Parameterized domination in circle graphs

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Outline

1. Motivation
2. Hardness results
   - Independent dominating set
   - Acyclic dominating set
   - Tree dominating set
3. Sketch of the algorithms
4. Conclusions

Next section is...

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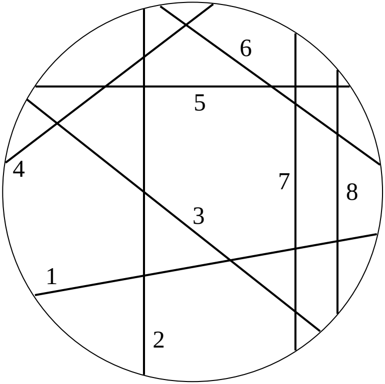
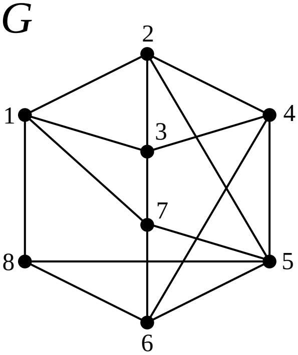
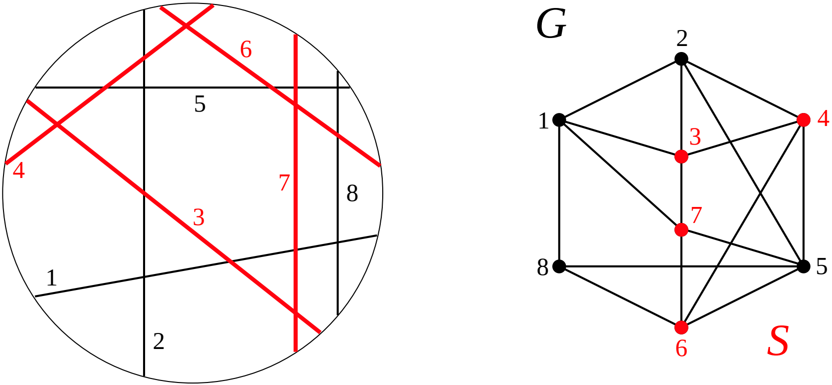
Circle graphs and domination





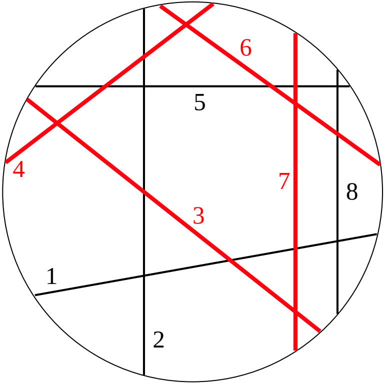
Circle graph: intersection graph of chords in a circle.

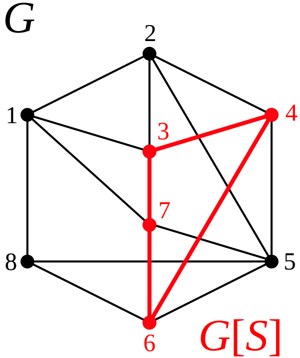
Circle graphs and domination



DOMINATING SET: $S \subseteq V(G)$ s.t. each $v \in V(G) \setminus S$ has a neighbor in S.

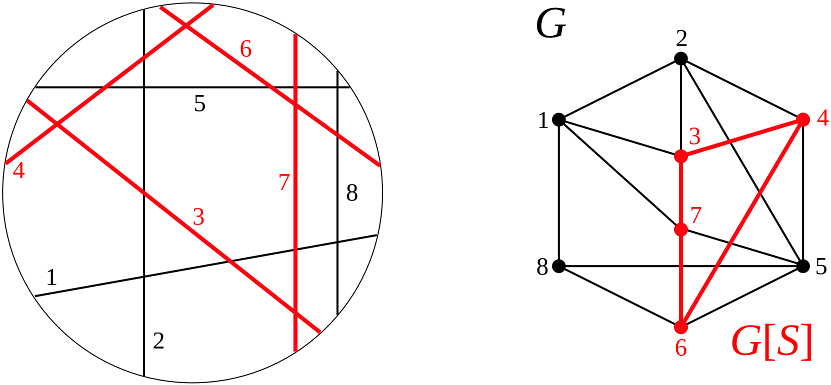
Circle graphs and domination





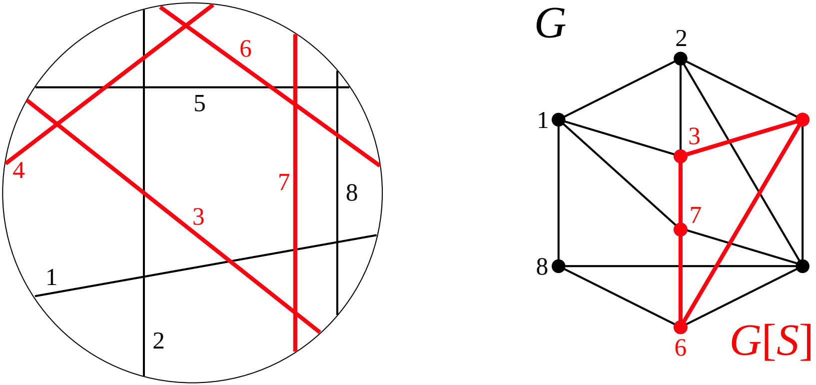
We will impose extra conditions that $G[S]$ must satisfy.

Circle graphs and domination



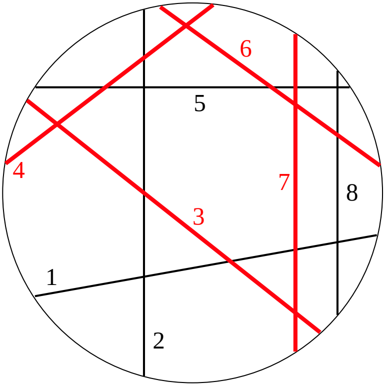
CONNECTED DOMINATING SET: $G[S]$ is connected.

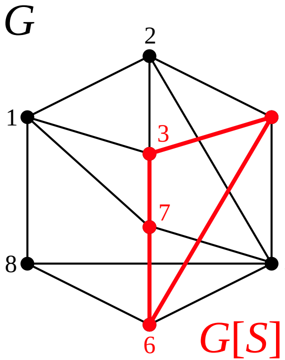
Circle graphs and domination



TOTAL DOMINATING SET: $G[S]$ has no isolated vertices.

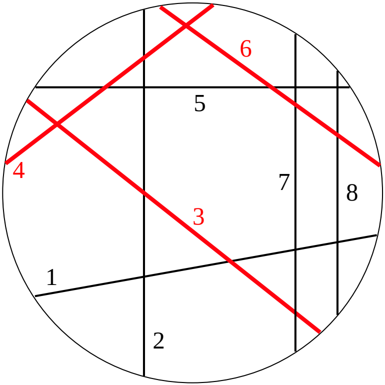
Circle graphs and domination

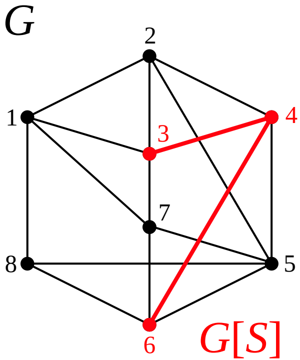




Acyclic Dominating set: $G[S]$ has no cycles.

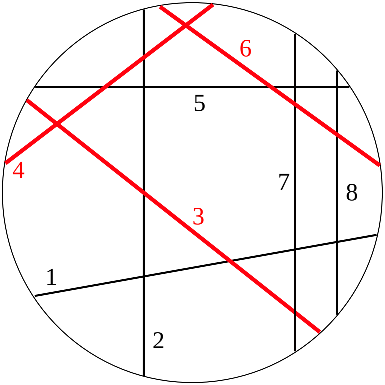
Circle graphs and domination

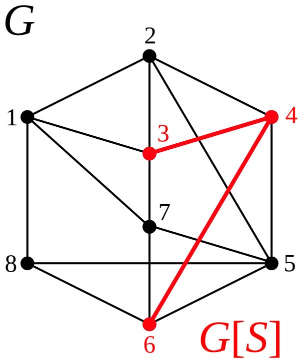




ACYCLIC DOMINATING SET: $G[S]$ has no cycles.

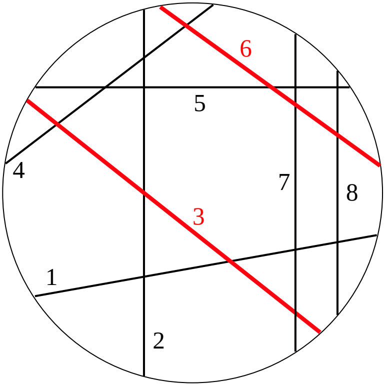
Circle graphs and domination

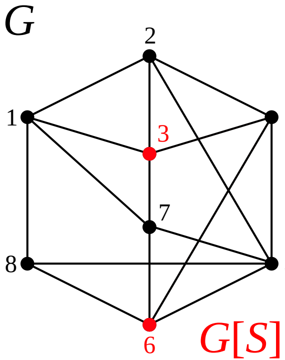




INDEPENDENT DOMINATING SET: $G[S]$ has no edges.

Circle graphs and domination





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State of the art in circle graphs

- Circle graphs can be recognized in $\mathcal{O}(n^2)$ time. [Spinrad. 1994]
- MAXIMUM CLIQUE and MAXIMUM INDEPENDENT SET can be solved in $\mathcal{O}(n^3)$ time. [Gavril. 1973]
- TREEWIDTH can be solved in $\mathcal{O}(n^3)$ time. [Kloks. 1996]
- 3-COLORABILITY can be solved in $\mathcal{O}(n \log n)$ time. [Unger. 1988]
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- ACYCLIC DOMINATING SET is in P in interval and proper circular-arc graphs. [Hedetniemi, Hedetniemi, Rall. 2000]
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What about the parameterized complexity of these domination problems, when parameterized by the solution size?

- Algorithm with running time $\mathcal{O}(n^{4k^2+3})$ for domination in polygon graphs. Therefore, DOMINATING SET is in XP. [Elmallah, Stewart. 1993]

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Our results (in circle graphs)

1. DOMINATING SET,
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   INDEPENDENT DOMINATING SET,
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   are $W[1]$-hard, parameterized by the size of the solution.

2. Whereas both CONNECTED and ACYCLIC DOMINATING SET are both $W[1]$-hard, CONNECTED ACYCLIC DOM. SET is in P.

3. If T is a given fixed tree, the problem of deciding whether a circle graph has a dominating set isomorphic to T is
   - NP-complete, when T is part of the input.
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   are $W[1]$-hard, parameterized by the size of the solution.

2. Whereas both Connected and Acyclic Dominating Set are both $W[1]$-hard, Connected Acyclic Dom. Set is in P.

3. If T is a given fixed tree, the problem of deciding whether a circle graph has a dominating set isomorphic to T is
   - NP-complete, when T is part of the input.
   - FPT, when parameterized by $|V(T)|$.

Parameterized complexity in one slide

- Idea: given an NP-hard problem, fix one parameter of the input to see if the problem gets more "tractable".

  Example: the size of a VERTEX COVER.

- Given a (NP-hard) problem with input of size n and a parameter k, a fixed-parameter tractable (FPT) algorithm runs in

  $$f(k) \cdot n^{\mathcal{O}(1)}, \quad \text{for some function } f.$$

  Examples: k-VERTEX COVER, k-LONGEST PATH.

- Barometer of intractability:

  $$\text{FPT} \subseteq W[1] \subseteq W[2] \subseteq W[3] \subseteq \cdots \subseteq XP$$

- The higher a problem is located in the W-hierarchy, the more unlikely it is to be in FPT.

Next section is...

Multicolored Clique

k-COLORED CLIQUE

Instance:	A graph $G = (V, E)$ and a coloring of V using k colors.
Parameter:	k.
Question:	Does there exist a clique of size k in G containing exactly one vertex from each color?

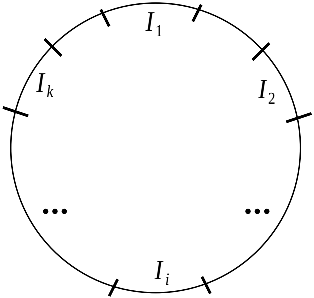
W[1]-hard in general graphs. [Fellows, Hermelin, Rosamond, Vialette. 2009]

Next subsection is...

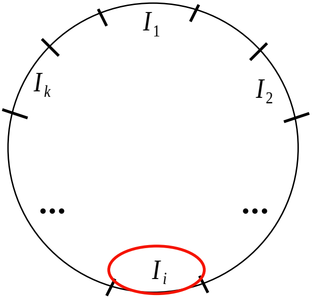
INDEPENDENT DOMINATING SET

Parameterized reduction from k-COLORED CLIQUE in a general graph G to finding an INDEPENDENT DOMINATING SET of size at most $2k$ in a circle graph H.

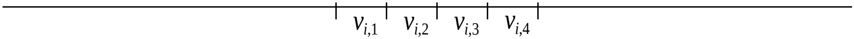
Let $C_1, \ldots, C_k \subseteq V(G)$ be the color classes of G, and note that WMA that $G[C_i]$ is an independent set for $1 \leq i \leq k$.



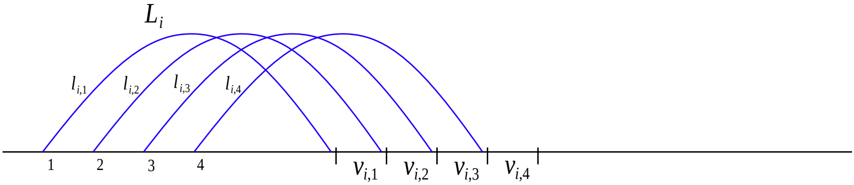
Let $I_1, \ldots, I_k$ be a collection of k disjoint intervals in the circle.



For $i = 1, \ldots, k$, we proceed to construct an induced subgraph H_i of H.

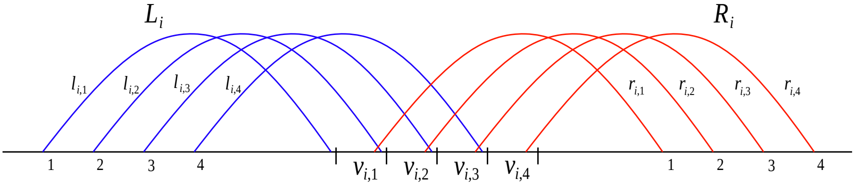


Let $v_{i,1}, \ldots, v_{i,t}$ be the vertices belonging to the color class $C_i \subseteq V(G)$.



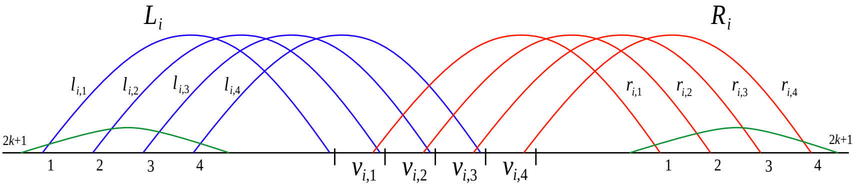
We add a clique L_i with t chords $l_{i,1}, \ldots, l_{i,t}$ in this way.

Independent Dominating Set

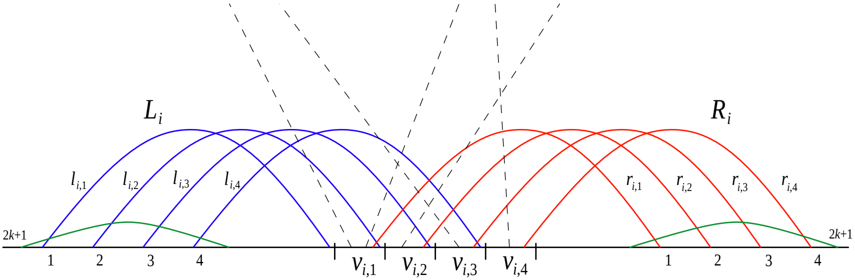


Symmetrically, we add a clique R_i with t chords $r_{i,1}, \ldots, r_{i,t}$ in this way.

Independent Dominating Set

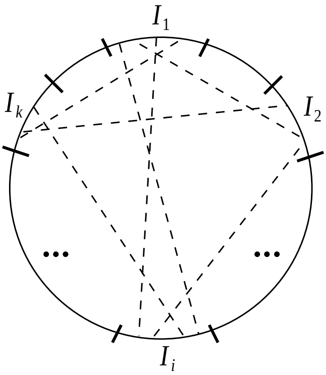


We also add two sets of $2k+1$ parallel chords in this way.



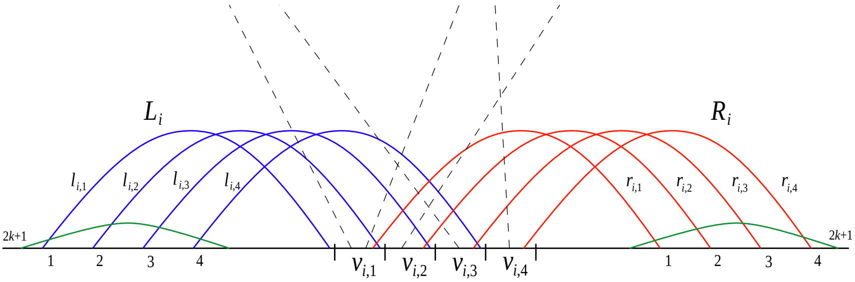
For each pair $v_{i,p}, v_{j,q} \in V(G)$ such that $i \neq j$ and $\{v_{i,p}, v_{j,q}\} \notin E(G)$, we add to H a chord between $v_{i,p}$ in H_i and $v_{j,q}$ in H_j, in this way.

Independent Dominating Set

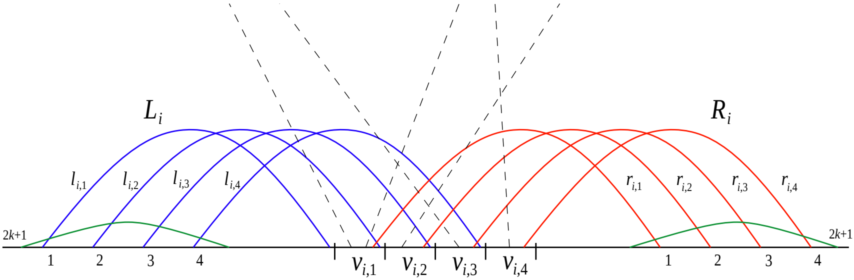


This completes the construction of the circle graph H.

Independent Dominating Set

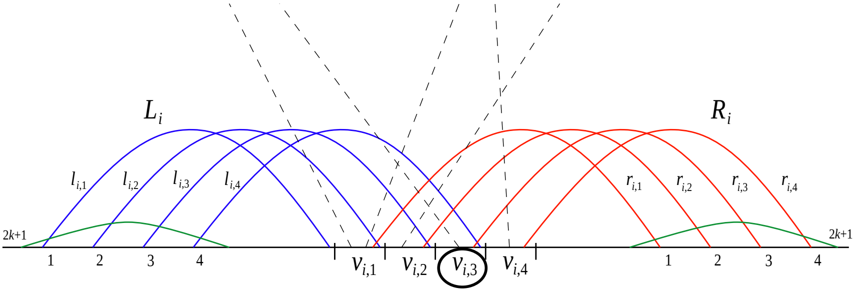


We now claim that G has a k-colored clique if and only if H has an independent dominating set of size at most $2k$.



Let first K be a k-colored clique in G containing, for $i = 1, \ldots, k$, a vertex v_{i,j_i} in H_i.

Independent Dominating Set



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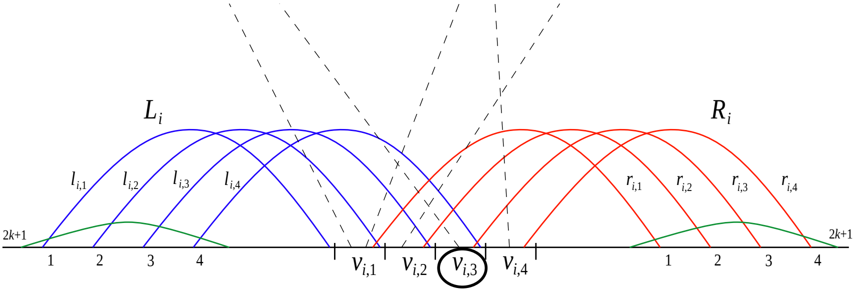
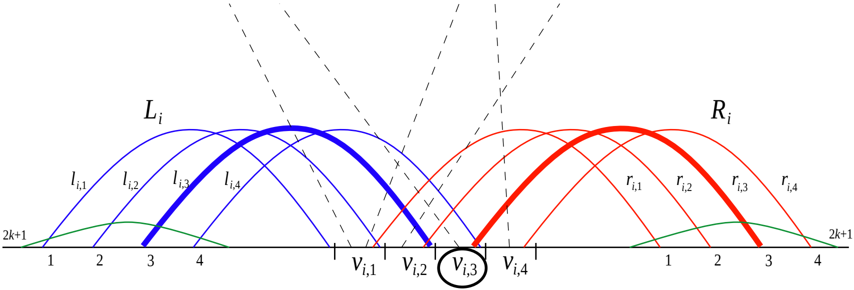


Let us obtain from K an independent dominating set S in H: For $i = 1, \ldots, k$, the set S contains the two chords l_{i,j_i} and r_{i,j_i} from H_i.

Independent Dominating Set



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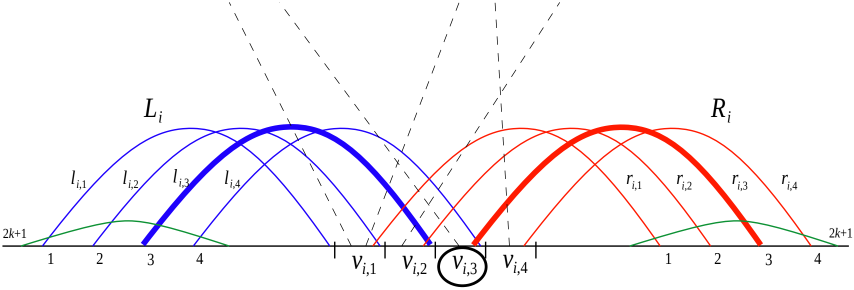
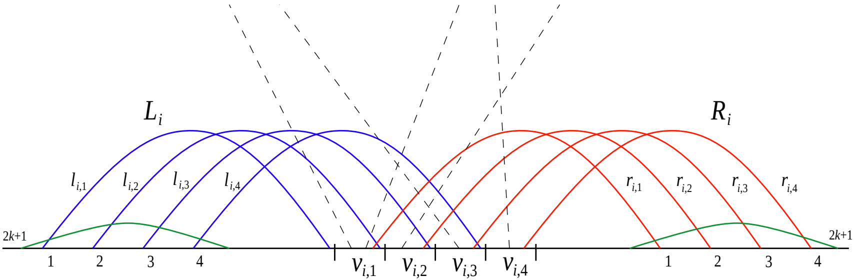


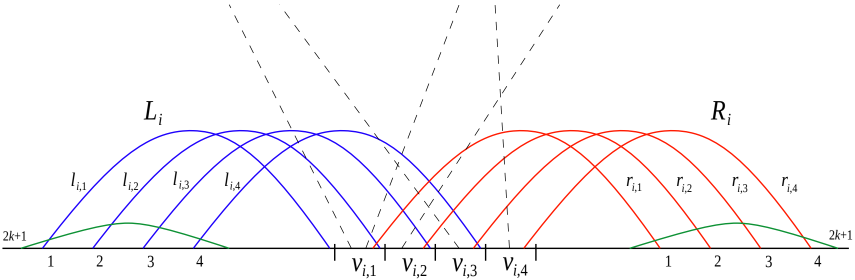
Since K is a clique in G, it follows that S is an independent dominating set of H of size $2k$.

Independent Dominating Set



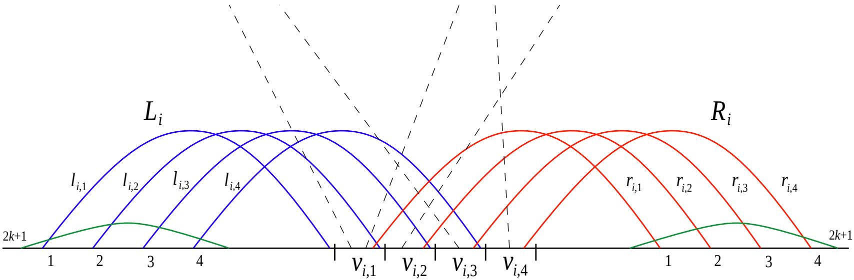
Conversely, assume that H has an independent dominating set S with $|S| \leq 2k$.

Independent Dominating Set

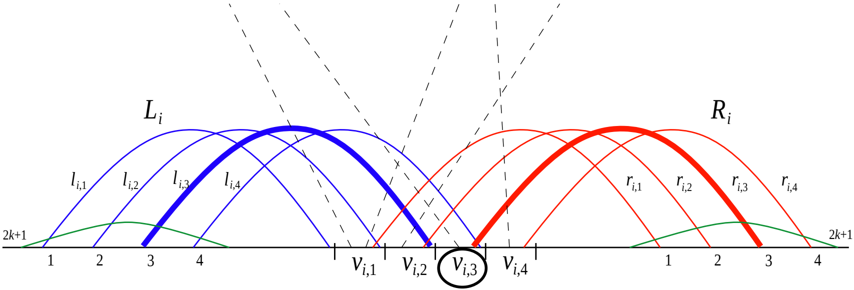


Because of the sets of $2k+1$ parallel chords, ≥ 1 of the chords in L_i and ≥ 1 of the chords in R_i must belong to S, so $|S| \geq 2k$.

Independent Dominating Set

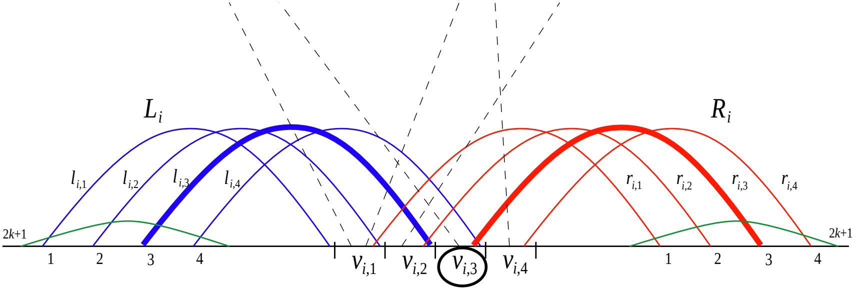


It follows that $|S| = 2k$ and that S contains in H_i, for $i = 1, \ldots, k$, a pair of non-crossing chords in L_i and R_i.



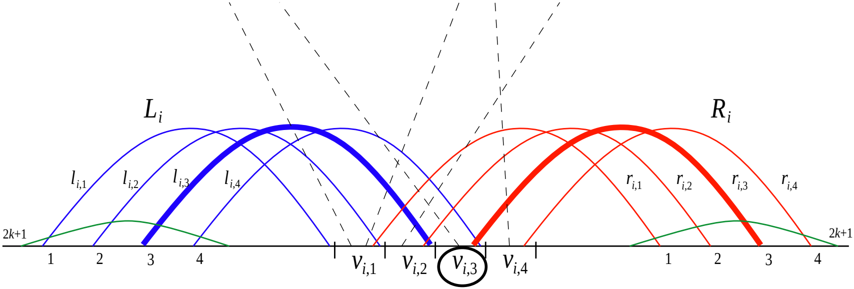
In each H_i, the two chords belonging to S must leave uncovered at least one of intervals $v_{i,1}, \ldots, v_{i,t}$.

Independent Dominating Set



Hence, a k-colored clique in G can by obtained by selecting in each H_i any of the uncovered vertices.

INDEPENDENT DOMINATING SET



Theorem

INDEPENDENT DOMINATING SET *is* $W[1]$*-hard in circle graphs.*

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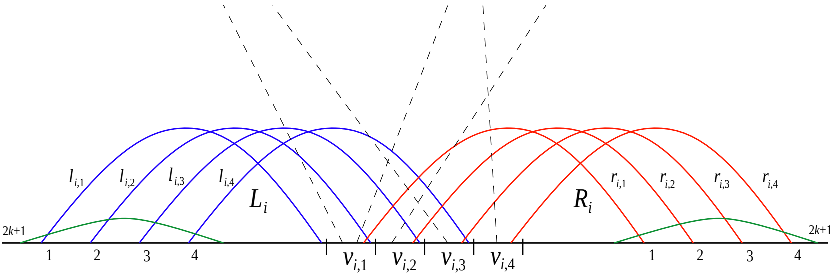
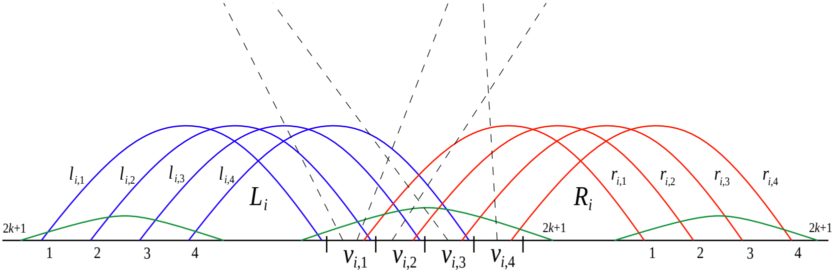
Acyclic Dominating Set

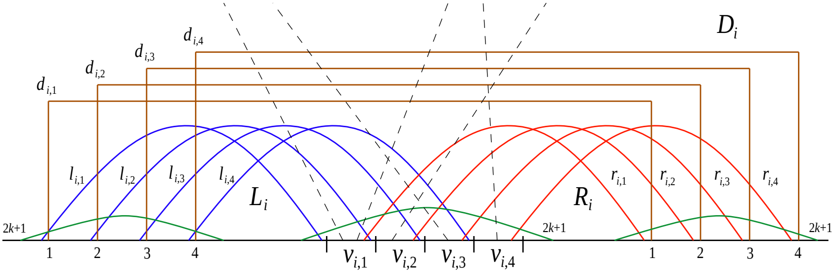


In order to prove that Acyclic Dominating Set is $W[1]$-hard in circle graphs, we modify the previous construction as follows.

ACYCLIC DOMINATING SET

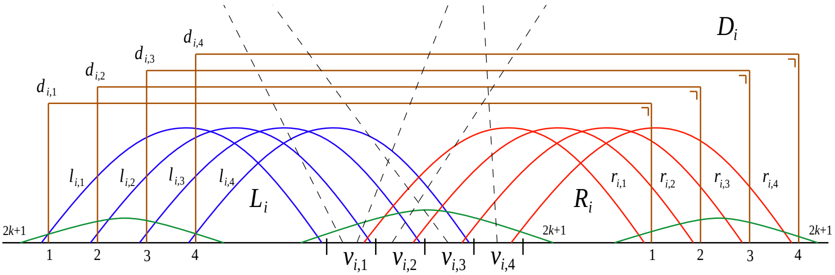


We add another set of $2k + 1$ parallel chords, in this way. We call these three sets of $2k + 1$ chords parallel chords.



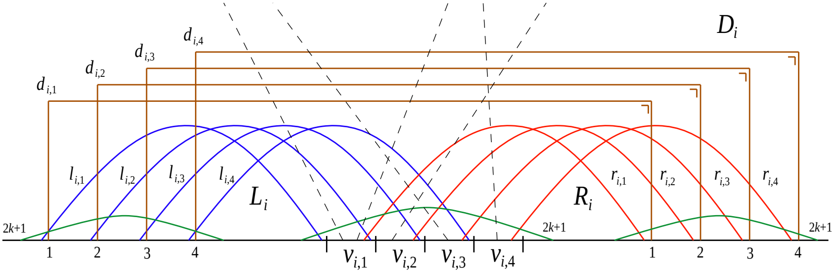
Furthermore, we add a new clique with t chords $d_{i,1}, \ldots, d_{i,t}$, in this way.

Acyclic Dominating Set



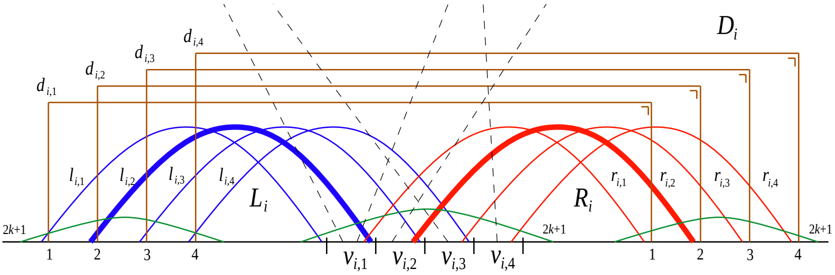
Finally, for each such chord $d_{i,j}$ we add a parallel twin chord, denoted by $d'_{i,j}$. We call these $2t$ chords distance chords, denoted D_i.

Acyclic Dominating Set



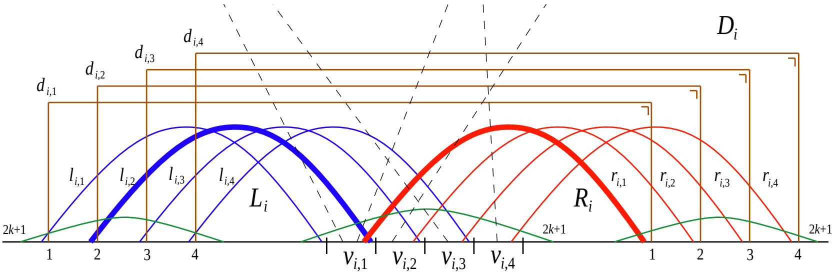
We now claim that G has a k-colored clique if and only if H has an acyclic dominating set of size at most $2k$.

Acyclic Dominating Set



Key point: a pair of chords l_{i,j_1} and r_{i,j_2} dominates all the distance chords in H_i if and only if l_{i,j_1} and r_{i,j_2} do not cross.

Acyclic Dominating Set



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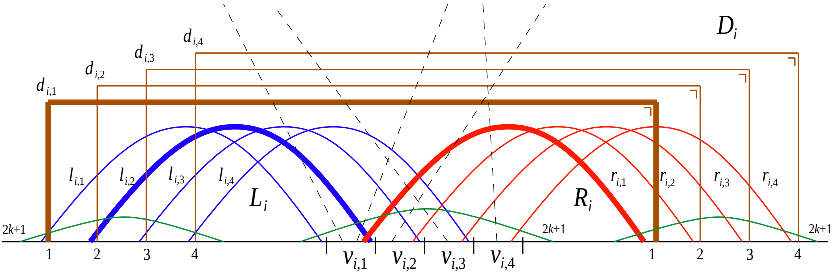
Acyclic Dominating Set



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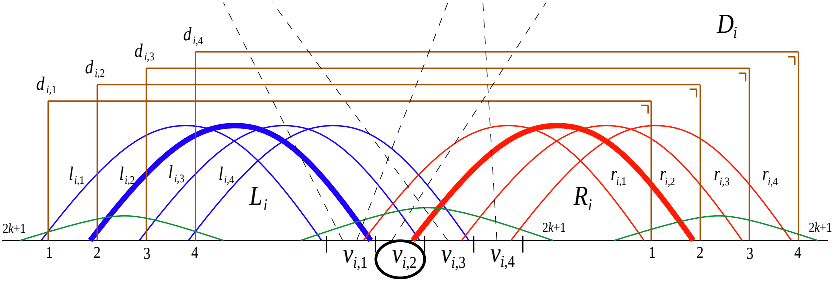
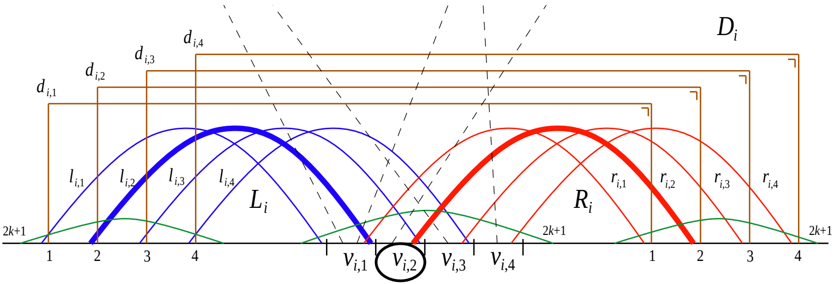
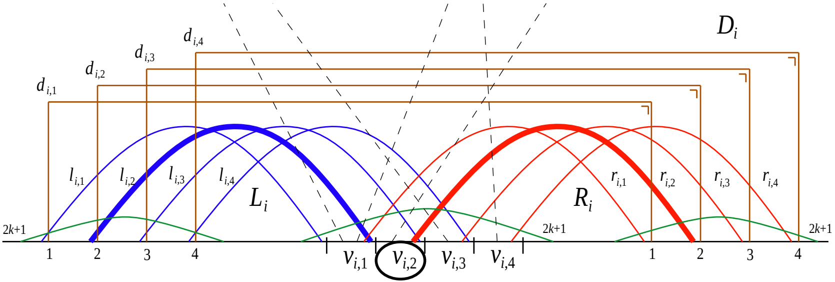


Let first K be a k-colored clique in G. We define an independent (hence, acyclic) dominating set S of size $2k$ as before.

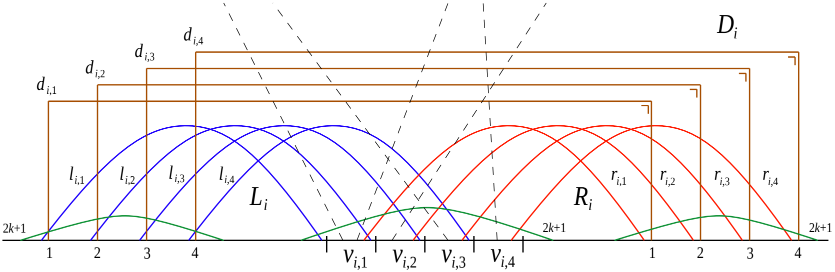
ACYCLIC DOMINATING SET



Conversely, assume that H has an acyclic dominating set S with $|S| \leq 2k$.

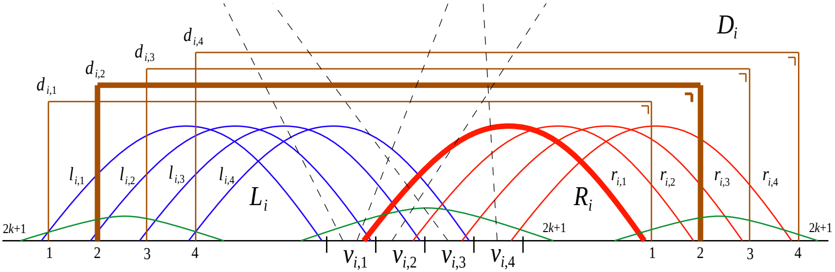


Objective: in each H_i, S contains a pair of non-crossing chords in L_i and R_i.



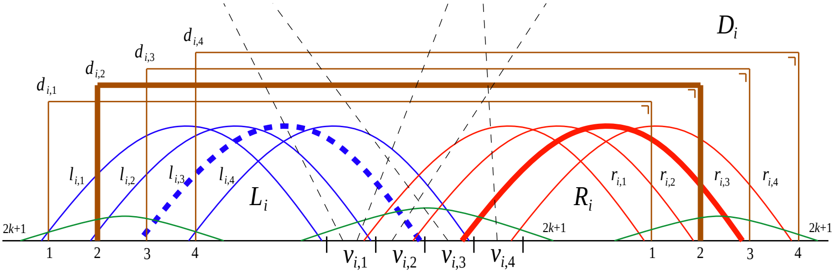
Assume first that S contains no transversal chord. Then it must contain exactly two chords u, v in each H_i.

Acyclic Dominating Set



We need to distinguish several cases. For instance, if $u \in D_i$ and $v \in L_i$, then some chord of H_i is not dominated. OK!

Acyclic Dominating Set



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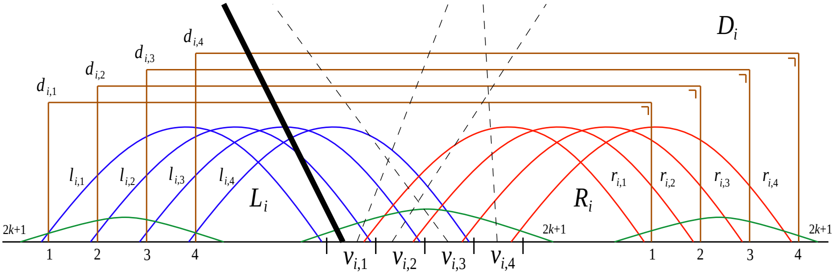
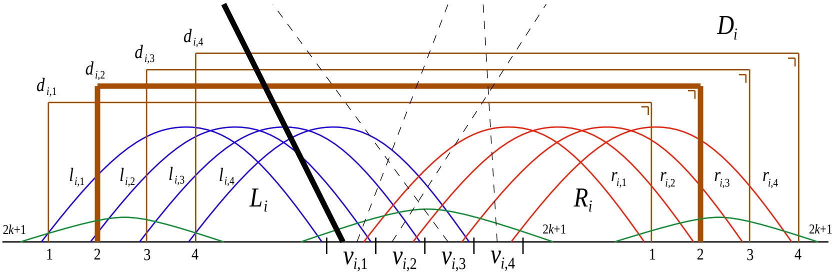


Otherwise, S contains some transversal chord, going from an interval I_i to another interval I_j.



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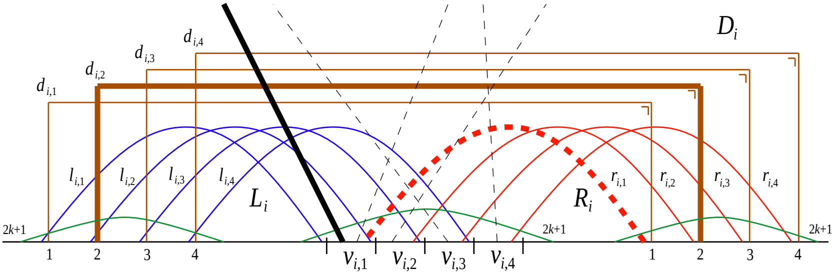
Acyclic Dominating Set



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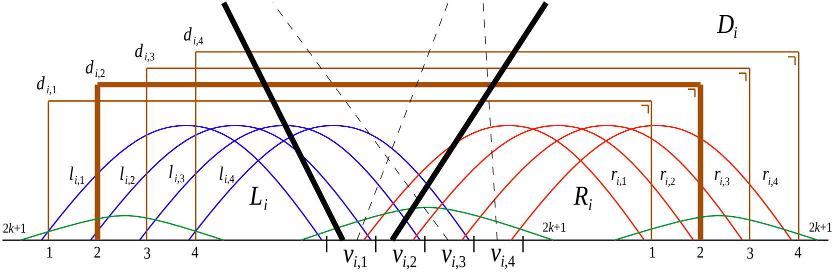
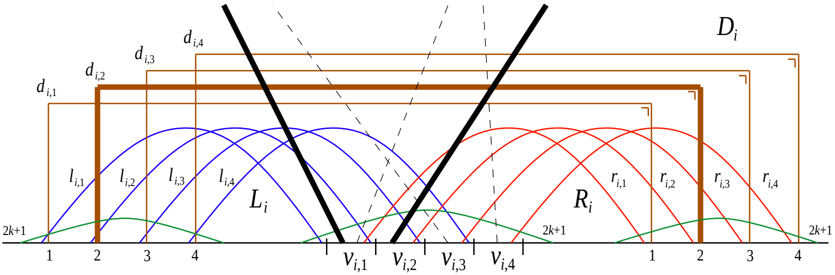


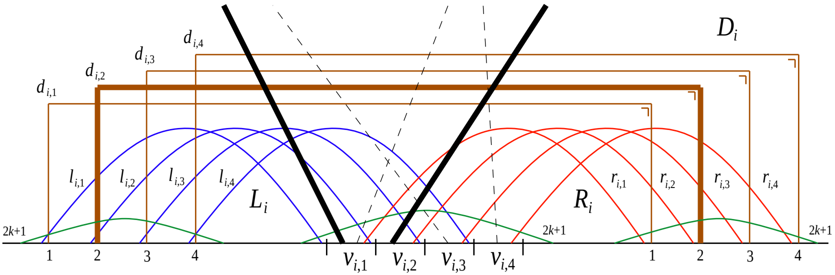
Then the intervals containing some transversal chord must contain two transversal chords.

Acyclic Dominating Set



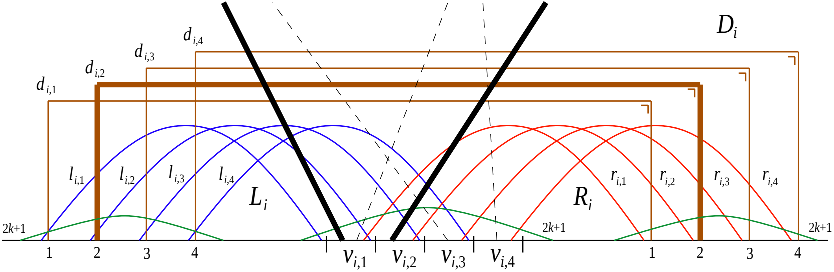
We conclude that $H[S]$ has a connected component with minimum degree at least two, and therefore $H[S]$ contains a cycle.

Acyclic Dominating Set



But this is a contradiction to the assumption that $H[S]$ is an acyclic dominating set!

ACYCLIC DOMINATING SET



Theorem

ACYCLIC DOMINATING SET *is* $W[1]$*-hard in circle graphs.*

Next subsection is...

1. Motivation
2. Hardness results
   - Independent dominating set
   - Acyclic dominating set
   - Tree dominating set
3. Sketch of the algorithms
4. Conclusions

T-Dominating Set

Theorem

Acyclic Dominating Set *is $W[1]$-hard in circle graphs.*

Theorem

Connected Dominating Set *is $W[1]$-hard in circle graphs.*

Theorem

Connected Acyclic Dominating Set *is in P in circle graphs.*

T-Dominating Set
Instance: A graph $G = (V, E)$ and a tree T.
Question: Has G a dominating set S such that $G[S] \simeq T$?

Theorem

T-Dominating Set *is NP-complete in circle graphs, and FPT when parameterized by $|V(T)|$.*

T-Dominating Set

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CONNECTED DOMINATING SET *is* $W[1]$*-hard in circle graphs.*

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Theorem

T-Dominating Set *is* NP*-complete in circle graphs, and FPT when parameterized by $|V(T)|$.*

NP-completeness of T-DOMINATING SET

3-PARTITION

Instance:	A multiset $I = \{a_1, \ldots, a_n\}$ of $n = 3m$ integers.
Question:	Can I be partitioned into m triples that all have the same sum B?

Strongly NP-complete, even if every $a_i \in (B/4, B/2)$. [Garey, Johnson. 1979]

NP-completeness of T-DOMINATING SET

$I = \{1,1,1,1,2,2,2,2,3\}$

$\{1,2,2\}$ $\{1,2,2\}$ $\{1,1,3\}$

$m = 3$ $B = 5$

Let $I = \{a_1, \dots, a_n\}$ be an instance of 3-PARTITION, in which the a_i's are between $B/4$ and $B/2$, and let $B = \sum_{i=1}^{n} a_i / m$ be the desired sum.

NP-completeness of T-Dominating Set

$I = \{1,1,1,1,2,2,2,2,3\}$

$\{1,2,2\}$ $\{1,2,2\}$ $\{1,1,3\}$

$m = 3$ $B = 5$

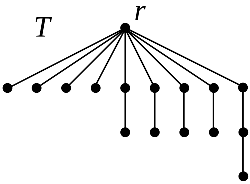
We proceed to define a tree T and to build a circle graph G that has a T-dominating set S if and only if I is a Yes-instance of 3-Partition.

NP-completeness of T-Dominating Set

$I = \{1,1,1,1,2,2,2,2,3\}$

$\{1,2,2\}$ $\{1,2,2\}$ $\{1,1,3\}$

$m = 3$ $B = 5$



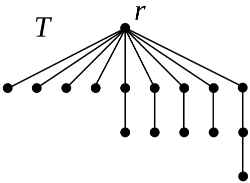
Let T be the rooted tree obtained from a root r to which we attach a path with a_i vertices, for $i = 1, \ldots, n$.

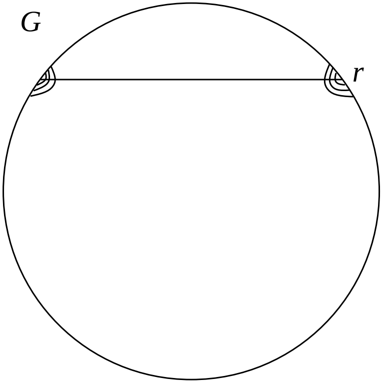
NP-completeness of T-DOMINATING SET

$I = \{1,1,1,1,2,2,2,2,3\}$

$\{1,2,2\}$ $\{1,2,2\}$ $\{1,1,3\}$

$m = 3$ $B = 5$





To build G, we start with a chord r (the root of T), and in each endpoint of r we add $n + 1$ **parallel chords** intersecting only with r.

NP-completeness of T-DOMINATING SET

$I = \{1,1,1,1,2,2,2,2,3\}$

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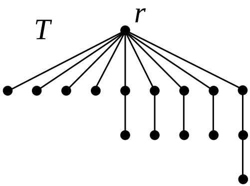
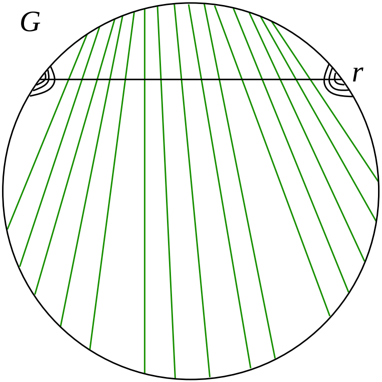
$m = 3$ $B = 5$





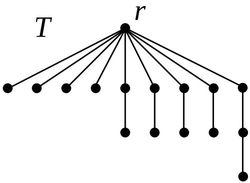
Now we add mB parallel chords $g_1, \ldots, g_{mB}$ intersecting only with r.
These chords are called branch chords.

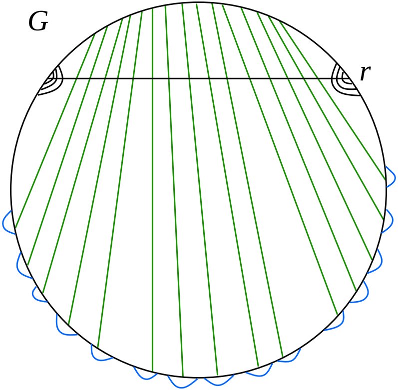
NP-completeness of T-DOMINATING SET

$I = \{1,1,1,1,2,2,2,2,3\}$

$\{1,2,2\}$ $\{1,2,2\}$ $\{1,1,3\}$

$m = 3$ $B = 5$





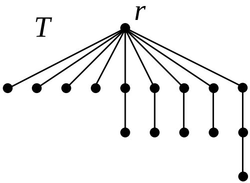
For $i = 1, \ldots, mB$, we add a chord b_i incident only with g_i. These chords are called pendant chords.

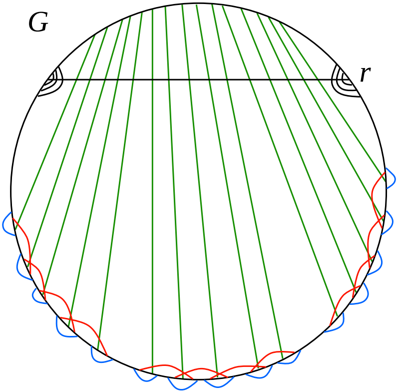
NP-completeness of T-DOMINATING SET

$I = \{1,1,1,1,2,2,2,2,3\}$

$\{1,2,2\}$ $\{1,2,2\}$ $\{1,1,3\}$

$m = 3$ $B = 5$





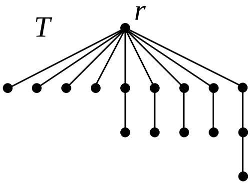
Finally, for $i \in \{1, 2, 3, \ldots, mB - 1\} \setminus \{B, 2B, 3B, \ldots, (m-1)B\}$, we add a chain chord r_i, in this way.

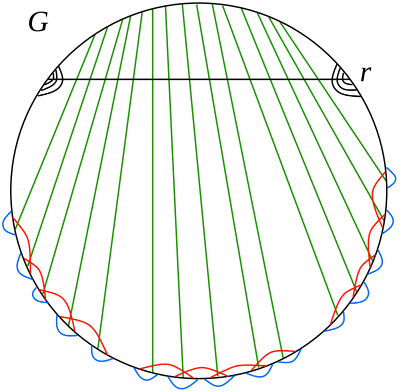
NP-completeness of T-DOMINATING SET

$I = \{1,1,1,1,2,2,2,2,3\}$

$\{1,2,2\}$ $\{1,2,2\}$ $\{1,1,3\}$

$m = 3$ $B = 5$





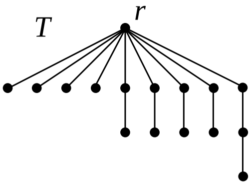
First, if I be a YES-instance of 3-PARTITION, we define a T-dominating set S in G, in the following way.

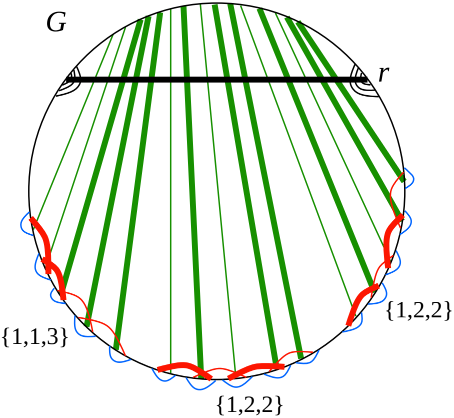
NP-completeness of T-DOMINATING SET

$I = \{1,1,1,1,2,2,2,2,3\}$

$\{1,2,2\}$ $\{1,2,2\}$ $\{1,1,3\}$

$m = 3$ $B = 5$





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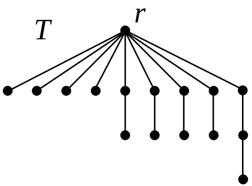
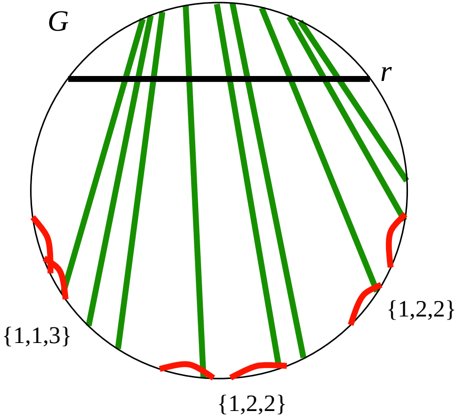
$m = 3$ $B = 5$





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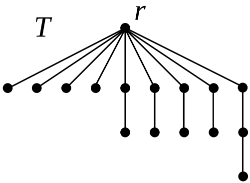
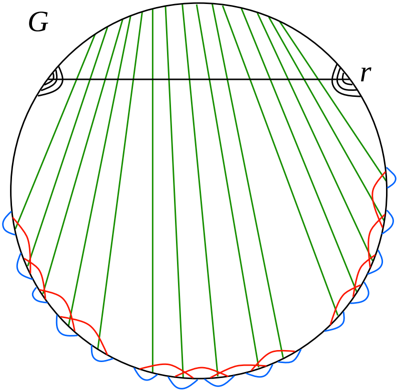
NP-completeness of T-DOMINATING SET

$I = \{1,1,1,1,2,2,2,2,3\}$

$\{1,2,2\}\ \ \{1,2,2\}\ \ \{1,1,3\}$

$m = 3 \qquad B = 5$





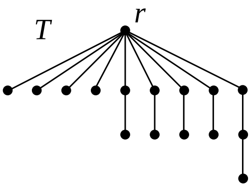
Conversely, let S be a T-dominating set S in G. By the parallel chords, necessarily r belongs to S.

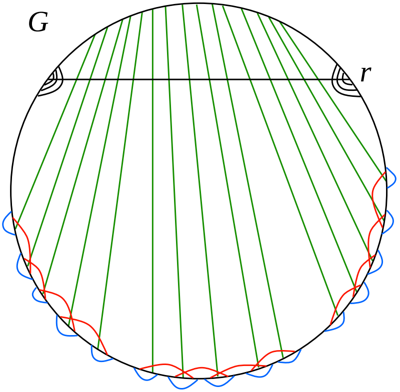
NP-completeness of T-DOMINATING SET

$I = \{1,1,1,1,2,2,2,2,3\}$

$\{1,2,2\}$ $\{1,2,2\}$ $\{1,1,3\}$

$m = 3$ $B = 5$





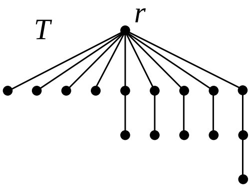
As $|S| = mB + 1$ and the number of pendant chords in G is mB, it follows by construction that S contains no pendant chord.

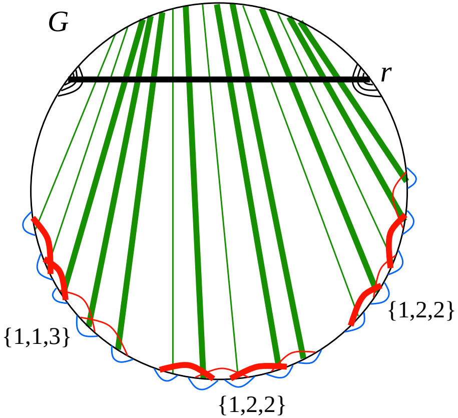
NP-completeness of T-DOMINATING SET

$I = \{1,1,1,1,2,2,2,2,3\}$

$\{1,2,2\}\ \ \{1,2,2\}\ \ \{1,1,3\}$

$m = 3 \qquad B = 5$





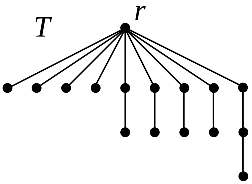
As the a_i's are strictly between $B/4$ and $B/2$, each **block** has exactly 3 branch chords in S.

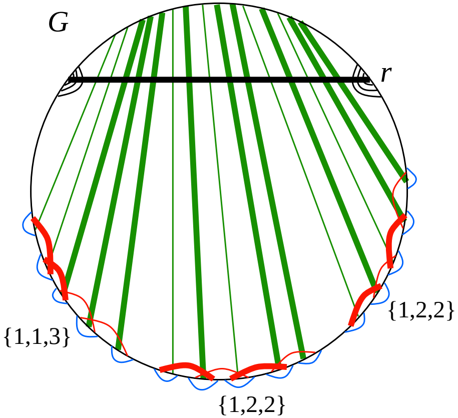
NP-completeness of T-DOMINATING SET

$I = \{1,1,1,1,2,2,2,2,3\}$

$\{1,2,2\}$ $\{1,2,2\}$ $\{1,1,3\}$

$m = 3$ $B = 5$





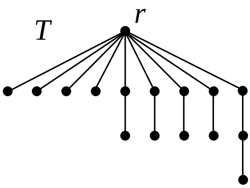
The fact that chain edges are missing between consecutive blocks assures the existence of a 3-partition of I.

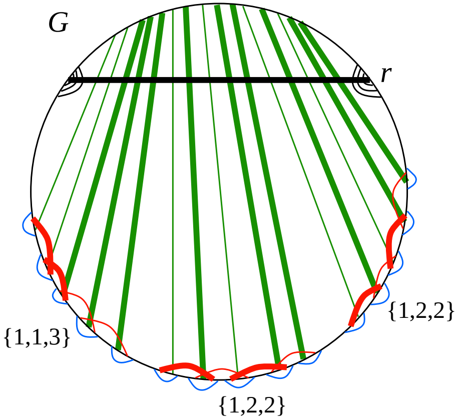
NP-completeness of T-Dominating Set

$I = \{1,1,1,1,2,2,2,2,3\}$

$\{1,2,2\}$ $\{1,2,2\}$ $\{1,1,3\}$

$m = 3$ $B = 5$





Theorem

T-Dominating Set is NP-complete in circle graphs.

Next section is...

1. Motivation
2. Hardness results
   - Independent dominating set
   - Acyclic dominating set
   - Tree dominating set
3. Sketch of the algorithms
4. Conclusions

Main ideas of the algorithms

Theorem

Deciding whether a circle graph has a dominating set isomorphic to some tree can be done in polynomial time.

- Idea: By dynamic programming, we compute all partial solutions whose extremal endpoints define a prescribed quadruple in the circle.
- Running time: $\mathcal{O}(k \cdot n^8)$.

Theorem

Deciding whether a circle graph has a dominating set isomorphic to a fixed tree T is FPT, when parameterized by $|V(T)|$.

- Idea: in the previous algorithm, we further impose that a partial solutions consists of a prescribed subset of subgraphs of T.
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Conclusions and further research

- We also proved that finding a dominating set isomorphic to some path can be solved in polynomial time in circle graphs.

- This result can be extended to a dominating set isomorphic to some graph with pathwidth bounded by a fixed constant ℓ.

★ Can this result be extended to graphs of bounded treewidth?

★ Polynomial kernel when parameterized by treewidth, or by vertex cover? (not plausible in general graphs, even if it is FPT by Courcelle)

- It can be easily seen that DOMINATING CLIQUE can be solved in polynomial time in circle graphs.

- DOMINATING CLIQUE is $W[1]$-hard in 3-interval graphs. [Jiang, Zhang, IPEC'11]

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Gràcies!