Graph modification of bounded size to minor-closed classes as fast as vertex deletion

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ESA 2025, Warsaw, Poland





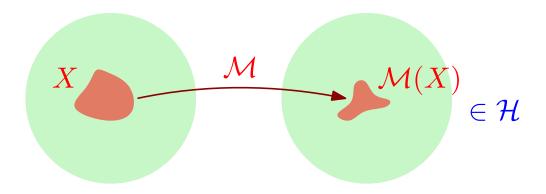


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Modification $+$ Target class $=$ Problem		
vertex deletion	edgeless graphs	Vertex Cover
	forests	FEEDBACK VERTEX SET
	bipartite graphs	Odd Cycle Transversal
edge addition + deletion	union of cliques	Cluster Editing
edge contraction	planar graphs	CONTRACTION TO PLANAR

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Highly prolific field:

299 papers mentionned just for edge-modifications in

[A survey of parameterized algorithms and the complexity of edge-modification, Crespelle, Drange, Fomin, Golovach, 2023]

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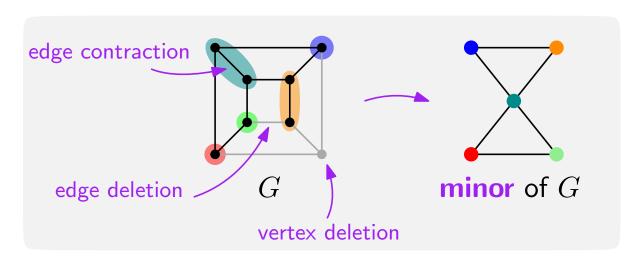
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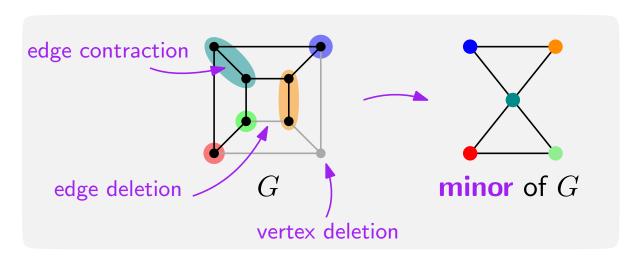
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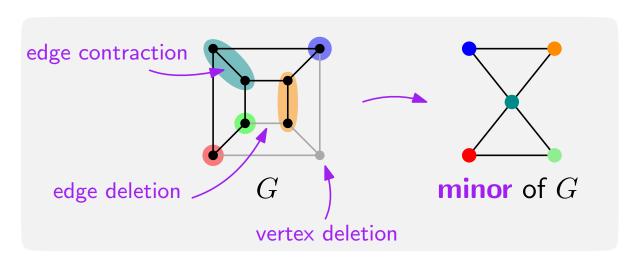
Holy grail:

Instead of solving modification problems one by one, can we provide a meta-algorithm solving as many problems as possible at once?

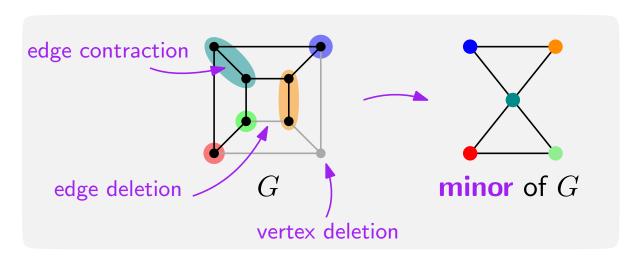




Minor-closed graph class ${\cal H}$



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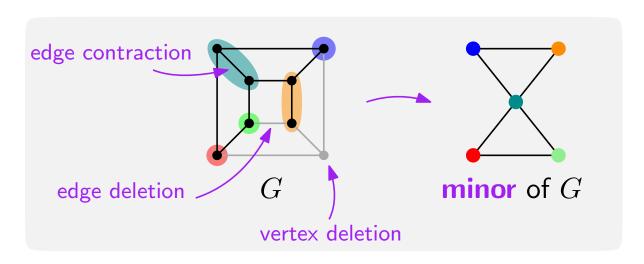


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edgeless graphs, forests,

planar graphs, graphs

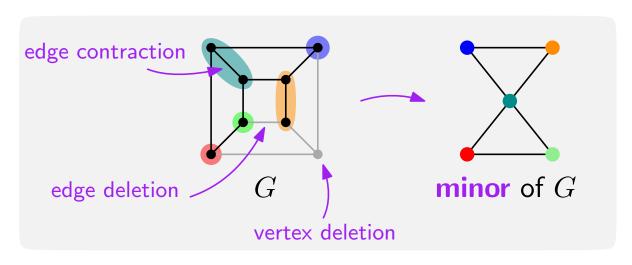
embeddable on a surface, ...



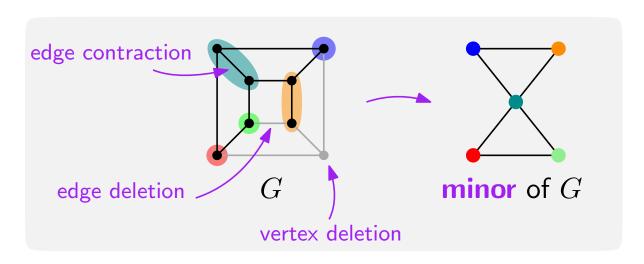
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Minor-closed graph class $\mathcal H$ If $G\in\mathcal H$, then minors of G in $\mathcal H$. [Robertson, Seymour, 2004] Obs(Planar)= $\mathcal H$ has a finite number of minor-obstructions.



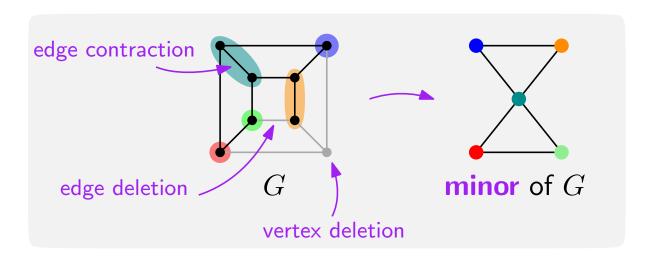
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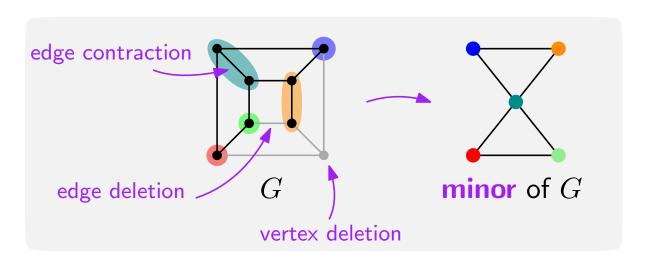
[Korhonen, Pilipczuk, Stamoulis, 2024]

Checking whether H is a minor of G can be done in time $\mathcal{O}_{H}(n^{1+o(1)})$.



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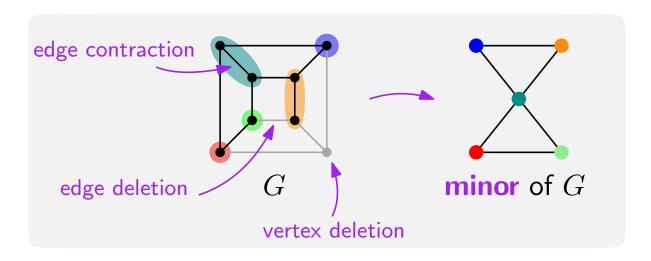
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 \rightarrow Vertex / Edge Deletion to \mathcal{H} in time $f_{\mathcal{H}}(\mathbf{k}) \cdot n^{1+o(1)}$.

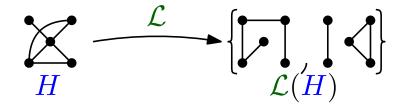


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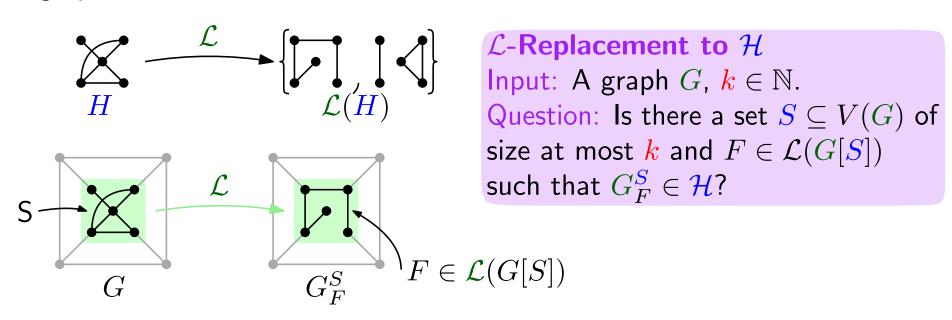
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 $ightharpoonup ext{VERTEX} / ext{EDGE DELETION TO } \mathcal{H} ext{ in time } f_{\mathcal{H}}(k) \cdot n^{1+o(1)}.$ because yes-instances of k-Vertex / Edge Deletion to \mathcal{H} are minor-closed.

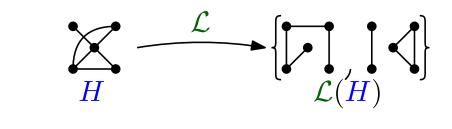
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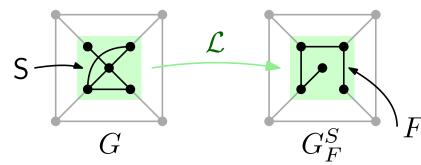


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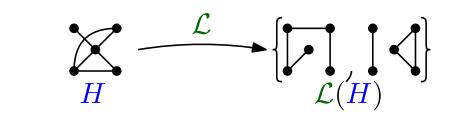
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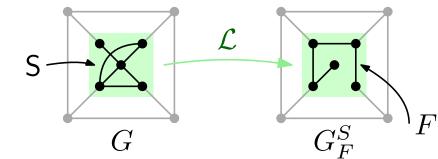
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[Fomin, Golovach, Thilikos, 2019]

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EDGE DELETION TO PLANAR

PLANAR COMPLETION TO A SUBGRAPH

MATCHING DELETION TO PLANAR

PLANAR SUBGRAPH ISOMORPHISM

[S., Stamoulis, Thilikos, 2025] Given a formula $\varphi \in \text{CMSO/tw} + \text{dp}$, and a graph G that is H-minor-free, one can check whether $G \models \varphi$ in time $f(|\varphi|, |H|) \cdot n^2$.

[S., Stamoulis, Thilikos, 2025]

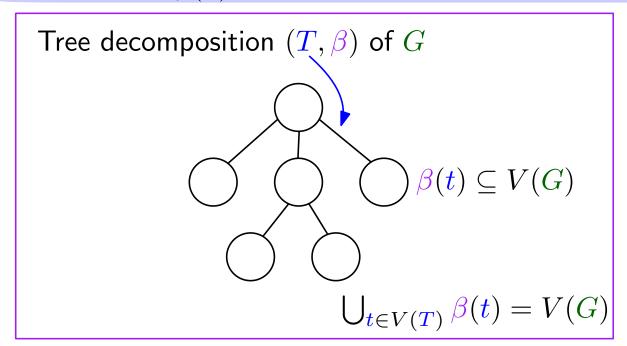
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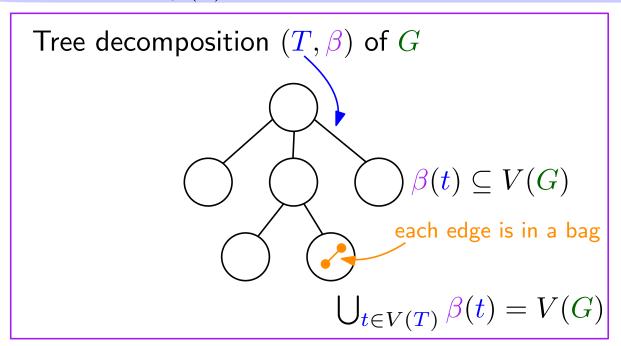
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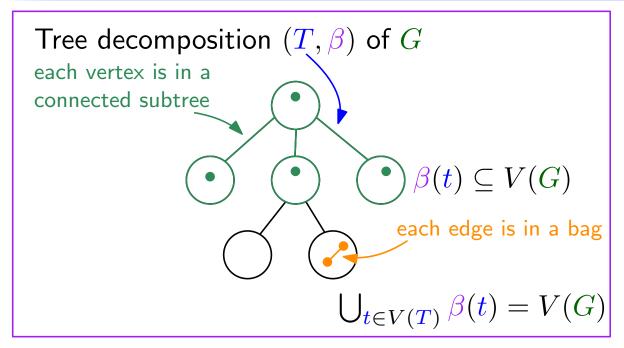
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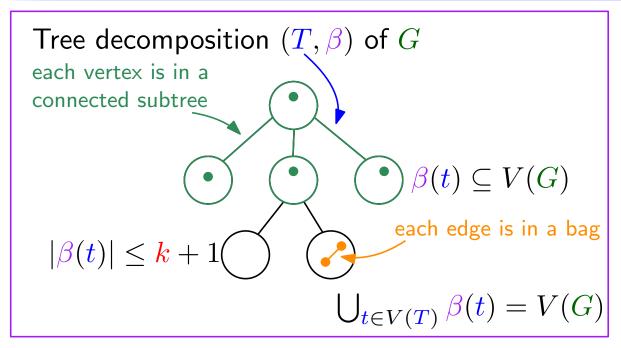
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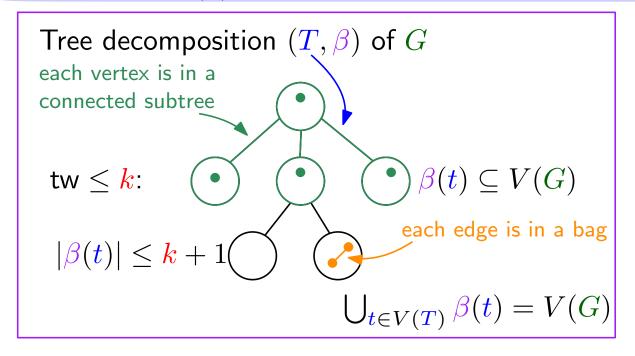
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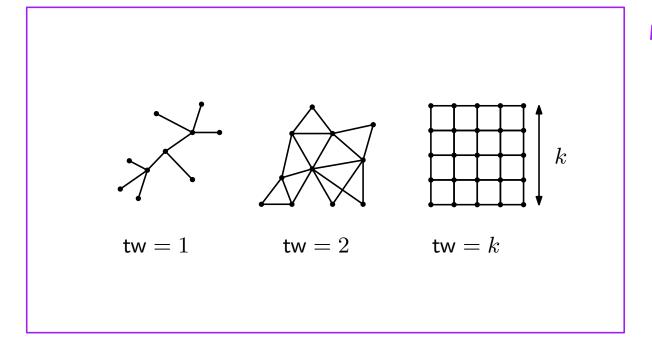
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Graph modification problems to a minor-closed graph class where the modification involves a vertex set of "annotated treewidth $\leq k$ " can be solved in time $f(k) \cdot n^2$.

very bad (not even explicit!)

Meta-algorithm on modifications and target classes

[S., Stamoulis, Thilikos, 2025]

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Natural goal: efficient parametric dependence on k, for particular (still relevant) cases of the modification operation.

H minor-closed

[Baste, S., Thilikos, 2018-2020]

[S., Stamoulis, Thilikos, 2020-2021]

[Morelle, S., Stamoulis, Thilikos, 2023]

[Morelle, S., Thilikos, ESA 2025]

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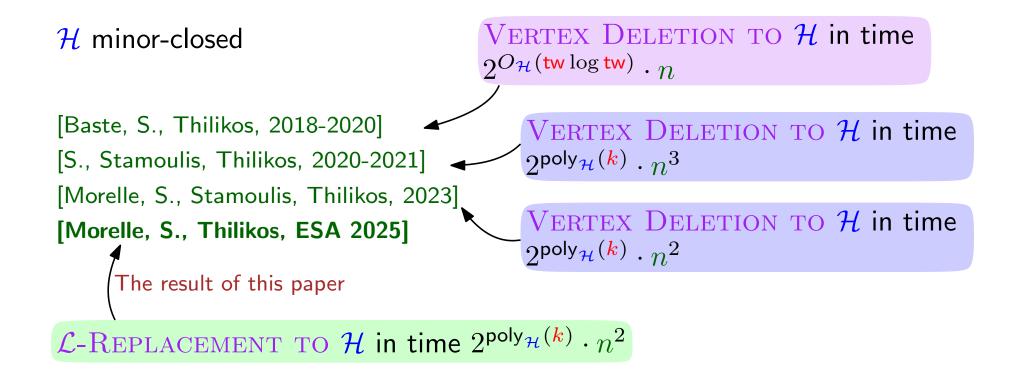
VERTEX DELETION TO \mathcal{H} in time $2^{O_{\mathcal{H}}(\mathsf{tw}\log\mathsf{tw})} \cdot n$

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Our result can be seen as a rare example of an efficient meta-algorithm for graph modification problems to minor-closed graph classes.

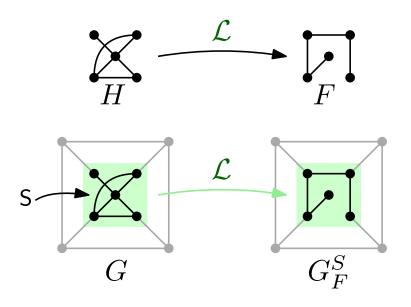
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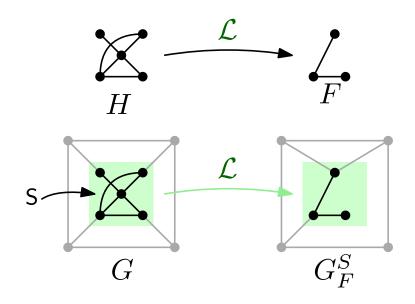
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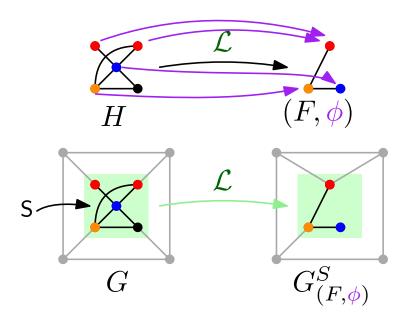
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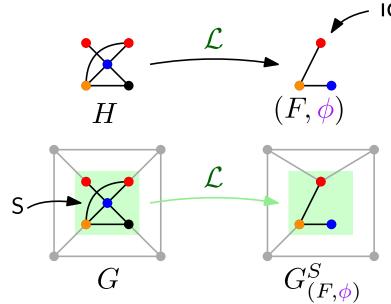
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identification

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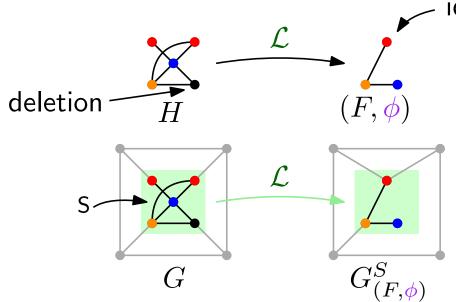
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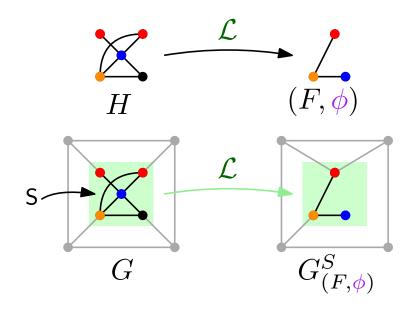
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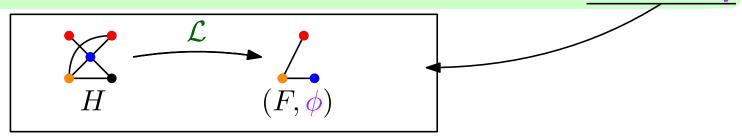
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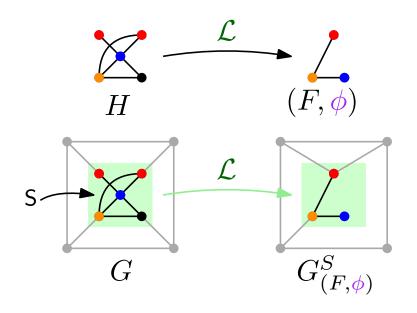
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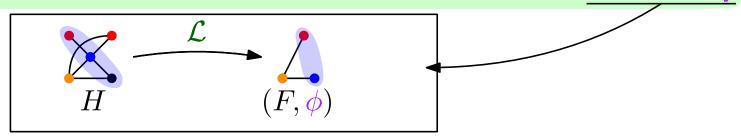


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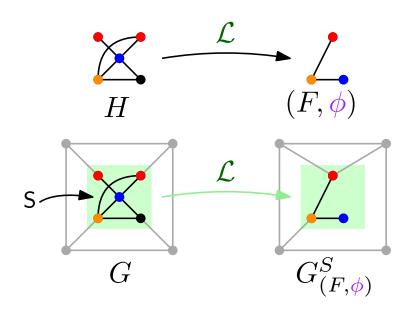
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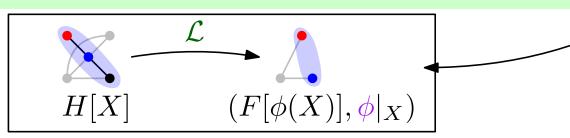
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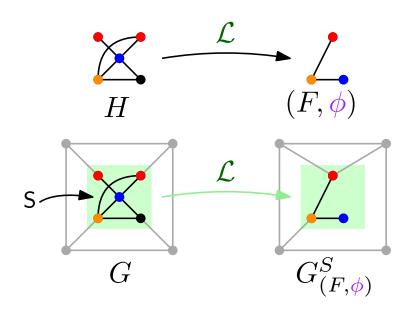
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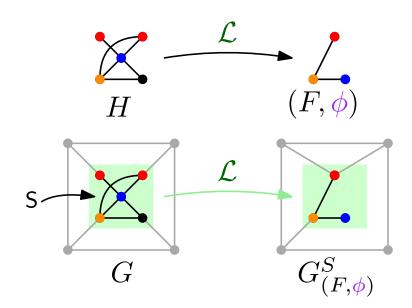
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L hereditary:

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- Edge Deletion to ${\cal H}$
- ullet Edge Contraction to ${\cal H}$
- Subgraph Complementation to ${\cal H}$
- Vertex Identification to ${\cal H}$
- Matching Contraction to ${\cal H}$
- ullet Independent Set Deletion to ${\cal H}$



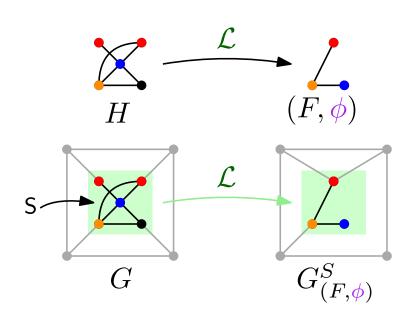
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L non-hereditary:

- deleting exactly k vertices/edges
- Planar Subgraph Isomorphism



The Irrelevant Vertex technique

The Irrelevant Vertex technique ← originates from [Robertson, Seymour, 1995]

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[S., Stamoulis, Thilikos, 2022] for VERTEXDeletion to \mathcal{H}

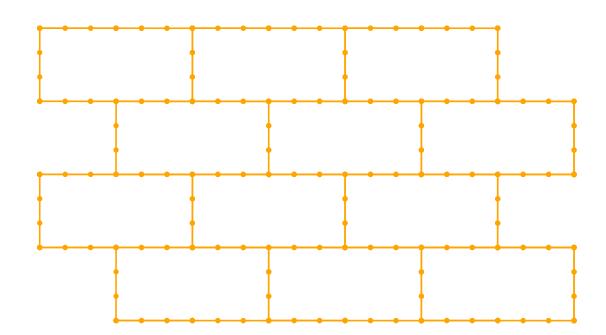
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A wall:

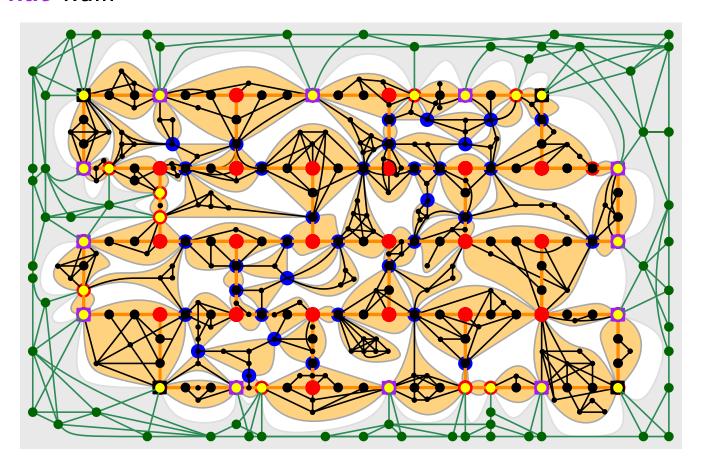


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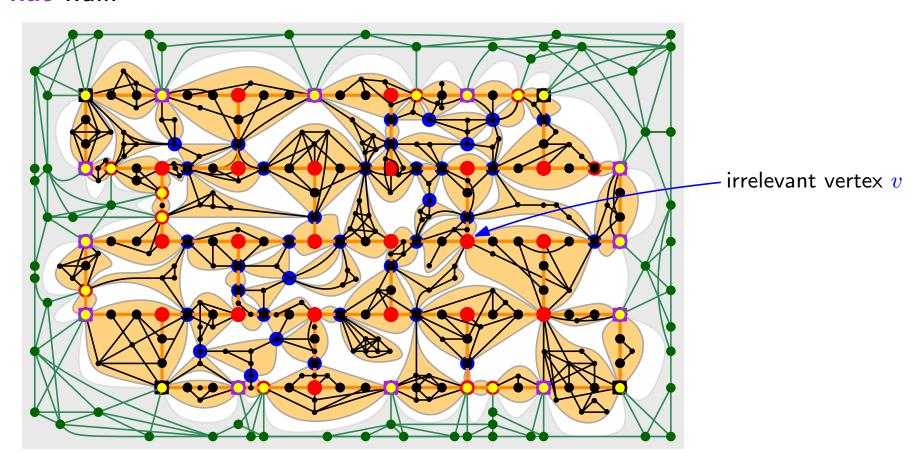
A **flat** wall:



The Irrelevant Vertex technique

Given a graph G and a big enough flat wall W in G, one can find a vertex v such that (G, k) and (G - v, k) are equivalent instances of the problem.

A **flat** wall:



Given a graph G and integers t and r, one can find either:

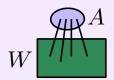
• a K_t -minor in G,



• a tree decomposition of G of width $f(t) \cdot r$, or



• a set $A \subseteq V(G)$ of size at most f(t) and a flat wall W of G - A of height r.



Given a graph G and integers t and r, one can find either:

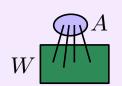
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Irrelevant Vertex technique: find an irrelevant vertex v, or branch to identify some vertex v that is in the solution.

Given a graph G and integers t and r, one can find either:

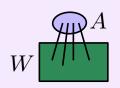
• a K_t -minor in G,



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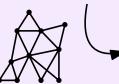
Irrelevant Vertex technique: find an irrelevant vertex v, or branch to identify some vertex v that is in the solution.

Given a graph G and integers t and r, one can find either:

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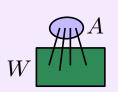


• a tree decomposition of G of width $f(t) \cdot r$, or



Courcelle's theorem: Every problem expressible in CMSO logic is solvable in time $f(tw) \cdot n$.

• a set $A \subseteq V(G)$ of size at most f(t) and a flat wall W of G-A of height r.



Irrelevant Vertex technique: find an irrelevant vertex v, or branch to identify some vertex v that is in the solution.

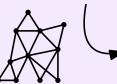
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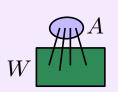
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Conclude



Courcelle's theorem: Every problem expressible in CMSO logic is solvable in time $f(tw) \cdot n$.

• a set $A \subseteq V(G)$ of size at most f(t) and a flat wall W of G-A of height r.



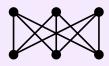
Irrelevant Vertex technique: find an irrelevant vertex v, or branch to identify some vertex v that is in the solution.

Given a graph G and integers t and r, one can find either:

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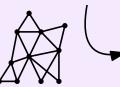


 $s_{\mathcal{H}} = \max \text{ size of an obstruction of } \mathcal{H}$



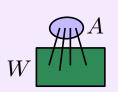
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Irrelevant Vertex technique: find an irrelevant vertex v, or branch to identify some vertex v that is in the solution.

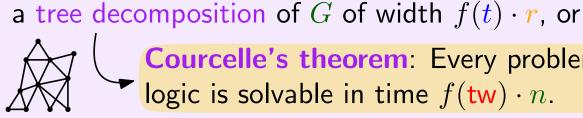
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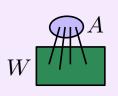
 $s_{\mathcal{H}} = \max \text{ size of an obstruction of } \mathcal{H}$ $t = s_{\mathcal{H}} + k$

Conclude



Courcelle's theorem: Every problem expressible in CMSO logic is solvable in time $f(tw) \cdot n$.

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Irrelevant Vertex technique: find an irrelevant vertex v, or branch to identify some vertex v that is in the solution.

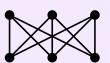
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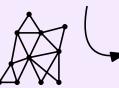
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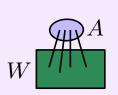
• a tree decomposition of G of width $f(t) \cdot r$, or





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Irrelevant Vertex technique: find an irrelevant vertex v, or branch to identify some vertex v that is in the solution.

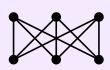
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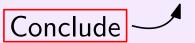
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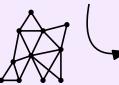
$$t = s_{\mathcal{H}} + k$$
 — no-instance



in time $f_{\mathcal{H}}(\mathbf{k}) \cdot n$

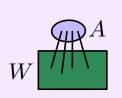
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Irrelevant Vertex technique: find an irrelevant vertex v, or branch to identify some vertex v that is in the solution.

Recurse on
$$(G - v, k)$$
 \longrightarrow in time $2^{f_{\mathcal{H}}(k)} \cdot n$

The Flat Wall theorem

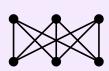
Given a graph G and integers t and r, one can find either:

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$$t = s_{\mathcal{H}} + k$$
 — no-instance



in time $f_{\mathcal{H}}(\mathbf{k}) \cdot n$

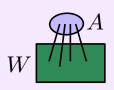
• a tree decomposition of G of width $f(t) \cdot r$, or





Courcelle's theorem: Every problem expressible in CMSC logic is solvable in time $f(tw) \cdot n$.

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Irrelevant Vertex technique: find an irrelevant vertex v, or branch to identify some vertex v that is in the solution,

Recurse on
$$(G - v, k)$$
 \longrightarrow in time $2^{f_{\mathcal{H}}(k)} \cdot n$

The Flat Wall theorem

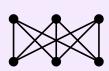
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 $s_{\mathcal{H}} = \max \text{ size of an obstruction of } \mathcal{H}$

$$t = s_{\mathcal{H}} + k$$
 — no-instance



in time $f_{\mathcal{H}}(\mathbf{k}) \cdot n$

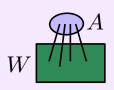
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Courcelle's theorem: Every problem expressible in CMSC logic is solvable in time $f(tw) \cdot n$.

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Irrelevant Vertex technique: find an irrelevant vertex v, or branch to identify some vertex v that is in the solution,

Recurse on
$$(G - v, k)$$
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The Flat Wall theorem

Given a graph G and integers t and r, one can find either:

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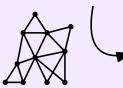
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 $t = s_{\mathcal{H}} + k$ — no-instance

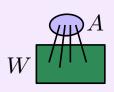
• a tree decomposition of G of width $f(t) \cdot r$, or





 \mathcal{L} -Replacement to \mathcal{H} in time $2^{O_{\mathcal{H}}(k^2+(k+\mathsf{tw})\log(k+\mathsf{tw}))} \cdot n$

• a set $A \subseteq V(G)$ of size at most f(t) and a flat wall W of G-A of height r.



Irrelevant Vertex technique: find an irrelevant vertex v, or branch to identify some vertex v that is in the solution.

Recurse on
$$(G - v, k)$$
 \longrightarrow in time $2^{f_{\mathcal{H}}(k)} \cdot n$

bad!

One can solve \mathcal{L} -REPLACEMENT TO \mathcal{H} in time $2^{\text{poly}_{\mathcal{H}}(k)} \cdot n^2$ for \mathcal{L} hereditary.

One can solve \mathcal{L} -REPLACEMENT TO \mathcal{H} in time $2^{\text{poly}_{\mathcal{H}}(\mathbf{k})} \cdot n^2$ for \mathcal{L} hereditary.

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 $\mathcal{H} = \mathcal{P}$ planar: $s_{\mathcal{H}} = 6 \rightarrow \text{already very big!}$

One can solve \mathcal{L} -REPLACEMENT TO \mathcal{H} in time $2^{\text{poly}_{\mathcal{H}}(\pmb{k})} \cdot n^2$ for \mathcal{L} hereditary. $k^{2^{2^s\mathcal{H}^{24}}}$

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One can solve \mathcal{L} -REPLACEMENT TO \mathcal{P} in time $2^{\mathcal{O}(\mathbf{k}^9)} \cdot n^2$ for \mathcal{L} hereditary.

One can solve \mathcal{L} -REPLACEMENT TO \mathcal{H} in time $2^{\text{poly}_{\mathcal{H}}(\pmb{k})} \cdot n^2$ for \mathcal{L} hereditary. $k^{2^{2^s\mathcal{H}^{24}}} < \blacksquare$

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Irrelevant vertex technique

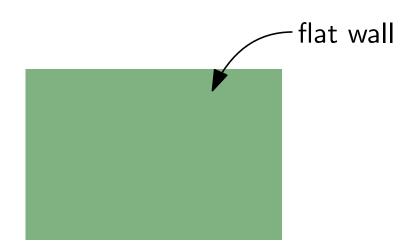
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Irrelevant vertex technique

General case:



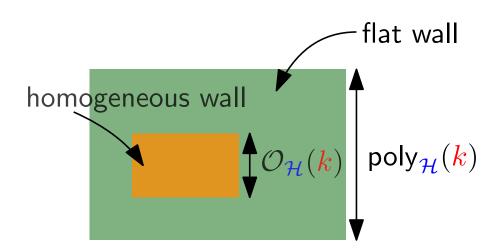
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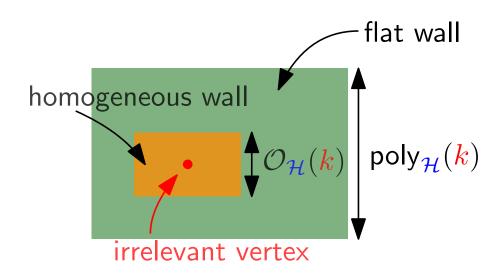
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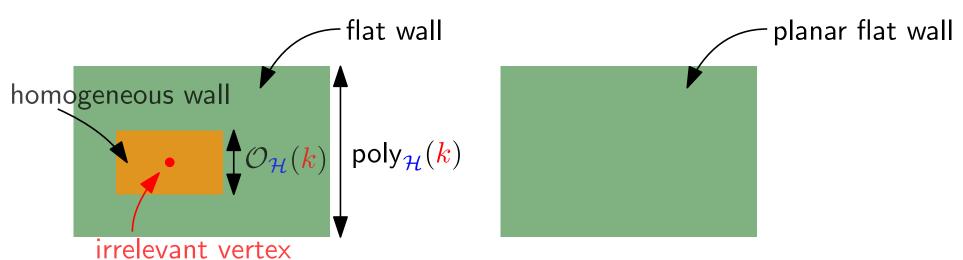
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One can solve \mathcal{L} -Replacement to \mathcal{P} in time $2^{\mathcal{O}(k^9)} \cdot n^2$ for \mathcal{L} hereditary.

Irrelevant vertex technique

General case: Planar case:



One can solve \mathcal{L} -Replacement to \mathcal{H} in time $2^{\text{poly}_{\mathcal{H}}(\pmb{k})} \cdot n^2$ for \mathcal{L} hereditary. $k^{2^{2^s\mathcal{H}^{24}}}$

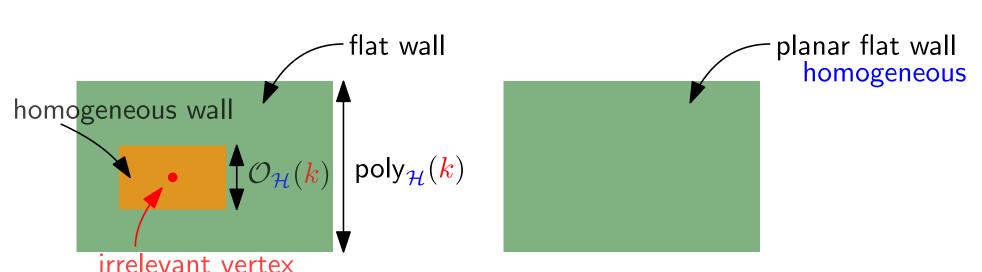
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Irrelevant vertex technique

General case:

Planar case:



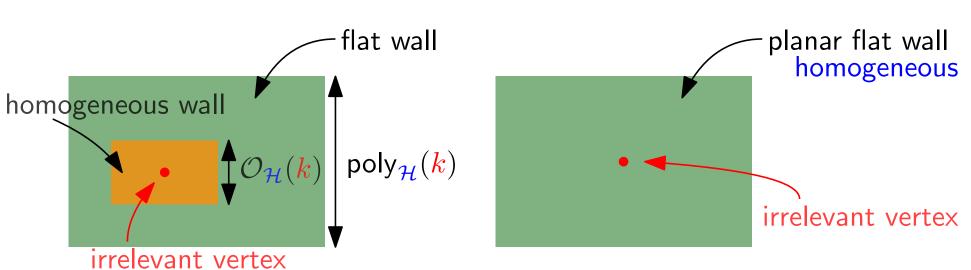
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One can solve \mathcal{L} -REPLACEMENT TO \mathcal{P} in time $2^{\mathcal{O}(k^9)} \cdot n^2$ for \mathcal{L} hereditary.

works also for the class of graphs embeddable on a surface Σ

One can solve \mathcal{L} -REPLACEMENT TO \mathcal{H} in time $2^{\text{poly}_{\mathcal{H}}(\mathbf{k})} \cdot n^2$ for \mathcal{L} hereditary.

One can solve \mathcal{L} -Replacement to \mathcal{P} in time $2^{\mathcal{O}(k^9)} \cdot n^2$ for \mathcal{L} hereditary.

One can solve \mathcal{L} -REPLACEMENT TO \mathcal{H} in time $2^{\text{poly}_{\mathcal{H}}(\mathbf{k})} \cdot n_{\mathcal{I}}^2$ for \mathcal{L} hereditary.

Can we improve?

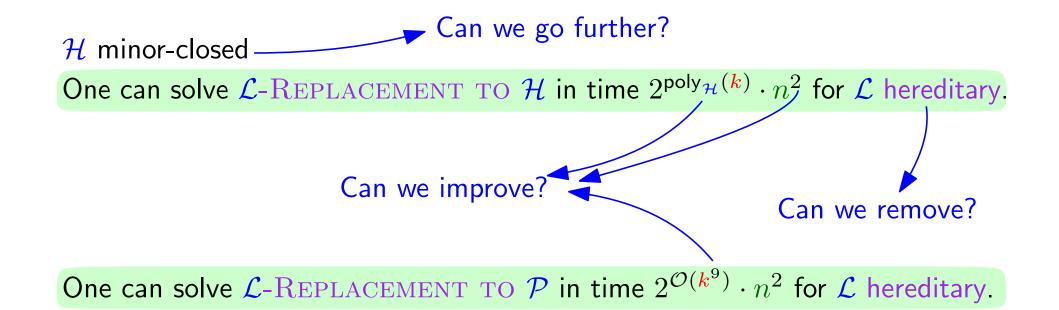
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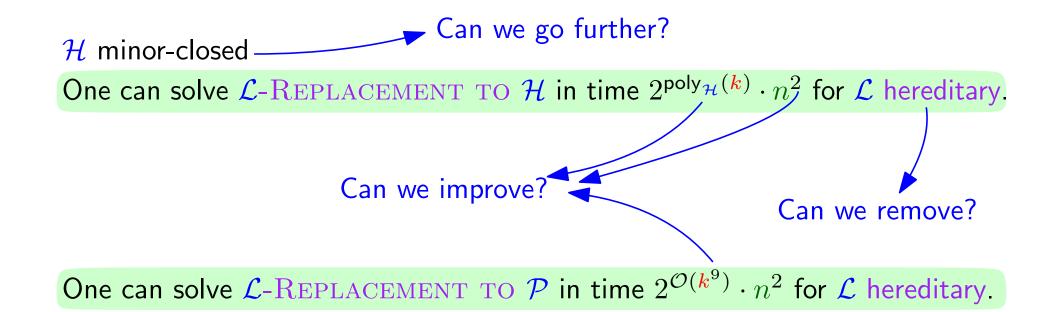
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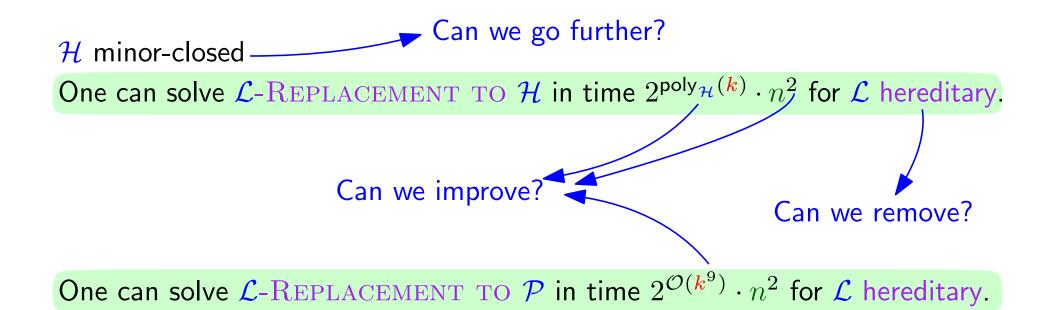
One can solve \mathcal{L} -Replacement to \mathcal{P} in time $2^{\mathcal{O}(k^9)} \cdot n^2$ for \mathcal{L} hereditary.



k: bound on the size of the vertex set involved in the modification

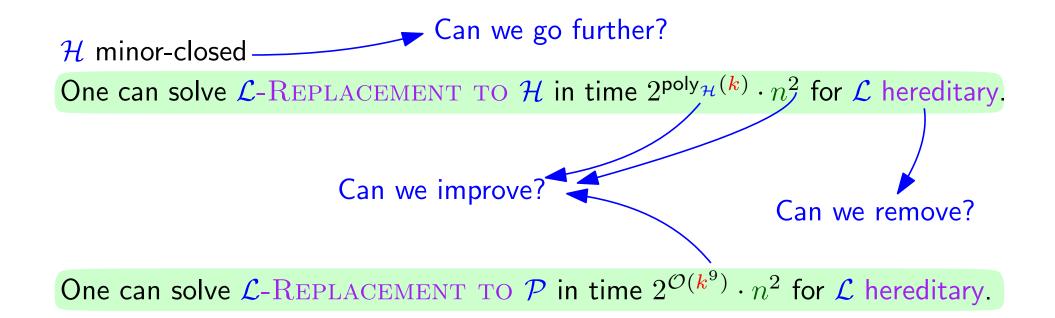


k: bound on the size of the vertex set involved in the modification treewidth instead?



k: bound on the size of the vertex set involved in the modification treewidth instead?

ELIMINATION DISTANCE TO \mathcal{H} $\mathcal{H}\text{-Treewidth}$



k: bound on the size of the vertex set involved in the modification treewidth instead?

ELIMINATION DISTANCE TO \mathcal{H} $\mathcal{H}\text{-Treewidth}$

Thanks!