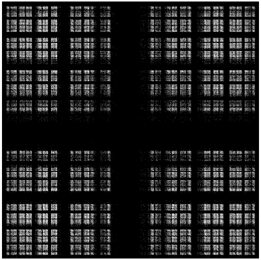

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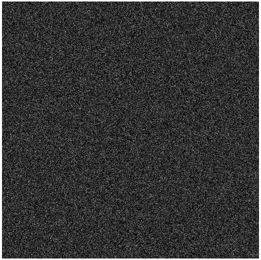

successful whitening: XOR of 3 (apparently biased) data flows looks pretty random

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You cannot trust blindly the standard implementations of randomness tests
(mathematically unsound tests, errors in the code).

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- try new approaches to whitening
- apply (certainly non-perfect) pseudo-random generators to produce useful random objects (e.g., error correcting codes)

Internship proposal:

Generation of random bits is a classical problem known in the context of pseudo-random generators and also in connection with of truly random physical processes (there exist electronic devices that produce random bits using an unpredictable physical noise or intrinsically nondeterministic quantum phenomena). However, the quality of physical generators of random bits remains badly founded and poorly tested. The first objective of this project is an experimental study of the validity and quality of several physical random numbers generators.

When we talk about the quality of random or pseudo-random generators, we have to use randomness tests. The second objective of the project is an inventory and revision of statistical tests for random and pseudo-random generators. We suggest to improve the quality of statistical tests and develop new techniques of “whitening” that improves the quality of non-ideal sources of random bits. Another axis of the project is a conversion of various probabilistic proofs into unconventional randomness tests.

Prerequisites: Basic knowledge of probability and statistics, and solid programming skills. The main tools in the project are pretty standard: C / gcc / Linux. The project requires not only writing your own code but also reading and maintaining the code that already exists.