Designing Hypergraph Layouts to GMPLS Routing Strategies

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Outline

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1. Introduction
   - Concepts and Motivations
   - The Label Stack
2. Model
3. Hardness Results
4. Approximation Algorithms
5. The Case of the Path
6. Conclusions

Concepts in brief... GMPLS/AOPS

About GMPLS switching...

- GMPLS = Generic MultiProtocol Label Switching.
- G/MPLS is a tag-switching technology (packet-based networks).
- Each packet is tagged (labeled), so it can be associated and treated as a single flow.
- Packet forwarding is based on the content of the label solely.

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About AOPS...

All-Optical Packet Switching (AOPS) is an all-optical hardware implementation of GMPLS switching for packet forwarding.

1. Label processing and packet forwarding decisions are all performed completely optically
2. No need for OEO regeneration... faster packet forwarding.
3. *One 'decoding' device is needed for each used label at each node*
4. Few labels...

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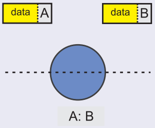

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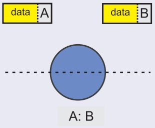

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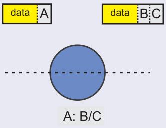

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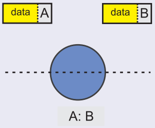

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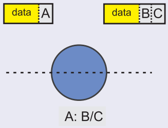

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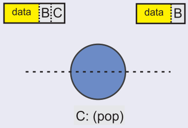

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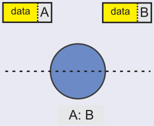

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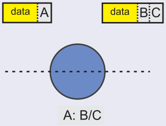

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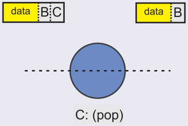

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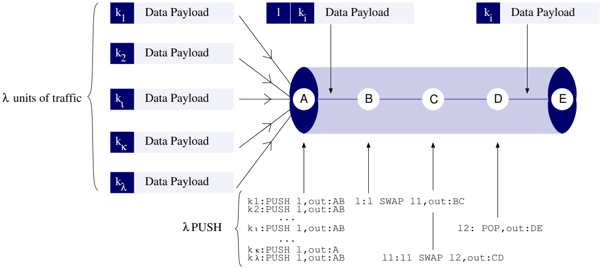
... but solely the top one can be processed!

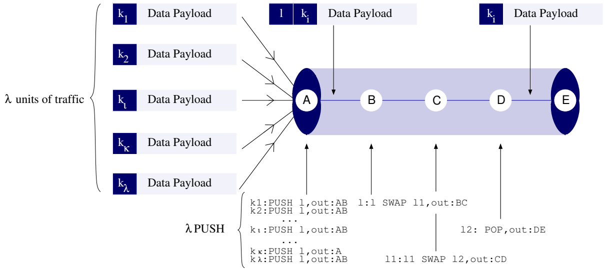
Label Stacking & Tunnels



By pushing a label, we can create a higher hierarchy LSP (Label Switched Path), called TUNNEL.

- Using a stack size of 2, the number of used labels can be decreased
- The larger the stack, fewer labels are needed...

Label Stacking & Tunnels



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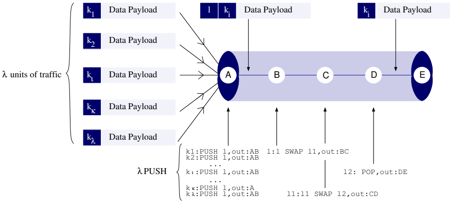
The problem we tackle is the minimization of the total number of labels using tunnels.

Outline

Some notation

- $G = (V, E)$ is the underlying digraph (which can be symmetric or not).
- $|V| = n$, and vertices are numbered $1, \ldots, n$.
- r_{ij} is the request from $i \in V$ to $j \in V$, with multiplicity m_{ij}. R is the set of all requests.
- $P(G)$ is the set of all simple dipaths in G.
- t stands for a tunnel, and T is the set of tunnels, that is $t \in T \subseteq P(G)$.
- ℓ is a length function on the arcs, that is $\ell : E \rightarrow \mathbb{R}^+$.
- for a tunnel t, $\ell(t) = \sum_{e \in t} \ell(e)$ is its length and $w(t)$ is the amount of traffic it carries.

Cost of a tunnel = number of labels



Let t be a tunnel.

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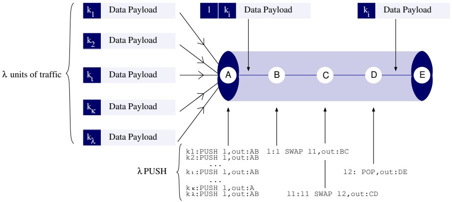


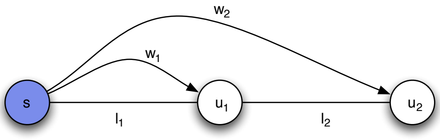
Let t be a tunnel. Its cost is defined as

$$c(t) = w(t) + (\ell(t) - 1).$$

- $w(t)$ is the number of forwarded traffic units (LSPs)
- $\ell(t)$ is the length of the tunnel (usually, the number of hops)

Cost Example - Scenario

We consider the case of a line network with one source and multiple destinations...

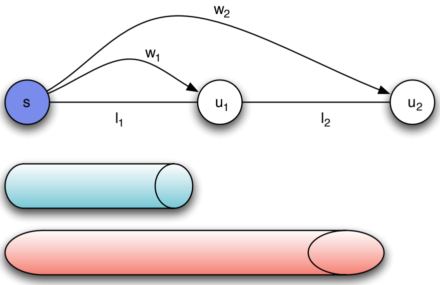


With no tunnels we need $l_1 \cdot (w_1 + w_2) + l_2 \cdot w_2$ labels...

With tunnels?? The optimal solution depends on the values of l_i and w_i...

Cost Example - Solutions

First Solution...



$$c(T_{(s,u_1)}) = w_1 + l_1 - 1$$
$$c(T_{(s,u_2)}) = w_2 + (l_1 + l_2) - 1$$

Total cost is: $w_1 + w_2 + 2l_1 + l_2 - 2$

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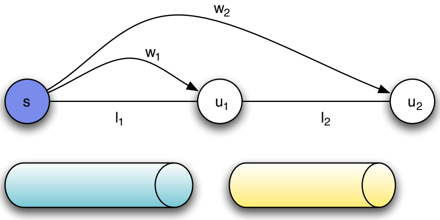
Second Solution...



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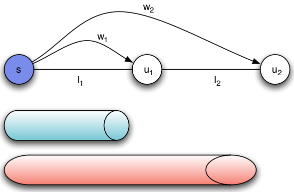
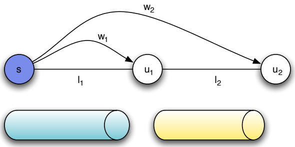
Cost Example - Solutions

Which one is the best solution?



If $l_1 \leq w_2$



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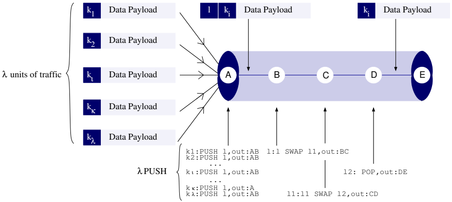
Cost of a tunnel = number of labels (II)



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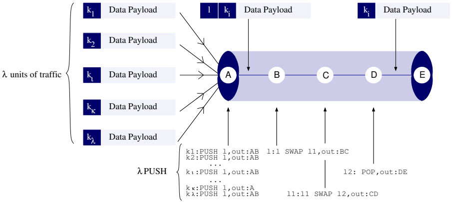

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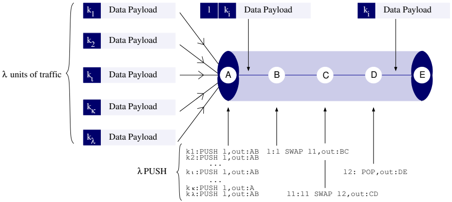

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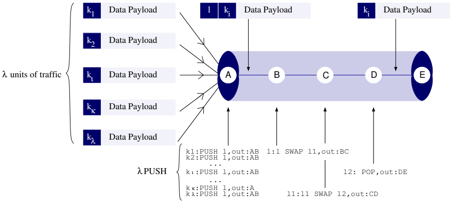

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This observation naturally leads to the definition of a **hypergraph layout**.

Hypergraph layout

Definition (Hypergraph layout)

Given a graph G and a set $T \subseteq P(G)$ of dipaths, the associated hypergraph layout $H(T)$ is the directed hypergraph with $V(H(T)) = V(G)$, and where for each tunnel $t \in T \subseteq P(G)$ there is a directed hyperarc in $H(T)$ connecting any vertex of t to the end of t.

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- The problem can then be simply expressed as finding a **feasible hypergraph layout of minimum cost**.

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Since $L(r_{ij}) \geq d_H(i,j)$, we conclude that **in an optimal solution the routing necessarily uses shortest dipaths in the hypergraph layout**.

Statement of the problem

It follows that the cost function can be rewritten w.l.o.g. as

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MINIMUM COST HYPERGRAPH LAYOUT

- Input: A digraph $G = (V, E)$ with a length function $\ell : E \to \mathbb{R}^+$, and a set R of traffic requests.
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If G is a **symmetric** digraph, the problem is denoted MINIMUM COST SYMMETRIC HYPERGRAPH LAYOUT.

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General (directed) network

Theorem

The Minimum Cost Hypergraph Layout *problem cannot be approximated within a factor* $C \log n$ *for some constant* $C > 0$*, even if the instance is a partial broadcast, unless* $\mathrm{P} = \mathrm{NP}$.

[1] Given a finite set $\mathcal{S}$ and a collection $\mathcal{C}$ of subsets of $\mathcal{S}$, the aim is to find a subcollection $\mathcal{C}'$ of $\mathcal{C}$ of minimum cardinality that covers all the elements of $\mathcal{S}$.

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- The reduction is from MINIMUM SET COVER[1].
- MINIMUM SET COVER is not approximable within a factor $C \log n$, for some constant $C > 0$, unless $\mathrm{P} = \mathrm{NP}$.
  [Raz and Safra, STOC 1997]

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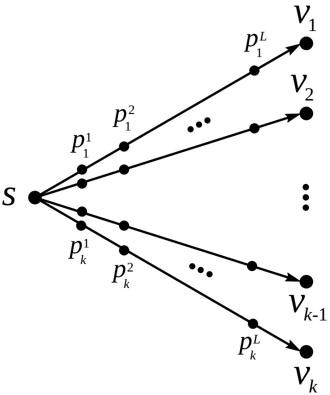
Idea of the reduction

To a SET COVER instance with sets $S_1, S_2, \ldots, S_k$, with $S_i \subseteq \{a_1, a_2, \ldots, a_n\}$, we associate the following graph:

- We start with a distinguished node s.

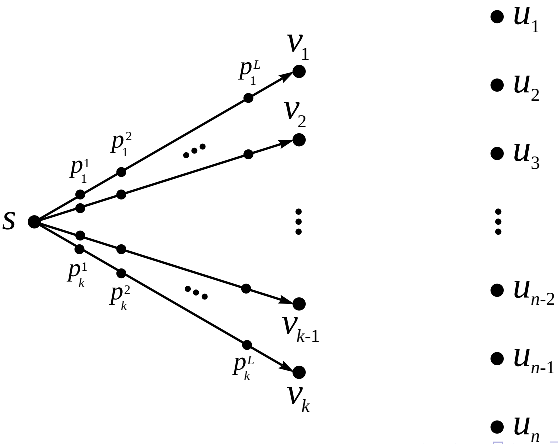
Idea of the reduction (II)

- For each set S_i we introduce a node v_i and a directed path of length $L+1$ (L is a big constant) from s to v_i through L new vertices $p_i^1, p_i^2, \ldots, p_i^L$.



Idea of the reduction (III)

- For each element a_j we introduce a vertex u_j and, for each vertex v_i we add the arcs (v_i, u_j) if $a_j \in S_i$.



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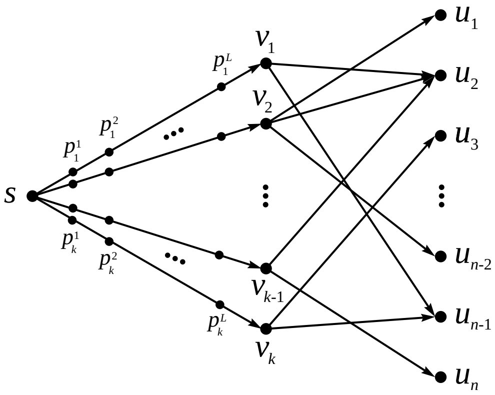

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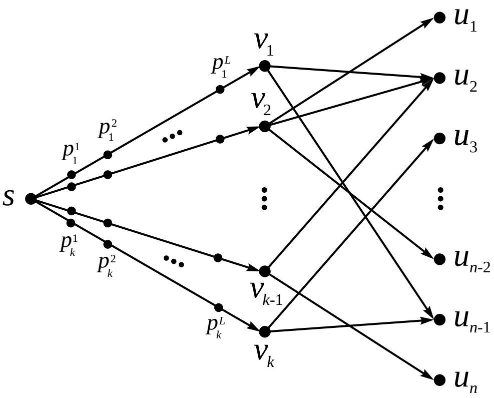

- The requests are from s to u_j, for $i = 1, \ldots, n$.

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Symmetric network

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The MINIMUM COST SYMMETRIC HYPERGRAPH LAYOUT *problem is* APX*-hard even if the instance is a partial broadcast. Therefore, it does not accept a* PTAS *unless* P=NP*.*

[2] Given an edge-weighted graph $G = (V, E)$ and a subset $S \subseteq V$, find a connected subgraph with minimum edge-weight containing all the vertices in S.

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- The reduction is from MINIMUM STEINER TREE[2].
- MINIMUM STEINER TREE is APX-hard, hence it does not accept a PTAS unless P = NP.
  [Bern and Plassmann, IPL 1989]

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Outline

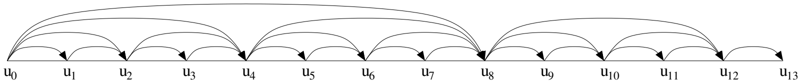
Case of the path

Proposition

When the network is a path, there exists a polynomial-time approximation algorithm for the Minimum Cost Hypergraph Layout *problem with an approximation ratio* $\mathcal{O}(\log n)$.

Idea: 2^k-hops long tunnels

We consider tunnels of length a power of 2.



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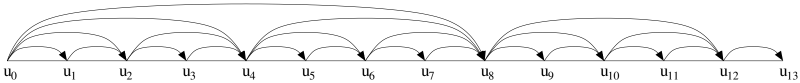

.... then, we route the demands using shortest paths in this layout.

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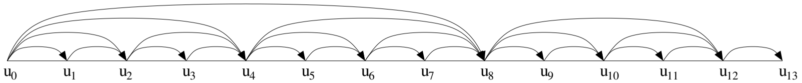

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In this layout:

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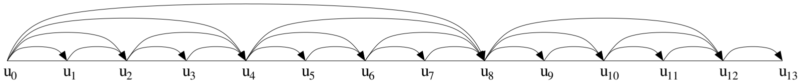

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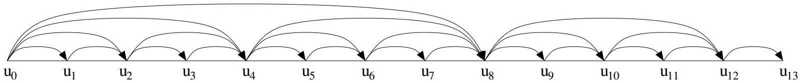

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$\Rightarrow$ $\mathcal{O}(\log n)$-approximation algorithm.

Outline

Case of the tree

Theorem (Bermond, Marlin, Peleg, and Pérennes, TCS 2003)

In a general tree on n nodes with all-to-all traffic, for each value of $c \in \{1, \ldots, n\}$ there exists a virtual layout allowing to route all traffic with **diameter** *at most $10c \cdot n^{\frac{1}{2c-1}}$ and* **load** *at most c. In addition, such a layout can be constructed in polynomial time.*

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Proposition

When the network is a tree, there exists a polynomial-time approximation algorithm for Minimum Cost Hypergraph Layout *problem with an approximation ratio $\mathcal{O}(\log n)$.*

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General graph

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- The we apply the algorithm for the tree to each connected component of $H \Rightarrow$ overall approximation factor $\mathcal{O}(\log n)$.

The case of the path with bounded number of sources

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- Here we extended the dynamic programming algorithm for any fixed number k of sources, with a running time of $n^{\mathcal{O}(k)}$.

Conclusions

- We modeled a problem raised by label minimization in GMPLS networks as a hypergraph layout problem.
- The problem seems closely related to classical VPL problems.
- We provided hardness results and approximations algorithms.
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- Does the problem remain NP-hard if the **routes** to be followed by the requests are part of the **input**?

Thanks!

Questions?