Finding subdigraphs in digraphs of bounded directed treewidth

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LAGOS 2025, Buenos Aires, Argentina

Undirected vs directed treewidth

Treewidth of **undirected** graphs: invariant that measures the structural similarity of a graph to a forest.

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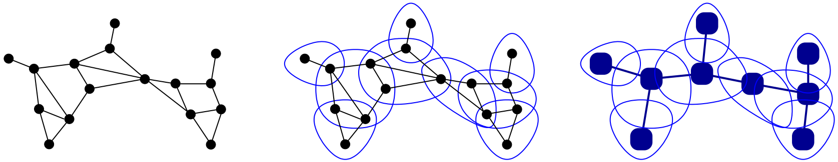

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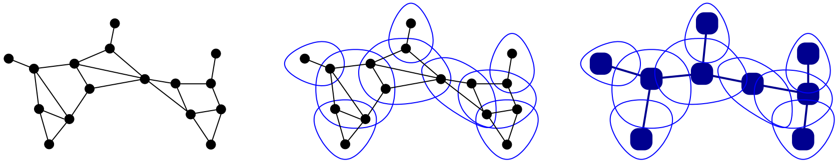

Treewidth of **directed** graphs: invariant that measures the structural similarity of a digraph to a DAG.

[Johnson, Robertson, Seymour, Thomas. 2001]

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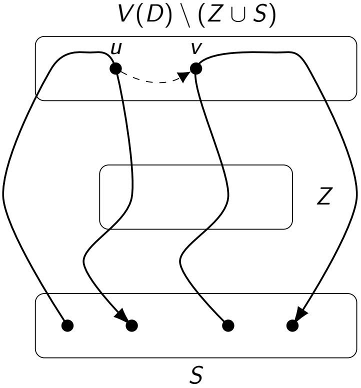
Let D be a digraph, $Z \subseteq V(D)$, and $S \subseteq V(D) \setminus Z$.

We say that S is Z-guarded if there is no directed walk in $D \setminus Z$ with first and last vertices in S that uses a vertex of $D \setminus (Z \cup S)$.



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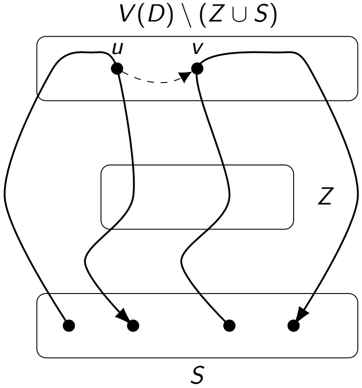
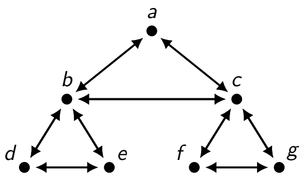


S is w-guarded if S is X-guarded by some $X \subseteq V(D)$ with $|X| \leq w$.

Directed treewidth ("distance to a DAG")



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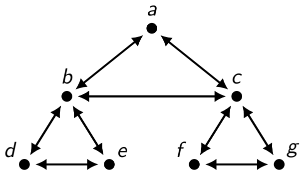

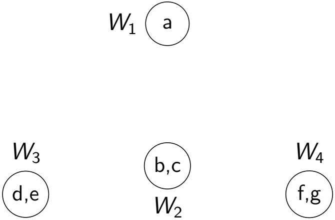

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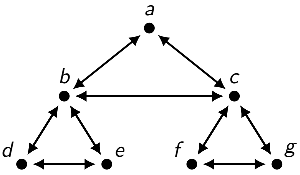

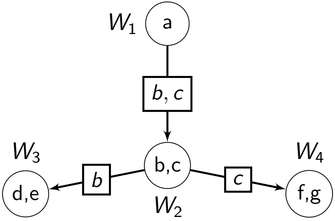

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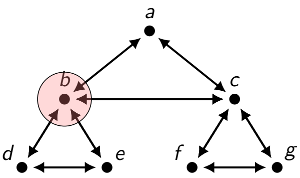

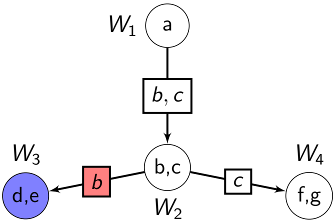

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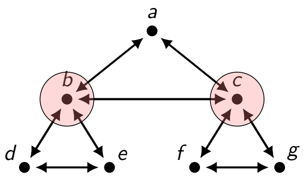

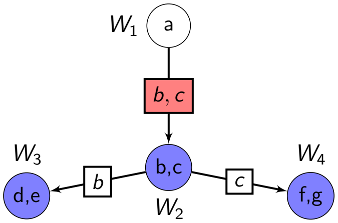

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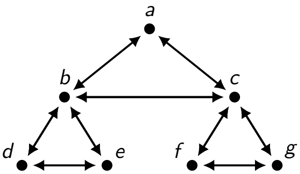

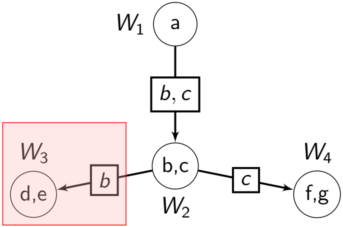

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- *Width* = $\boxed{\text{size of largest set of (bag} \cup \text{adjacent guards)} -1}$.
  - $= 2$ in the example.

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- para-NP-hard problem: NP-hard for a fixed value of the parameter.

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Undirected treewidth rules!

Theorem (Courcelle. 1990)

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What about directed treewidth (dtw)?

- Directed Hamiltonian Path is $W[2]$-hard parameterized by dtw.
- Max Directed Cut is NP-hard restricted to DAGs ($\mathrm{dtw} = 0$).

[Lampis, Kaouri, Mitsou. 2011]

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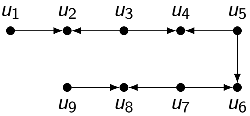

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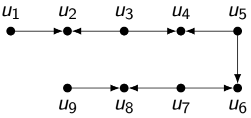

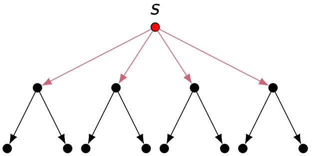

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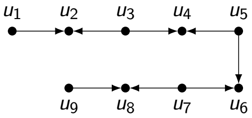

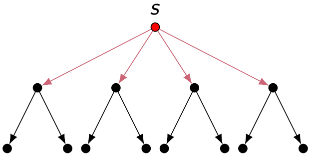

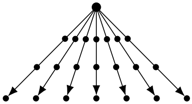

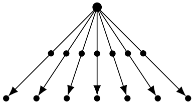

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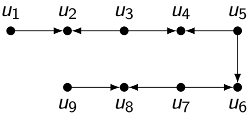

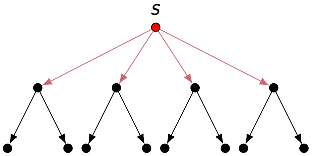

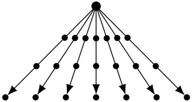

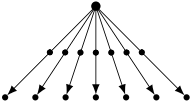

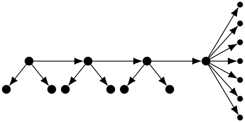

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Let H be a digraph that consists of the union of at most k directed paths or stars. Then deciding if a digraph D on n vertices and directed treewidth dtw contains H as a subdigraph can be solved in time $n^{\mathcal{O}(k+\mathsf{dtw})}$.

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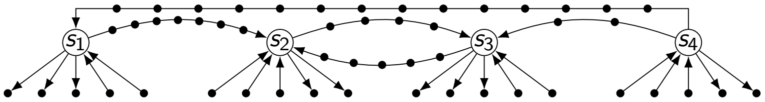

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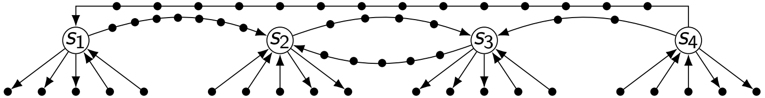

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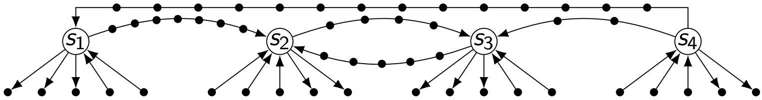

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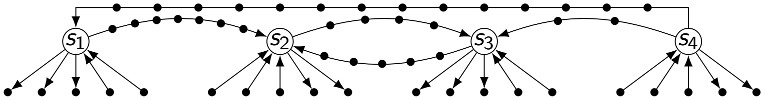

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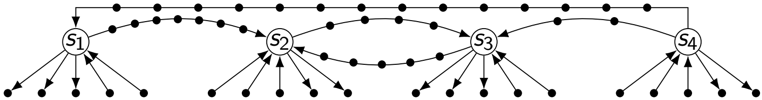

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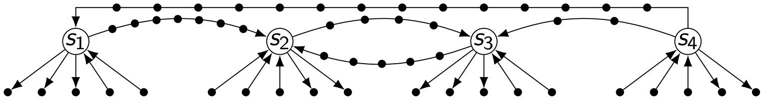

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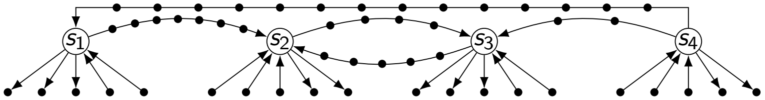

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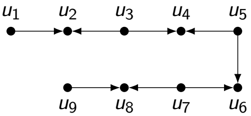
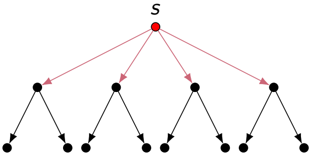
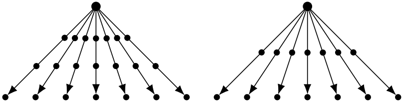
- Finally, we solve a generalized version of Directed k-Disjoint Paths where, in addition to the pairs of terminals, we are given, for each terminal s_i a set $X_{s_i} \subseteq N(s_i)$, with $|X_{s_i}| =$ allowed-for-stars(s_i), and we want to route the paths avoiding these sets.

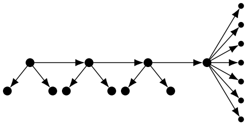
Our techniques: hardness results

We present several (more or less involved) NP-hardness reductions:









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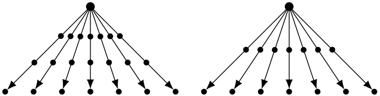

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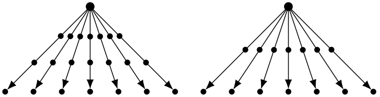

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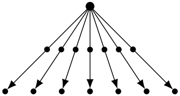

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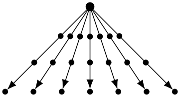

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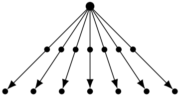

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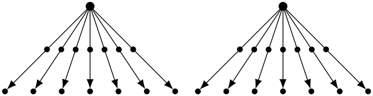

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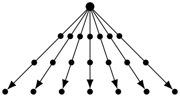

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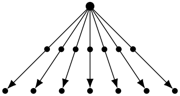

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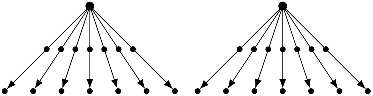

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Strong connection with notoriously open problem: Exact Matching.

Gràcies!