Optimal Erdős-Pósa property for pumpkins

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RSME 2012. *Santiago de Compostela*

Outline of the talk

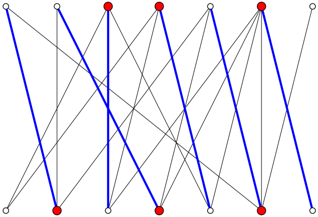
1. Motivation
2. Our result
3. Sketch of proof
4. Further research

Next section is...

Packing and covering

König's min-max theorem in bipartite graphs:

$$\textcolor{red}{\textsc{Min Vertex Cover}} = \textcolor{blue}{\textsc{Max Matching}}$$



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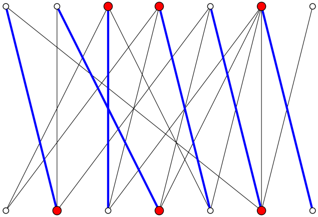

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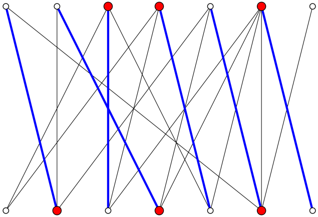

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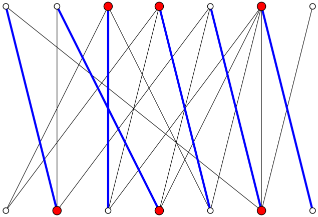

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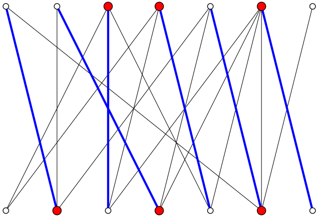

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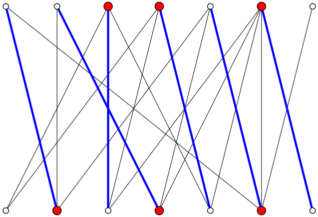

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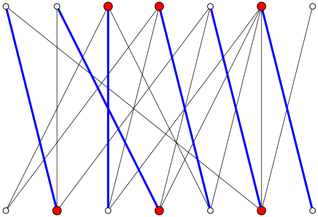

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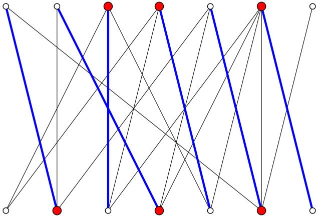

If there exists such f for all G, then $\mathcal{H}$ satisfies the **Erdős-Pósa property**.

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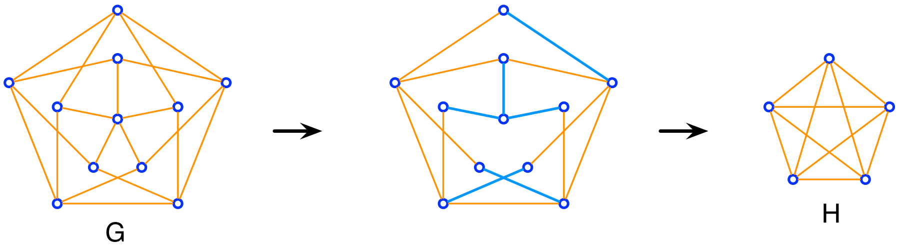
Minors and models in graphs



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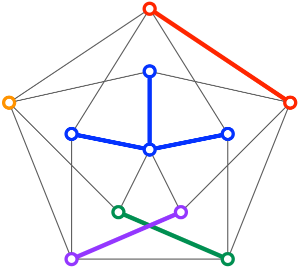
H-model in G: collection $\{S_u : u \in V(H)\}$ s.t.

- the S_u's are vertex-disjoint connected subgraphs of G, and
- there is an edge between S_u and S_v in G for every edge $uv \in E(H)$.



A K_5-model

The S_u's are called vertex images.

Packing and covering H-models

Let H be a **fixed** graph. For a graph G, we define:

$\nu_H(G)$:= packing number
$= $ max. number of vertex-disjoint H-models in G

$\tau_H(G)$:= covering number
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Clearly, $\tau_H(G) \geqslant \nu_H(G) \quad \forall G.$

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This is called the **Erdős-Pósa property of H-minors**.

Erdős-Pósa property of H-minors

Fundamental result:

$$\boxed{\tau_H(G) \leqslant f(\nu_H(G)) \quad \forall G \quad \Leftrightarrow \quad H \text{ is } \textbf{planar}}$$

[Robertson, Seymour ' 86]

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- Known upper bounds $\tau_H \leqslant f(\nu_H)$ are **huge**: $f(\nu_H) \in \Omega(2^{{\nu_H}^2})$.

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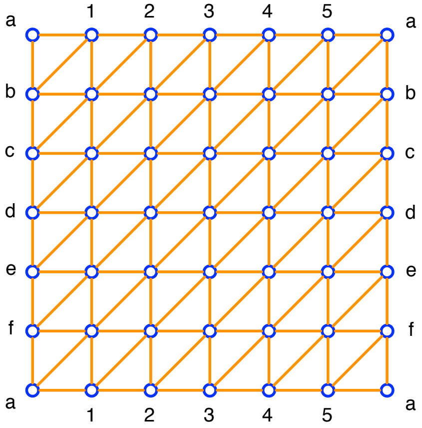
- **Natural objective**: optimize $f(\nu_H)$.

The property does NOT hold if H is not planar

$\boxed{H = K_5}$ ✗

Take a $\sqrt{n} \times \sqrt{n}$ triangulated toroidal grid G:



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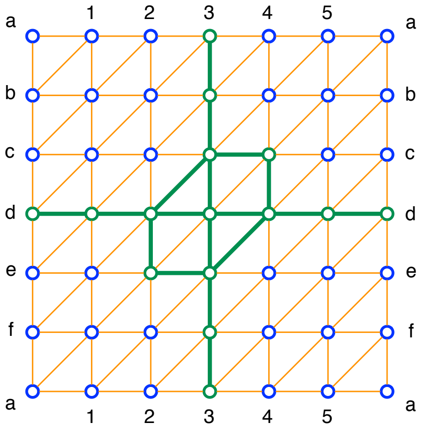

$$\nu_H(G) = 1 \quad \text{but} \quad \tau_H(G) = \Theta(\sqrt{n})$$

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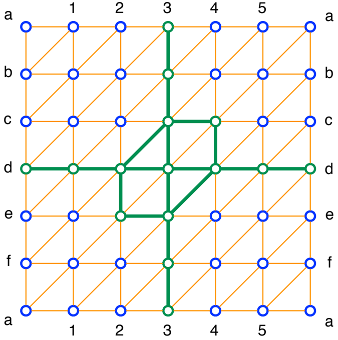
$\boxed{H = K_5 \text{ ✗}}$

$\boxed{H \text{ not planar ✗}}$



Therefore, the result of Robertson and Seymour is best possible.

Brief state of the art of Erdős-Pósa property for minors

- $\tau_H(G) \leqslant f(\nu_H(G)) \quad \forall G \quad \Leftrightarrow \quad H$ is **planar** [Robertson, Seymour ' 86]

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- $f(k) = O(k)$ when H is planar and G belongs to a minor-closed graph class (optimal). [Fomin, Saurabh, Thilikos '10]

Next section is...

Pumpkins 

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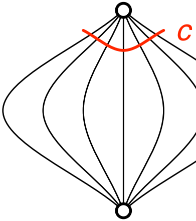
c-pumpkin:



Can be seen as a natural generalization of a cycle.

(N.B: “graph” = multigraph)

Graphs with no c-pumpkin minor

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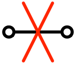
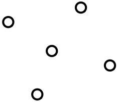
- $c = 1$: empty graphs





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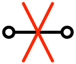

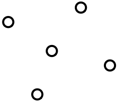

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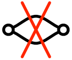

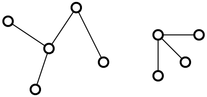

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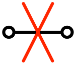
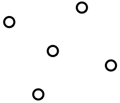

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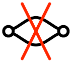
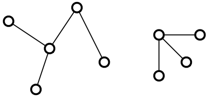
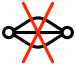
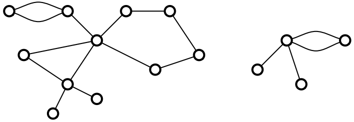



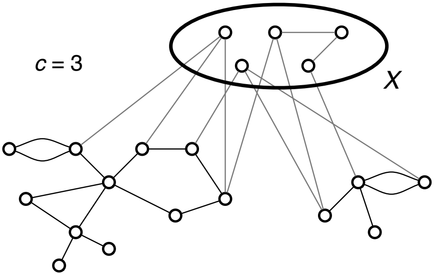
- $c = 3$: no two cycles share an edge

- etc.

Covering (or hitting) pumpkins

c-pumpkin hitting set:
vertex subset $X \subseteq V(G)$ s.t. $G - X$ has no c-pumpkin minor



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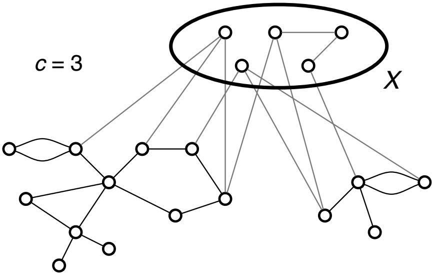
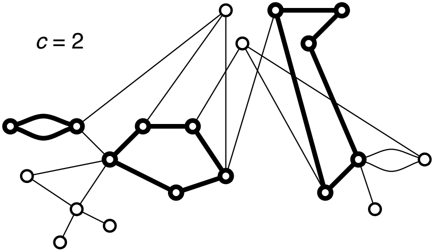


Hitting set number $\tau_c(G)$: min. size of a c-pumpkin hitting set

Packing pumpkins

c-pumpkin packing:
collection of vertex-disjoint subgraphs of G, each containing a c-pumpkin minor



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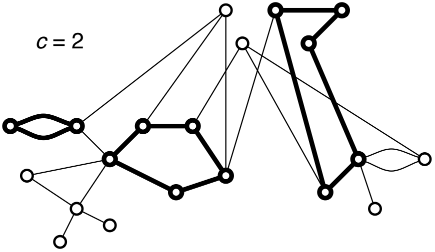

Packing number $\nu_c(G)$: max. cardinality of a c-pumpkin packing

A recent result and our main theorem

- A recent result on Erdős-Pósa property for pumpkins:

Theorem (Fomin, Lokshtanov, Misra, Philip, Saurabh '12)

For any fixed integer $c \geqslant 1$ and given an integer $k \geqslant 1$, every graph G either contains k vertex-disjoint c-pumpkins-models, or has a c-pumpkin hitting set of size at most $f(k) = O(k^2)$.

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⋆ Their proof uses tree decompositions and brambles.

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⋆ Our proof follows and generalizes Erdős-Pósa's proof for the case $c = 2$.

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Theorem: $\exists f(k)$ s.t. $\forall G, k$, either $\nu_c(G) \geqslant k$ or $\tau_c(G) \leqslant f(k)$.

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- Any c-pumpkin-minor-free graph H satisfies $\mathrm{tw}(H) \leqslant d$ for some constant d, as the c-pumpkin is planar. [Robertson, Seymour '86]
- Thus $\mathrm{tw}(G - X) \leqslant d$ for any c-pumpkin hitting set X, and therefore $\tau_c(G) = \Omega(n)$.

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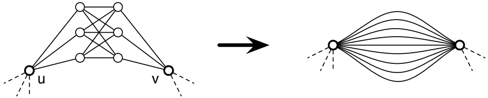
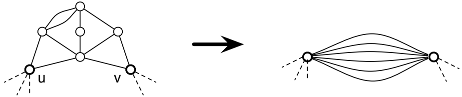
- On the other hand, every subgraph H of G containing a c-pumpkin-model has a cycle, so $V(H) = O(\log n)$, and therefore $\nu_c(G) = O(n/\log n)$.
- This implies that (easy to check) $\exists$ constant $b > 0$ such that $f(k) > b \cdot k \log k$ (i.e., $f(k) = \Omega(k \log k)$).

Next section is...

Useful reduction rules

We first need two reduction rules **R1** and **R2** dealing with 1-connected and 2-connected components without c-pumpkin minors, respectively, that preserve both $\nu_c(G)$ and $\tau_c(G)$:





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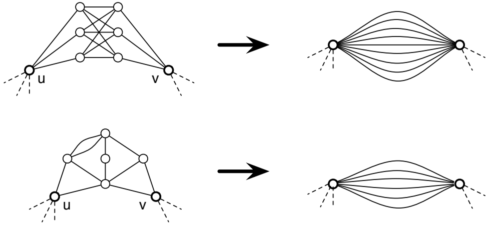

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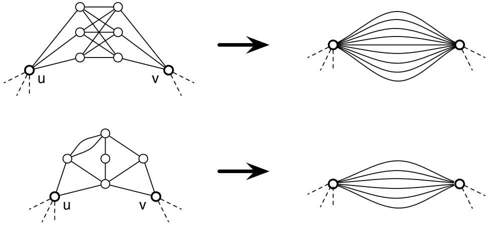

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Lemma

Let $c \geqslant 2$ be a fixed integer. Suppose that G^* results from the application of **R1** *or* **R2** *on a graph G. Then $\tau_c(G) = \tau_c(G^*)$ and $\nu_c(G) = \nu_c(G^*)$.*

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- A graph is c-reduced if rules **R1** or **R2** cannot be applied anymore.

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Main Lemma

If $|V(\overline{H})| \geqslant d \cdot k \log k$ for some constant d (depending only on c), then H contains k vertex-disjoint c-pumpkin-models.

Ingredients in the proof of the Main Lemma

- We prove it by induction on k, using that:

Lemma

Every n-vertex c-reduced graph G contains a c-pumpkin-model of size $O(\log n)$.

(Generalization of: If $\delta(G) \geqslant 3$, then $\mathrm{girth}(G) < 2 \log n$) [Alon, Hoory, Linial '02]

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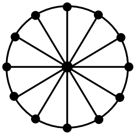
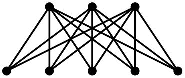
- Crucial: $\forall\ p \geqslant 0$, $\exists\ f(p)$ s.t. every 3-connected graph with $\geqslant f(p)$ vertices has a minor isomorphic to: [Oporowski, Oxley, Thomas '93]

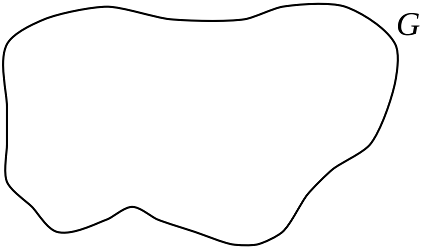




(Note that for $p \geqslant c$, both W_p and $K_{3,p}$ contain the c-pumpkin as a minor)

Outline of the overall proof

- Given G,



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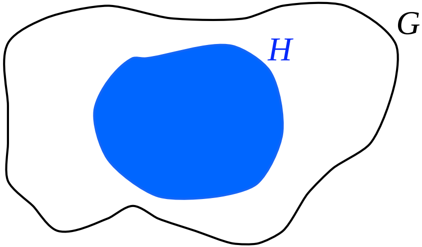

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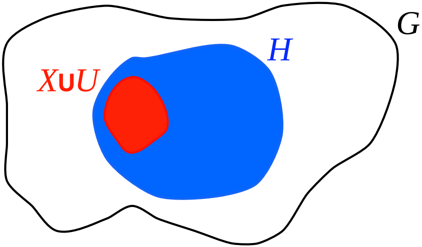

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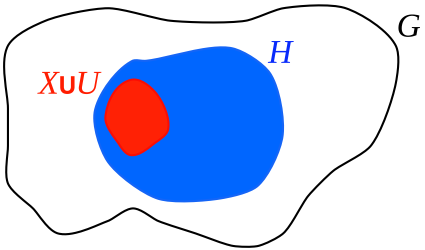

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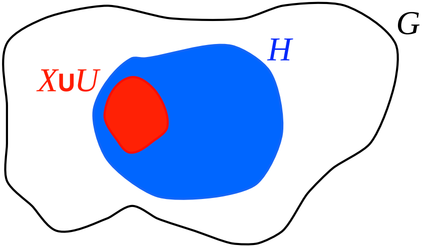

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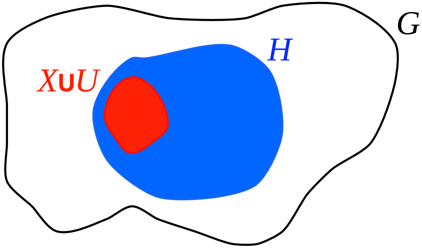

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- This follows from the Main Lemma applied to the graph $\overline{H}$.

Next section is...

Further research

Main open problem: H non-acyclic planar, $f_H(k) = O(??)$

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Approximation algorithms

Goal: Given a graph G, finding

- a c-pumpkin packing $\mathcal{M}$ and
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  s.t. $|X| \leqslant f(c, n) \cdot |\mathcal{M}|$ for some approximation ratio $f(c, n)$

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★ constant-factor (**deterministic**) approximation for the hitting version? (so far, such an algorithm is only known for $c \leqslant 3$) [Fiorini, Joret, Pietropaoli '10]

Gràcies!!