FPT algorithms for hitting forbidden minors

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Joint work with Julien Baste, Giannos Stamoulis, and Dimitrios M. Thilikos

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Graph Theory, Combinatorics and Algorithms
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Outline of the talk

1. Introduction
   - Parameterized complexity
   - Treewidth

2. Hitting forbidden minors
   - Parameterized by treewidth
   - Parameterized by solution size

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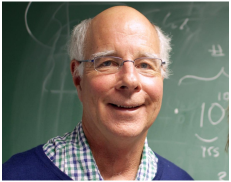
- Parameterized complexity: Topic of this talk...

The area of parameterized complexity

$\boxed{\text{Idea}}$ Measure the complexity of an algorithm in terms of the input size and an additional integer parameter.

This theory started in the late 80's, by Downey and Fellows:



Today, it is a well-established area with hundreds of articles published every year in the most prestigious TCS journals and conferences.

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These three problems are NP-hard, but are they $\boxed{\text{equally}}$ hard?

They behave quite differently...

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2. k-Clique: solvable in time $k^2 \cdot n^k$

3. Vertex k-Coloring: NP-hard for every fixed $k \geq 3$

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   $\boxed{\text{The problem is para-NP-hard}}$

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  maximum degree, or treewidth.

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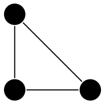
Treewidth via k-trees

Example of a 2-tree:



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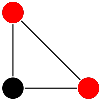

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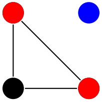

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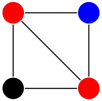

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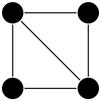

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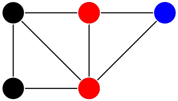

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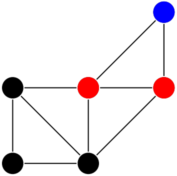

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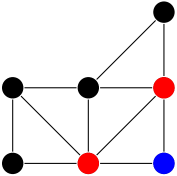

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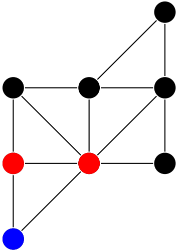

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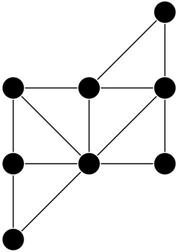

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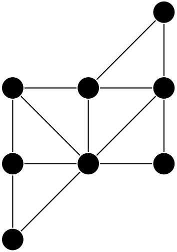

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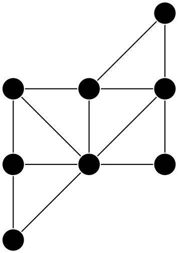

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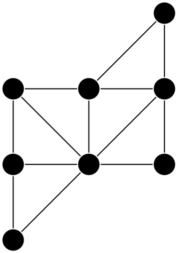

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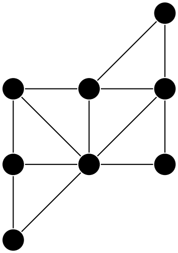

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Construction suggests the notion of tree decomposition: small separators.

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3. In many practical scenarios, it turns out that the treewidth of the associated graph is small (programming languages, road networks, ...).

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Graph logic that allows quantification over sets of vertices and edges.

Example: $\texttt{DomSet}(S)$: $[\ \forall v \in V(G) \setminus S, \exists u \in S : \{u, v\} \in E(G)\ ]$

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Theorem (Courcelle. 1990)

Every problem expressible in MSOL can be solved in time $f(\mathrm{tw}) \cdot n$ on graphs on n vertices and treewidth at most tw.

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Examples: Vertex Cover, Dominating Set, Hamiltonian Cycle, Clique, Independent Set, k-Coloring for fixed k, ...

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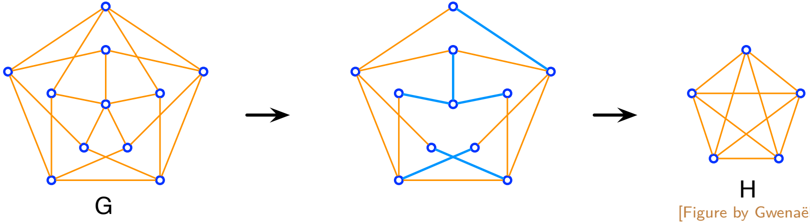
- $\mathcal{C}$ is a minor-closed graph class.

Graph minors



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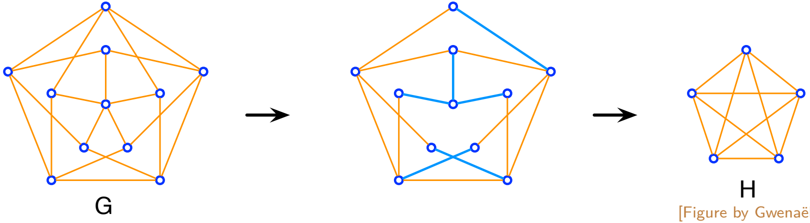

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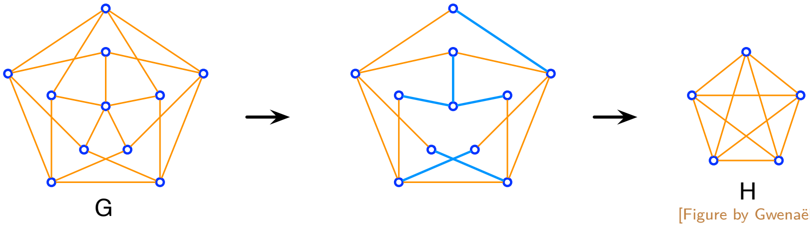

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Theorem (Robertson and Seymour. 1983-2004)

For every minor-closed graph class $\mathcal{C}$ there exists a finite collection $\mathcal{F}$ of forbidden minors such that, for every graph G,

$$G \in \mathcal{C} \iff F \not\leqslant_{\mathrm{m}} G \text{ for every } F \in \mathcal{F}.$$

Hitting forbidden minors

- If $\mathcal{C}$ = {edgeless graphs}, then $\mathcal{F} = \{K_2\}$.
- If $\mathcal{C}$ = {forests}, then $\mathcal{F} = \{K_3\}$.
- If $\mathcal{C}$ = {outerplanar graphs}, then $\mathcal{F} = \{K_4, K_{2,3}\}$.
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- $\mathcal{F} = \{K_2\}$: Vertex Cover.
- $\mathcal{F} = \{K_3\}$: Feedback Vertex Set.
- $\mathcal{F} = \{K_5, K_{3,3}\}$: Vertex Planarization.
- $\mathcal{F}$ = {diamond}: Cactus Vertex Deletion.

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Question:	Does G contain a set $S \subseteq V(G)$ with $\lvert S \rvert \leqslant k$ such that $G \setminus S$ does not contain any of the graphs in $\mathcal{F}$ as a minor?

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Hitting forbidden minors

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We consider the following two parameterizations of $\mathcal{F}$-M-Deletion:

1. Structural parameter: $\mathrm{tw}(G)$.
2. Solution size: k.

Next subsection is...

1. Introduction
   - Parameterized complexity
   - Treewidth
2. Hitting forbidden minors
   - Parameterized by treewidth
   - Parameterized by solution size

Theorem (Courcelle. 1990)

Every problem expressible in MSOL can be solved in time $f_{\mathcal{F}}(\mathrm{tw}) \cdot n$ on graphs on n vertices and treewidth at most tw.

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Very active area in parameterized complexity during the last decade.

Remark: Algorithms parameterized by treewidth appear very often as a "black box" in all kinds of parameterized algorithms.

What was known for particular collections $\mathcal{F}$

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- $\mathcal{F} = \{K_5, K_{3,3}\}$: VERTEX PLANARIZATION.
  Solvable in time $2^{\Theta(\mathrm{tw} \cdot \log \mathrm{tw})} \cdot n^{\mathcal{O}(1)}$. [Jansen, Lokshtanov, Saurabh. 2014 + Pilipczuk. 2015]

Work with Julien Baste and Dimitrios M. Thilikos (2016-)

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Determine, for every fixed $\mathcal{F}$, the (asymptotically) smallest function $f_{\mathcal{F}}$ such that $\mathcal{F}$-M-Deletion on n-vertex graphs can be solved in time

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[Baste, S., Thilikos. **Hitting minors on bounded treewidth graphs. I. General upper bounds**. 2020]

[Baste, S., Thilikos. **Hitting minors on bounded treewidth graphs. II. Single-exponential algorithms**. 2020]

[Baste, S., Thilikos. **Hitting minors on bounded treewidth graphs. III. Lower bounds**. 2020]

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Summary of our results

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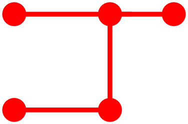
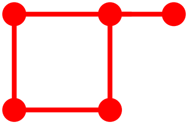
A dichotomy for hitting a connected minor





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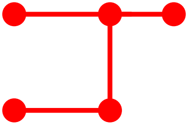 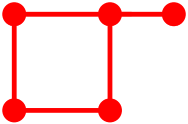

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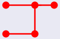
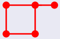
- $2^{\mathcal{O}(\mathrm{tw})} \cdot n^{\mathcal{O}(1)}$, *if* $H \leqslant_{\mathrm{c}}$  *or* $H \leqslant_{\mathrm{c}}$  .

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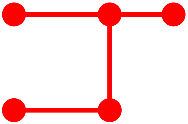

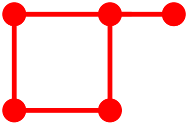

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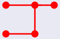
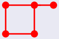
- $2^{\mathcal{O}(\mathrm{tw})} \cdot n^{\mathcal{O}(1)}$, *if* $H \leqslant_c$  *or* $H \leqslant_c$  .
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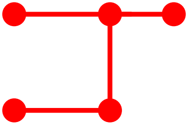

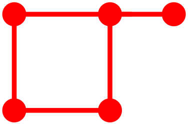

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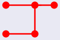
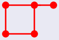
- $2^{\mathcal{O}(\mathrm{tw})} \cdot n^{\mathcal{O}(1)}$, *if* $H \leqslant_c$  *or* $H \leqslant_c$ .
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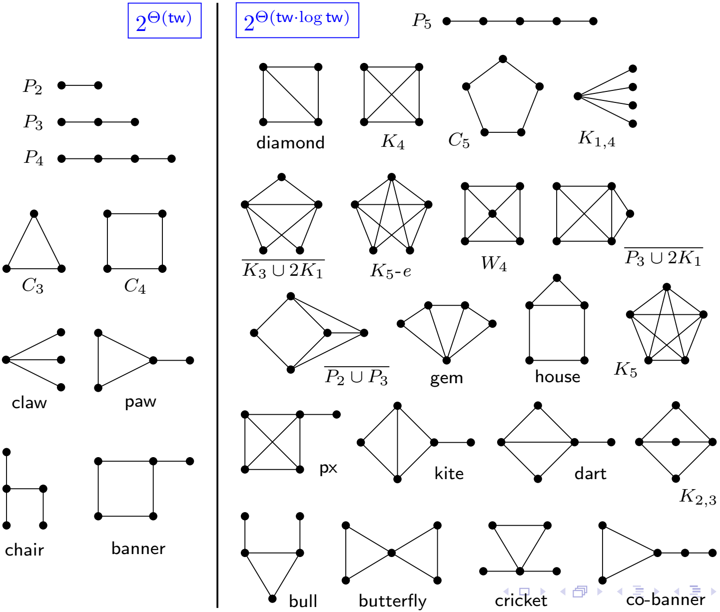
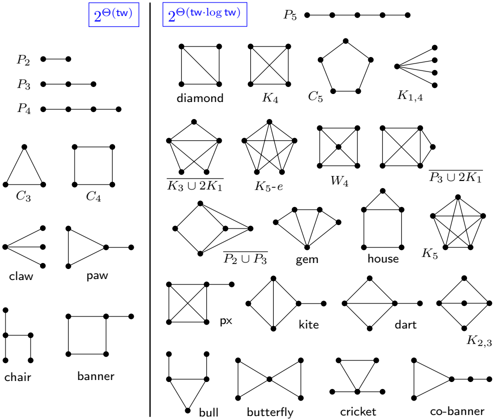
In both cases, the running time is asymptotically optimal under the ETH.

Complexity of hitting a single connected minor H



A compact statement for a single connected graph



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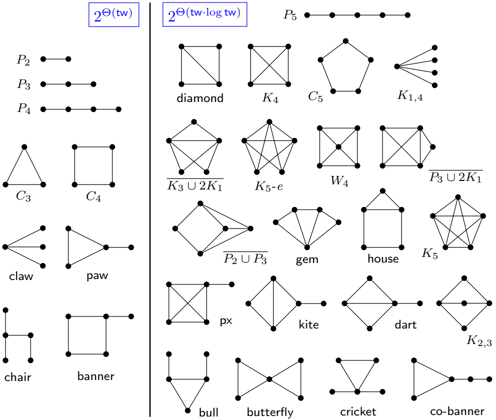

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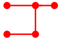
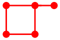
- All graphs on the left are contractions of  or 

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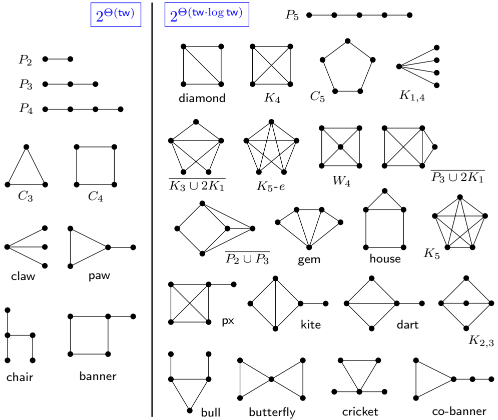

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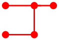
- All graphs on the left are contractions of  or 
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3. Lower bounds under the ETH
   - $2^{o(\mathrm{tw})}$ is "easy".
   - $2^{o(\mathrm{tw} \cdot \log \mathrm{tw})}$ is much more involved and we get ideas from:

     [Lokshtanov, Marx, Saurabh. 2011] [Marcin Pilipczuk. 2017] [Bonnet, Brettell, Kwon, Marx. 2017]

skip

We have three types of results

1. General algorithms
   - For every $\mathcal{F}$: time $2^{2^{\mathcal{O}(\mathrm{tw}\cdot\log\mathrm{tw})}} \cdot n^{\mathcal{O}(1)}$.
   - $\mathcal{F}$ planar: time $2^{\mathcal{O}(\mathrm{tw}\cdot\log\mathrm{tw})} \cdot n^{\mathcal{O}(1)}$.

   ★ $\mathcal{F}$ ~~planar~~: time $2^{\mathcal{O}(\mathrm{tw}\cdot\log\mathrm{tw})} \cdot n^{\mathcal{O}(1)}$.
   - G planar: time $2^{\mathcal{O}(\mathrm{tw})} \cdot n^{\mathcal{O}(1)}$.

2. Ad-hoc single-exponential algorithms
   - Some use "typical" dynamic programming.
   - Some use the rank-based approach. [Bodlaender, Cygan, Kratsch, Nederlof. 2013]

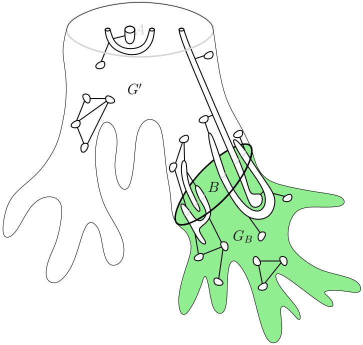
3. Lower bounds under the ETH
   - $2^{o(\mathrm{tw})}$ is "easy".
   - $2^{o(\mathrm{tw}\cdot\log\mathrm{tw})}$ is much more involved and we get ideas from:

   [Lokshtanov, Marx, Saurabh. 2011] [Marcin Pilipczuk. 2017] [Bonnet, Brettell, Kwon, Marx. 2017]

skip

Algorithm in time $2^{\mathcal{O}_{\mathcal{F}}(\mathsf{tw} \cdot \log \mathsf{tw})} \cdot n^{\mathcal{O}(1)}$ for any collection $\mathcal{F}$

[Fig. by Valentin Garnero]



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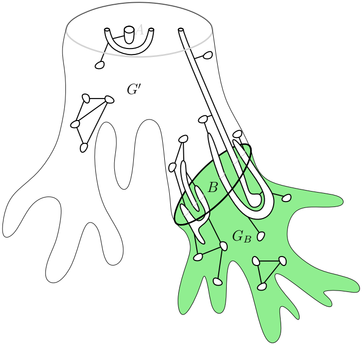

- For a fixed $\mathcal{F}$, we define an equivalence relation $\equiv^{(\mathcal{F},t)}$ on t-boundaried graphs:

$$G_1 \equiv^{(\mathcal{F},t)} G_2 \quad \text{if } \forall G' \in \mathcal{B}^t,$$
$$\mathcal{F} \leqslant_{\mathrm{m}} G' \oplus G_1 \iff \mathcal{F} \leqslant_{\mathrm{m}} G' \oplus G_2.$$

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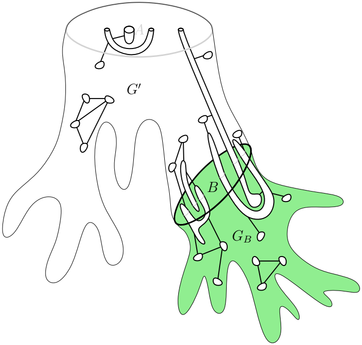

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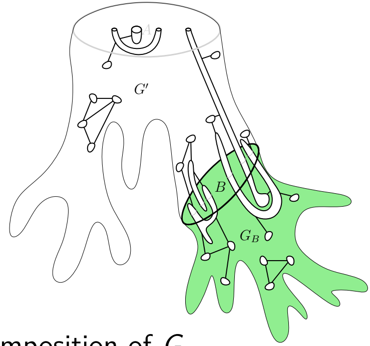

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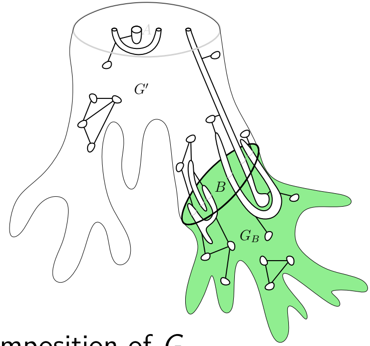

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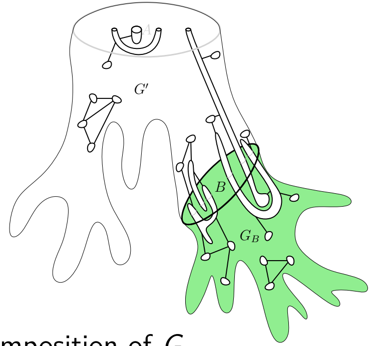

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- $\boxed{\text{Goal}}$ Bound the number of representatives: $|\mathcal{R}^{(\mathcal{F},t)}| = 2^{\mathcal{O}_{\mathcal{F}}(\text{tw}\cdot\log \text{tw})}$.

Bounding the set of representatives

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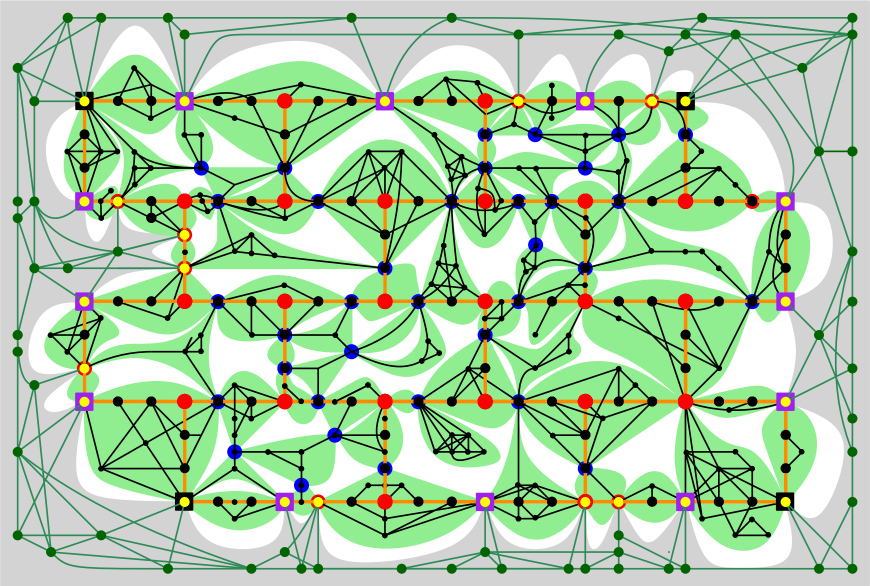
  As a representative R is $\mathcal{F}$-minor-free, if $\mathrm{tw}(R \setminus B) > c_{\mathcal{F}}$, $R \setminus B$ contains a large flat wall, where we can find an irrelevant vertex.

A flat wall can in fact be quite wild...



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[Figure by Dimitrios M. Thilikos]

Hard part: finding an irrelevant vertex inside a flat wall

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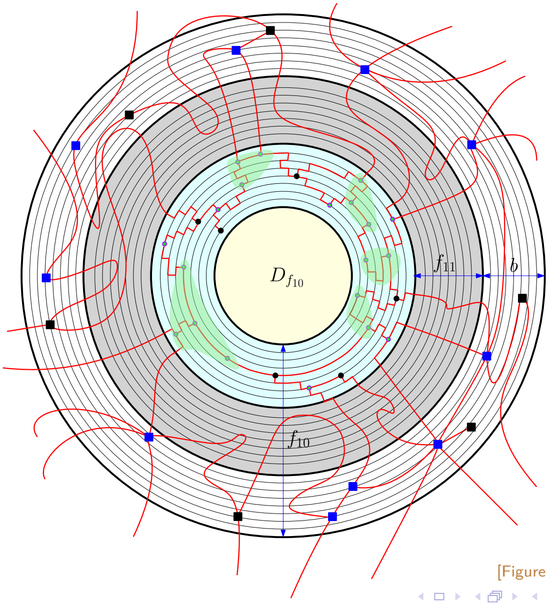

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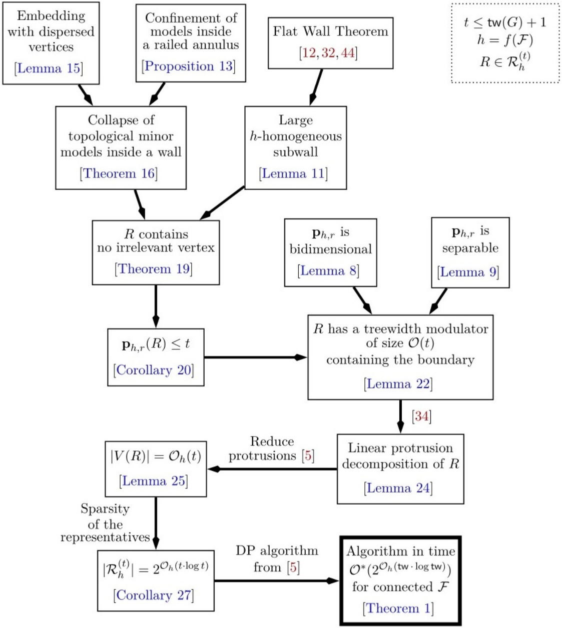
Diagram of the algorithm for a general collection $\mathcal{F}$



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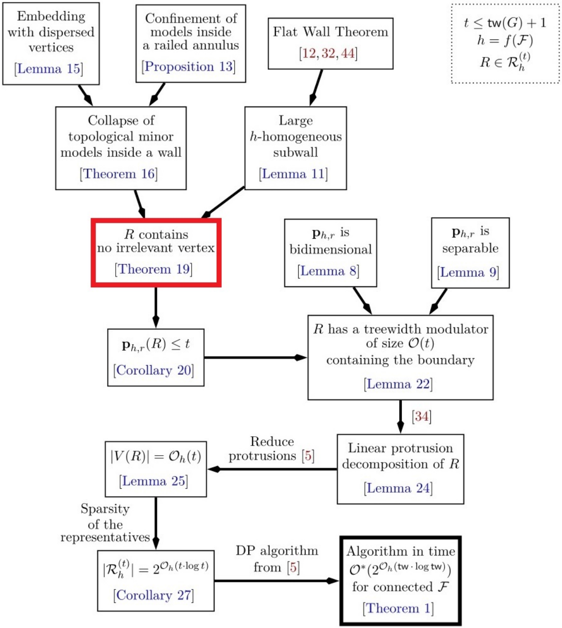

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Next subsection is...

1. Introduction
   - Parameterized complexity
   - Treewidth
2. Hitting forbidden minors
   - Parameterized by treewidth
   - Parameterized by solution size

We parameterize by the size of the desired solution

$\mathcal{F}$-M-DELETION

Input: A graph G and an integer k.

Parameter: k.

Question: Does G contain a set $S \subseteq V(G)$ with $|S| \leqslant k$ such that $G \setminus S$ does not contain any of the graphs in $\mathcal{F}$ as a minor?

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It is easy to see that, for every $k \geqslant 1$, the class of graphs

$$\mathcal{C}_k = \{G \mid (G, k) \text{ is a positive instance of } \mathcal{F}\text{-M-DELETION}\}$$

is minor-closed.

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But... only existential, non-uniform, $f(\mathcal{C}_k)$ astronomical.

Can we do better?

- The function $f(\mathcal{C}_k)$ is constructible. [Adler, Grohe, Kreutzer. 2008]

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- For some non-planar collections $\mathcal{F}$:
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- For every $\mathcal{F}$, some enormous explicit function $f_{\mathcal{F}}(k)$ can be derived from an FPT algorithm for hitting topological minors:

  $$f_{\mathcal{F}}(k) \cdot n^{\mathcal{O}(1)}.$$

  [Fomin, Lokshtanov, Panolan, Saurabh, Zehavi. 2020]

Our results

Theorem (S., Stamoulis, Thilikos. 2020)

For all $\mathcal{F}$, the $\mathcal{F}$-M-DELETION problem can be solved in time $2^{\text{poly}(k)} \cdot n^3$.

Here, $\text{poly}(k)$ is a polynomial whose degree depends on $\mathcal{F}$.

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Theorem (S., Stamoulis, Thilikos. 2020)

If $\mathcal{F}$ contains an apex graph, the $\mathcal{F}$-M-Deletion problem can be solved in time $2^{\text{poly}(k)} \cdot n^2$.

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skip

General scheme of the algorithm:

[whole slide shamelessly borrowed from Giannos Stamoulis]

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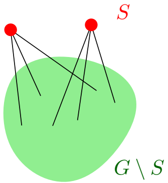

Iterative compression: given solution S of size $k+1$, search solution of size k.

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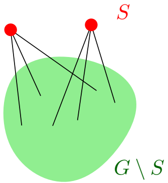

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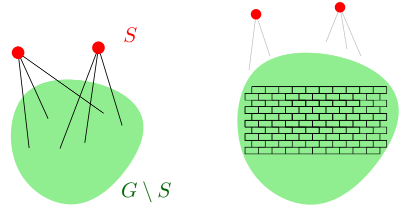

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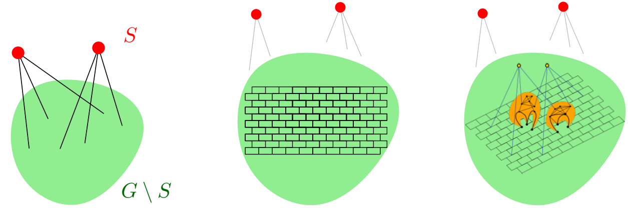

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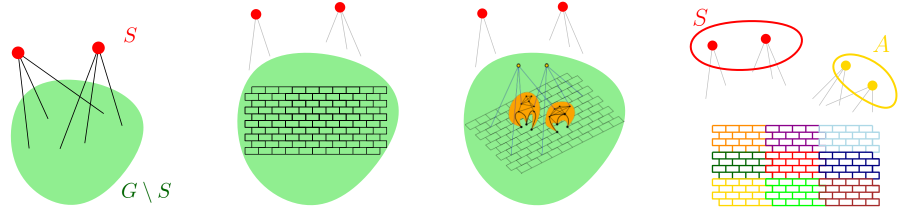

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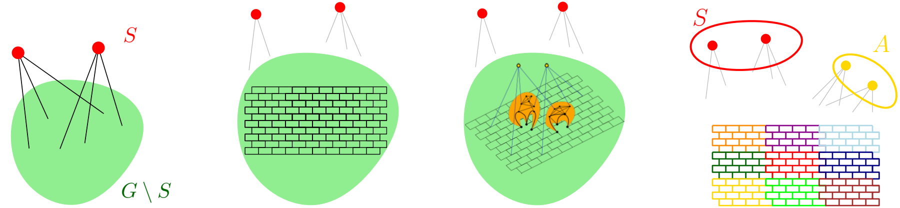

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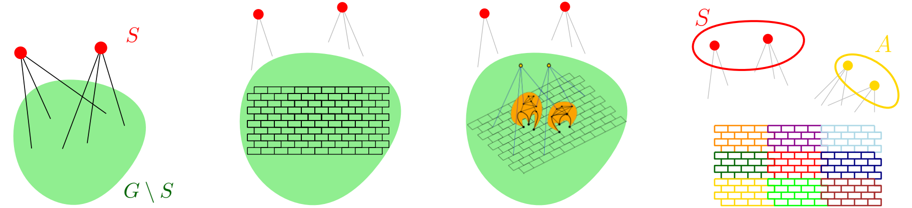

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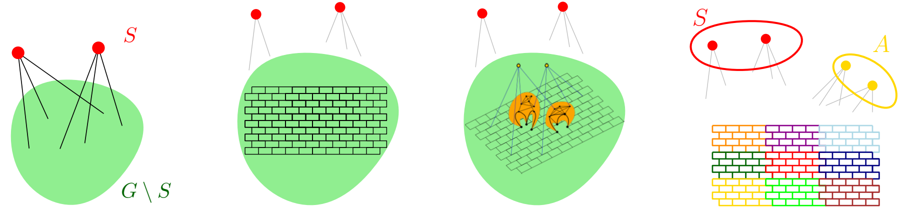

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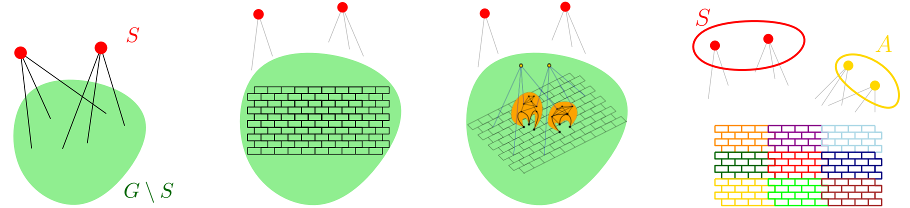
General scheme of the algorithm:

[whole slide shamelessly borrowed from Giannos Stamoulis]



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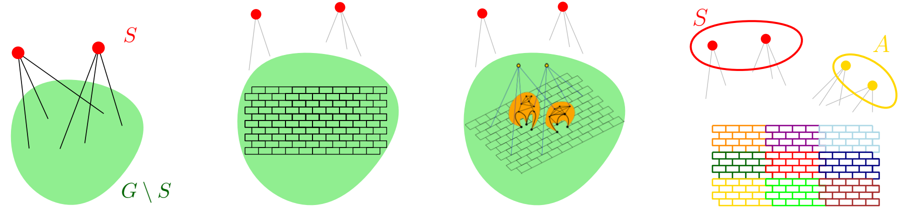

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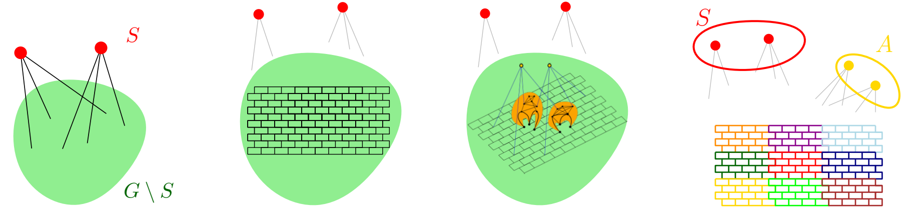

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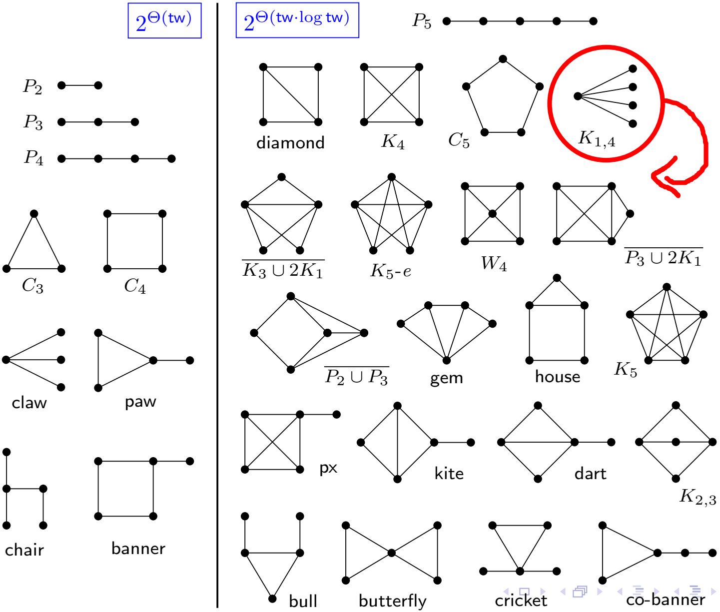
Is the price of homogeneity unavoidable?

skip

For topological minors, there is (at least) one change



Gràcies!

Toda raba!