Dual parameterization of WEIGHTED COLORING

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Outline of the talk

1. Introduction
2. Our results
3. Sketches of some proofs
4. Conclusions

Next section is...

1 Introduction

2 Our results

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4 Conclusions

Weighted Coloring

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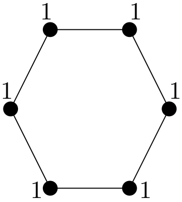
If $w(v) = 1$ for every $v \in V(G)$, then clearly $\sigma(G, w) = \chi(G)$.

Example

(G, w)



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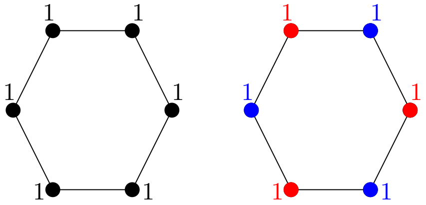

$\sigma(G, w) = 2$

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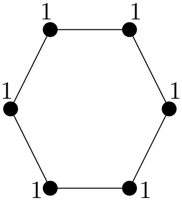

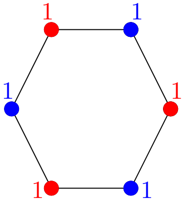

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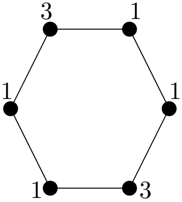
(G, w')



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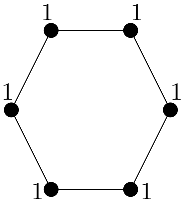

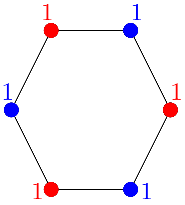

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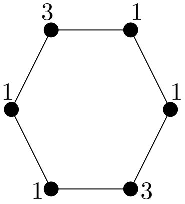

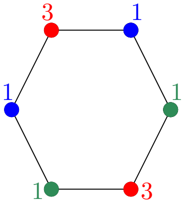

$\sigma(G, w') = 5$

What is known about Weighted Coloring

The Weighted Coloring problem was introduced by [Guan, Zhu. 1997] to study practical applications related to resource allocation.

If all the vertex weights are equal to one, then $\sigma(G, w) = \chi(G)$.
Thus, determining $\sigma(G, w)$ in an NP-hard problem.

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The problem is NP-hard even on:

- split graphs, interval graphs, bipartite graphs, and triangle-free planar graphs with bounded degree.

On the other hand, it is polynomial on

- cographs and some subclasses of bipartite graphs.

[de Werra, Demange, Monnot, Paschos. 2002]
[Escoffier, Monnot, Paschos. 2006]
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Complexity of WEIGHTED COLORING on trees (forests)

WEIGHTED COLORING can be solved on forests in time $n^{\mathcal{O}(\log n)} = 2^{\mathcal{O}(\log^2 n)}$ (quasi-polynomial). [Guan, Zhu. 1997] [Linhares, Reed. 2006]

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- PTAS on bounded treewidth graphs. [Escoffier, Monnot, Paschos. 2006]
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★ There is no algorithm solving WEIGHTED COLORING on n-vertex trees in time $n^{o(\log n)}$, under the ETH. [Araújo, Nisse, Pérennes. 2014]

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Weighted Coloring on forests is W[1]-hard parameterized by the size of a largest connected component. [Araújo, Baste, S. 2017]

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Input: A vertex-weighted graph (G, w) and a positive integer k.
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The dual parameterization has proved to be useful for VERTEX COLORING, GRUNDY COLORING, and b-COLORING.

[Chor, Fellows, Juedes. 2004]

[Havet, Sampaio. 2013]

Known results

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Escoffier showed that the problem is FPT, namely:

- Algorithm running in time $2^{\mathcal{O}(k \log k)} \cdot n^{\mathcal{O}(1)}$.
- Asked "what size of kernel can be achieved for the problem?"

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Our results for Dual Weighted Coloring

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   - Subclasses of split graphs, with lower bounds on the degrees.

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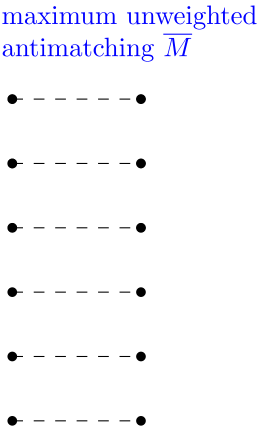
We start with an idea already used, in particular, by [Escoffier. 2016]

An exponential kernel for Dual Weighted Coloring



We compute (in poly time) a maximum unweighted antimatching $\overline{M}$.

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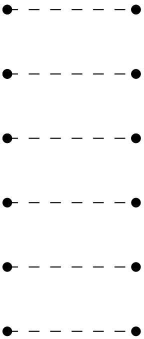

We may assume $|\overline{M}| \leq k - 1$

If $|\overline{M}| \geq k$, we are dealing with a YES-instance.

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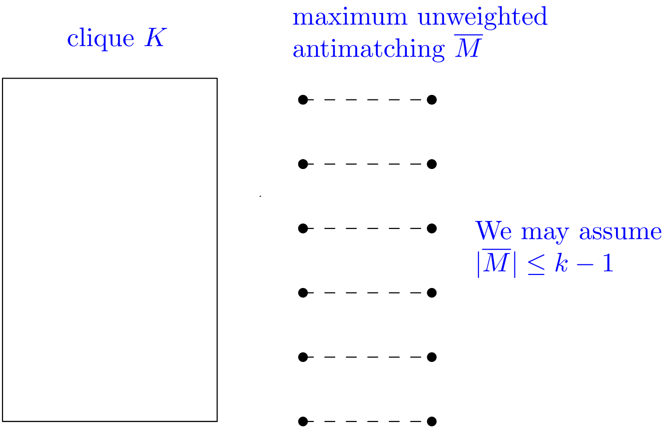

The complement of the antimatching $\overline{M}$ induces a clique K.

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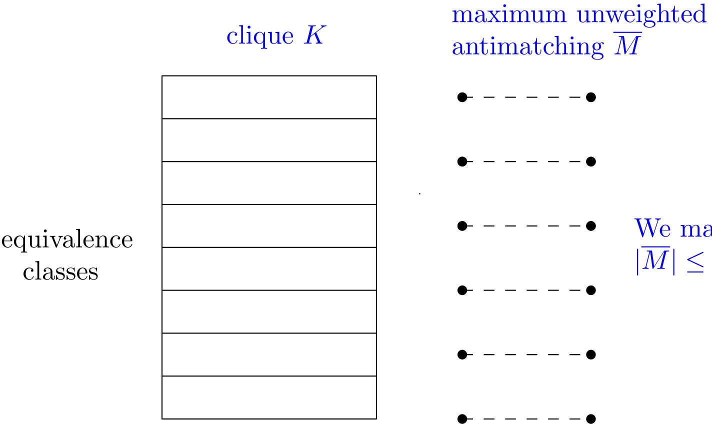

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We partition K into equivalence classes w.r.t. the neighborhood in $\overline{M}$.

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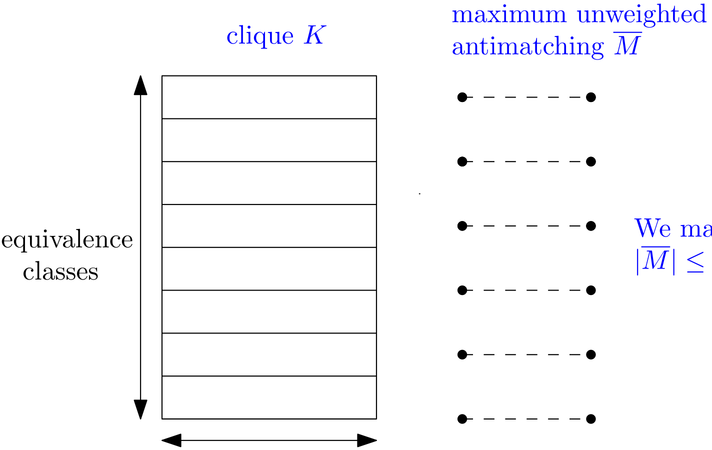

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We have to bound the number and the size of the equivalence classes.

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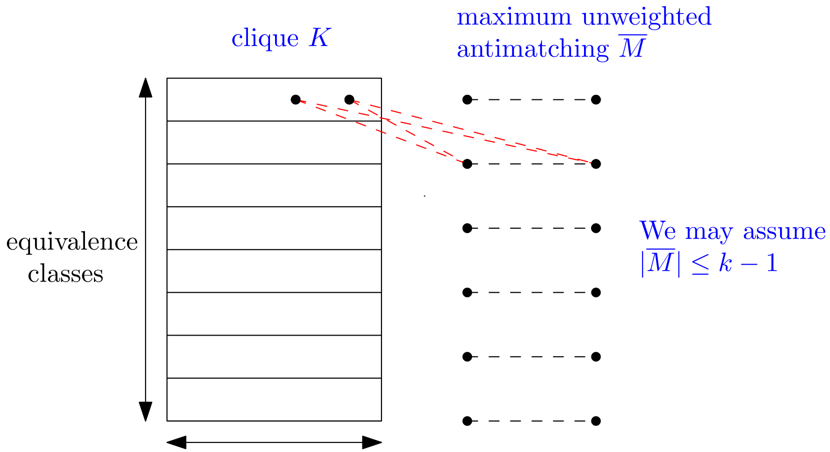

An equivalence class cannot miss both endvertices of a non-edge.

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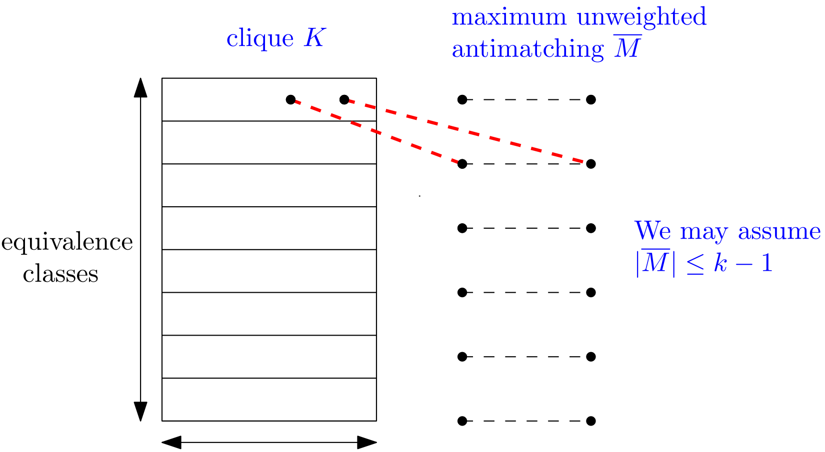

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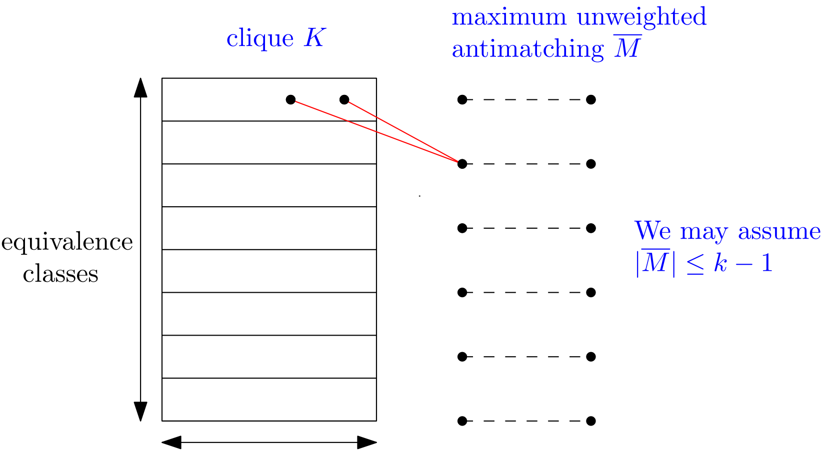

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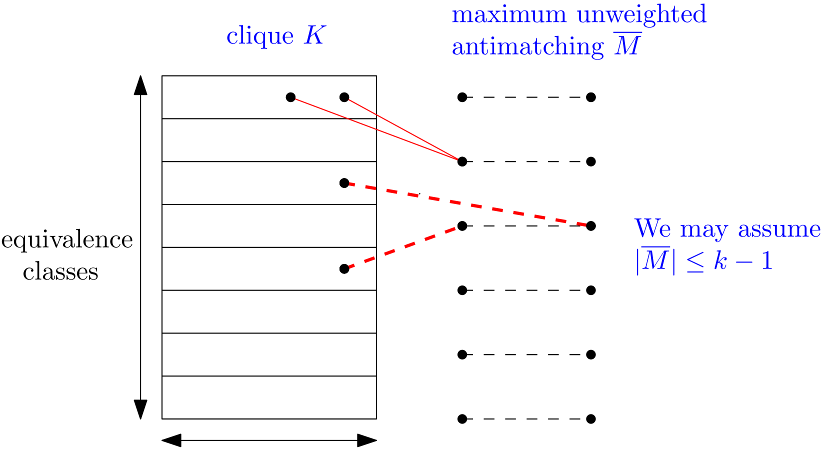

Two equivalence classes cannot miss different endvertices of a non-edge.

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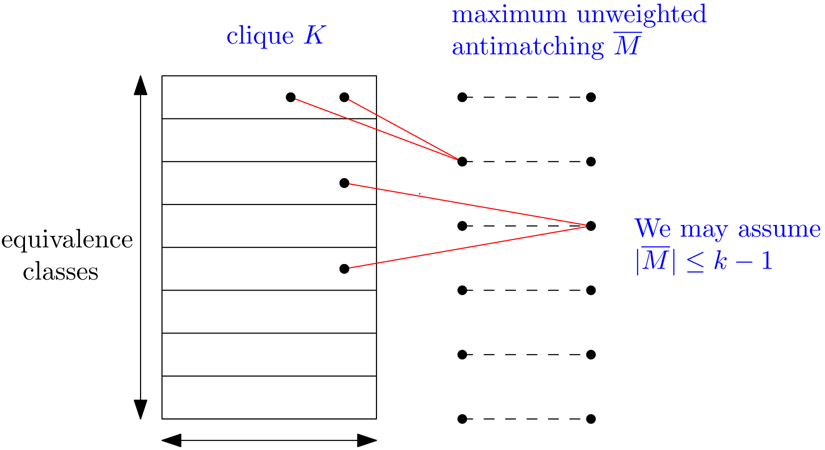

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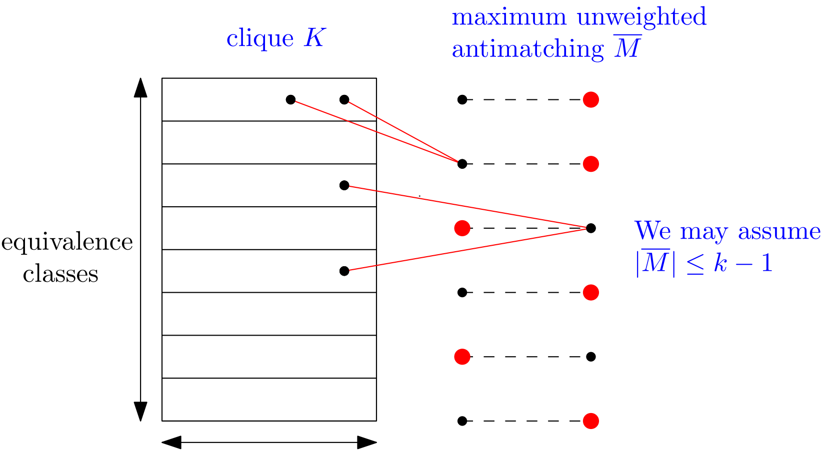

Every non-edge has a unique "potential non-neighbor" for the classes.

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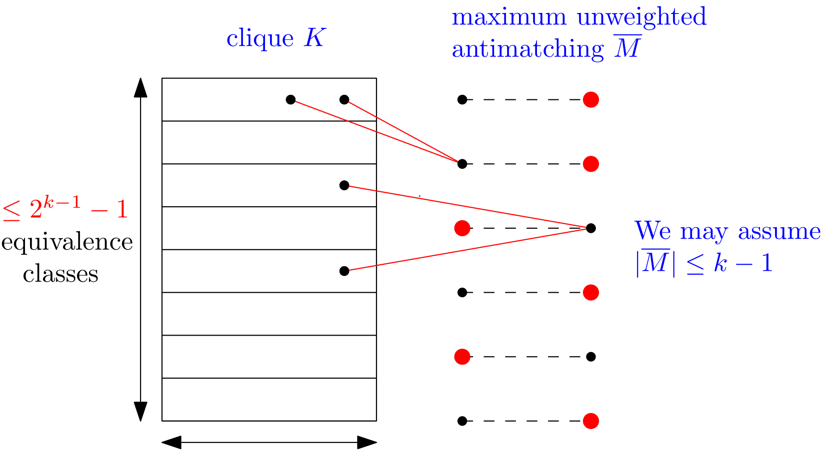

Hence, the number of equivalence classes is at most $2^{k-1} - 1$.

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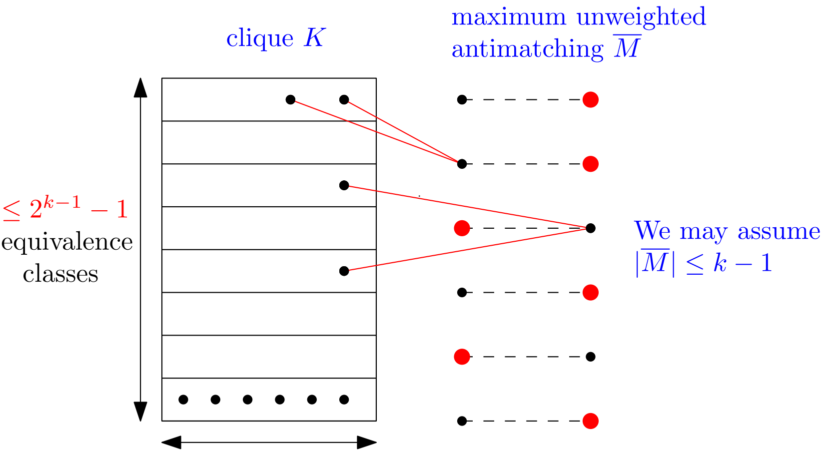

It remains to bound the size of each equivalence class.

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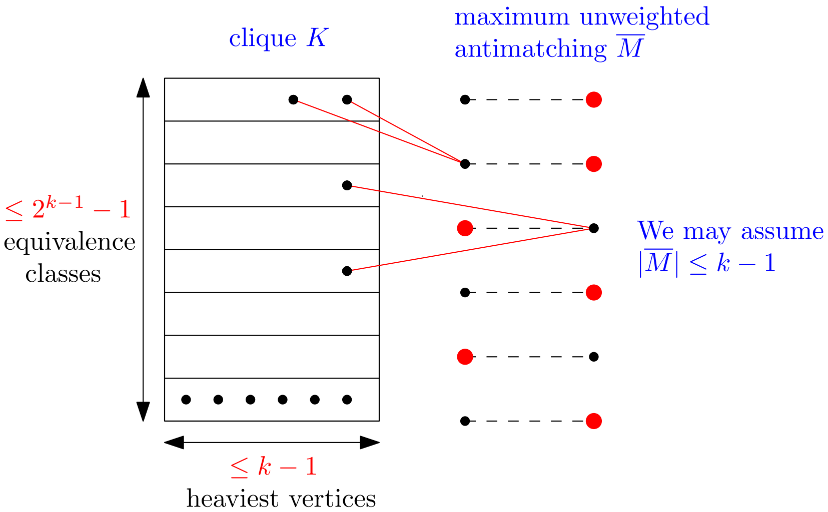

It is safe to keep only the $k-1$ heaviest vertices of each equivalence class.

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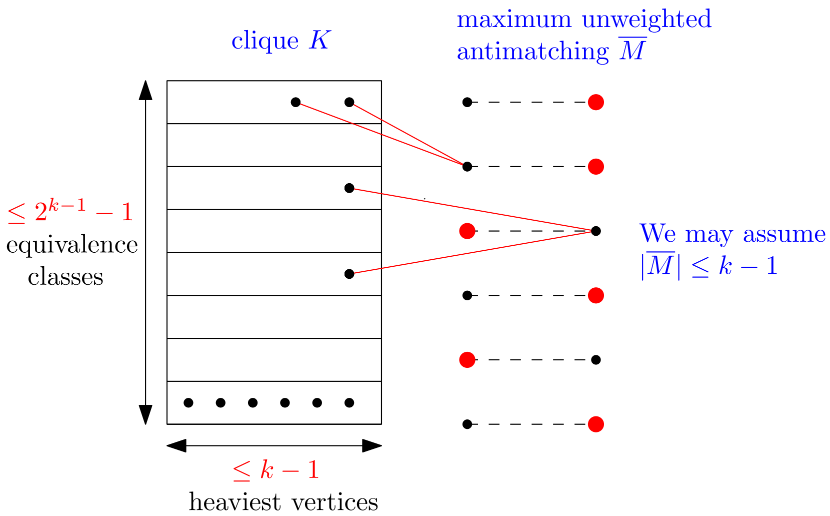

Thus, $|V(G)| \leq |V(\overline{M})| + |K| \ \leq \ 2(k-1) + (2^{k-1}-1)(k-1)$.

Ruling out polynomial kernels

Theorem

DUAL WEIGHTED COLORING *does not admit a polynomial kernel unless* NP $\subseteq$ coNP/poly*, even on split graphs with only two different weights.*

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We present a polynomial parameter transformation from Set Cover parameterized by the size of the universe, known not to admit polynomial kernels unless NP $\subseteq$ coNP/poly. [Dom, Lokshtanov, Saurabh. 2014]

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Let $A, B \subseteq \Sigma^* \times \mathbb{N}$ be two parameterized problems.

A polynomial parameter transformation from A to B is an algorithm s.t.:

Instance (x, k) of A $\xrightarrow{\text{polynomial time}}$ Instance (x', k') of B

1. (x, k) is a Yes-instance of $A \Leftrightarrow (x', k')$ is a Yes-instance of B.
2. $k' \leq \text{poly}(k)$.

[Bodlaender, Thomassé, Yeo. 2011]

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Our reduction is an appropriate modification of a reduction to prove the NP-hardness of Weighted Coloring on split graphs:

only the weights change.

[Demange, de Werra, Monnot, Paschos. 2002]

Polynomial parameter transformation from Set Cover

We are given an instance $(U, \mathcal{S}, k, \ell)$ of Set Cover,
where $\mathcal{S}$ is a family of sets over a universe U of size k (the parameter).
Question: $\exists$? $\mathcal{S}' \subseteq \mathcal{S}$ of at most ℓ sets covering all the elements of U.

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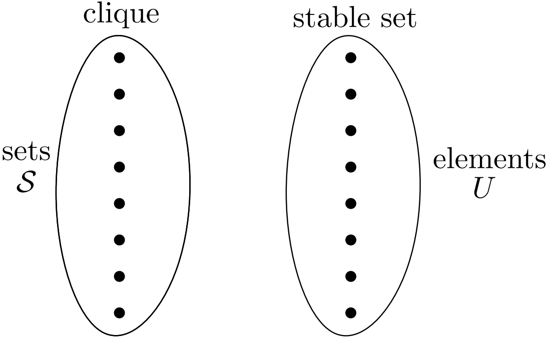

Polynomial parameter transformation from Set Cover

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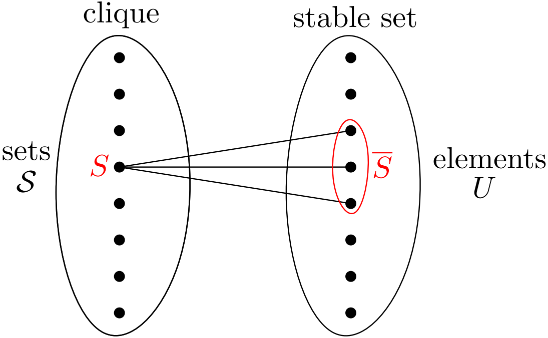

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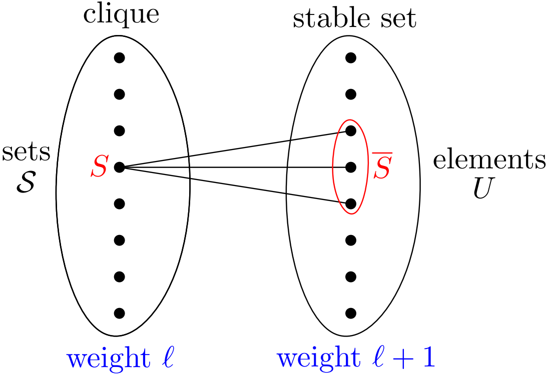

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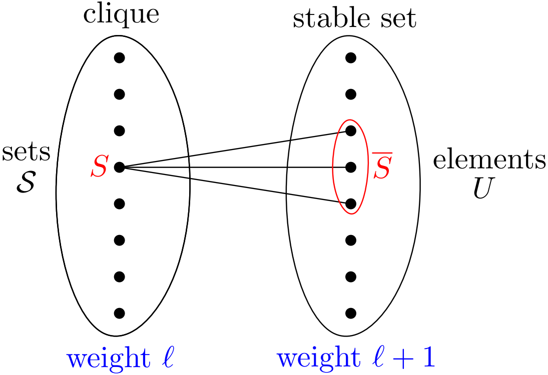

We build and instance (G, w, k'), with $k' = k(\ell + 1) - \ell = \mathcal{O}(k^2)$.

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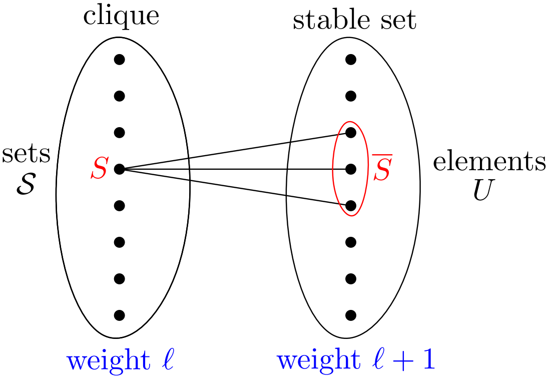

$(U, \mathcal{S}, k, \ell)$ satisfiable $\Leftrightarrow$ $\sigma(G, w) \leq \sum_{v \in V(G)} w(v) - k' = |\mathcal{S}| \cdot \ell + \ell$.

Dual Weighted Coloring seems to be strongly related to Set Cover...

Exploiting the relation with Set Cover

Dual Weighted Coloring seems to be strongly related to Set Cover... and also to d-Set Cover:

Proposition

Dual Weighted Coloring restricted to split graphs such that each vertex in the clique has at most d non-neighbors in the stable set, for some constant $d \geq 2$, admits a kernel with $\mathcal{O}(k^d)$ vertices.

Exploiting the relation with SET COVER

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DUAL WEIGHTED COLORING *restricted to split graphs such that each vertex in the clique has at most d non-neighbors in the stable set, for some constant $d \geq 2$, admits a kernel with $\mathcal{O}(k^d)$ vertices.*

Furthermore, for any $\varepsilon > 0$, a kernel with $\mathcal{O}(k^{\frac{d-3}{2}-\varepsilon})$ vertices does not exist unless $\mathsf{NP} \subseteq \mathsf{coNP}/\mathsf{poly}$.

Using the kernel lower bound for d-SET COVER. [Hermelin, Wu. 2012]

Next section is...

1 Introduction

2 Our results

3 Sketches of some proofs

4 Conclusions

Some questions for further research

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- Can the cubic kernel on interval graphs be improved?

- Identify other classes of (dense) graphs allowing for polynomial kernels.

Gràcies!

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