On approximating the d-girth of a graph

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Outline

1. Introduction
2. Summary of results
3. A simple approximation algorithm
4. A hardness result for planar graphs
5. Conclusions and further research

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Preliminaries

- Given a *minimization* problem Π, ALG is an α-approximation algorithm for Π (with $\alpha \geq 1$) if for any instance *I* of Π,

$$ALG(I) \leq \alpha \cdot OPT(I).$$

- Class APX (Approximable):

  an NP-hard optimization problem is in APX if it can be approximated within a constant factor.

  Example: MINIMUM VERTEX COVER

- Class PTAS (Polynomial-Time Approximation Scheme):

  an NP-hard optimization problem is in PTAS if it can be approximated within a constant factor $1+\varepsilon$, for all $\varepsilon > 0$.

  Example: MINIMUM VERTEX COVER in planar graphs

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  Example: MINIMUM VERTEX COVER in planar graphs

Let $d \geq 1$ be a fixed positive integer.

The d-GIRTH **problem**

Input: A simple undirected graph $G = (V, E)$.

Output: A subset $S \subseteq V$ such that $\delta(G[S]) \geq d$.

Objective: Minimize $|S|$.

- The optimum of the above problem is called the d-girth of G.
- For $d = 1$, the vertices of any edge are a trivial solution of size 2.
- For $d = 2$, it is exactly the GIRTH problem (find the length of a shortest cycle), which is in P.

  [Indeed, every graph H with $\delta(H) \geq 2$ contains a cycle.]

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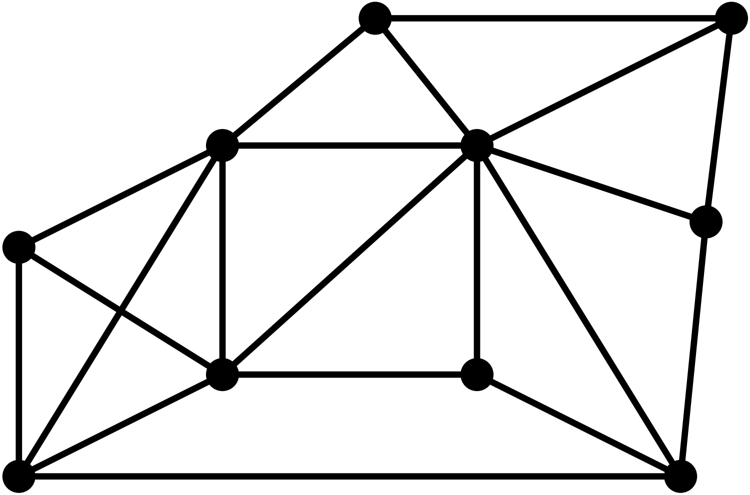

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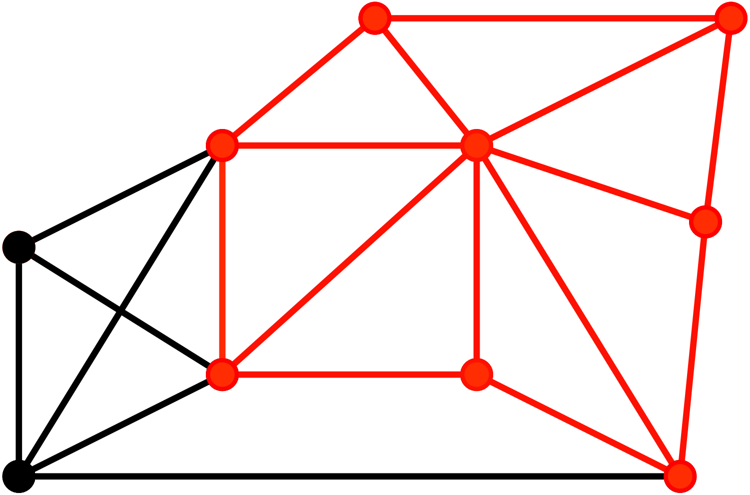

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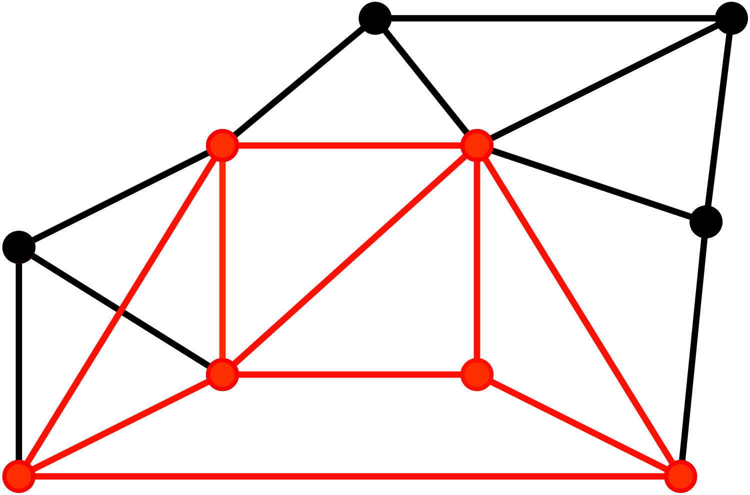

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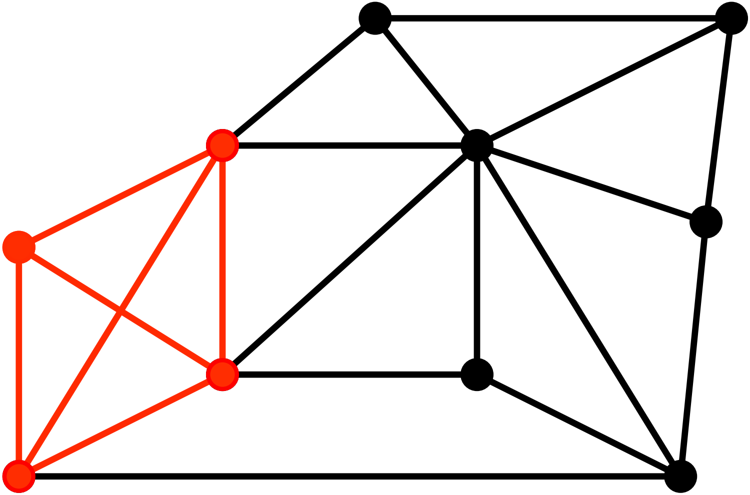

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The notion of *d*-girth was studied more than 20 years ago, from a purely combinatorial point of view:

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Results of the following type:

Theorem (Erdős, Faudree, Rousseau, Schelp'90)

Let $d \geq 2$ and $k > 1$ be given. Every n-vertex graph G with at least $\lceil d \cdot k \cdot n \rceil$ edges has a subgraph H with $\delta(H) \geq d$ and $|V(H)| \leq \lceil n/k \rceil$.

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Recently, first algorithmic studies of the problem:

[Amini, S., Saurabh. IWPEC'08]

- ⋆ $W[1]$-hard, taking as parameter the size of the solution.
- ⋆ FPT algorithms when the input graph has bounded local tree-width or excludes a fixed graph as a minor.

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- ⋆ The d-GIRTH problem is not in APX for any fixed $d \geq 3$.
- ⋆ $n/\log n$-approximation algorithm for minor-free graphs, using DP and a known structural result about minor-free graphs.
- ⋆ **Missing**: approximation algorithms in general graphs, and hardness results for sparse graphs.

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close relation with DENSE k-SUBGRAPH problem and TRAFFIC GROOMING problem in optical networks.

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⋆ Improved hardness results in general graphs:

for any $d \geq 3$ and any $\varepsilon > 0$, there is no polynomial-time algorithm for the d-GIRTH problem with approximation ratio $2^{\mathcal{O}(\log^{1-\varepsilon} n)}$ unless $\mathrm{NP} \subseteq \mathrm{DTIME}\left(2^{\mathcal{O}(\log^{1/\varepsilon} n)}\right)$.

[These hardness results hold even in graphs with degrees d or $d+1$]

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Algorithmic results

- General graphs:
  - ⋆ First approximation algorithm for the d-GIRTH problem: randomized algorithm with approximation ratio $n/\log n$.

    [The performance is not far from the best approximation algorithms for other very hard graph optimization problems like MAXIMUM CLIQUE, CHROMATIC NUMBER, LONGEST PATH, ...]
  - ⋆ another randomized algorithm with better performance in high-degree graphs.
  - ⋆ a deterministic algorithm for low-degree graphs.
- Planar graphs:
  - ⋆ deterministic approximation algorithm with ratio $n/\log n$.
  - ⋆ *subexponential* exact algorithm in time $2^{O(\sqrt{n} \cdot \log n)}$.

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An approximation algorithm for general graphs

while $G \neq \emptyset$

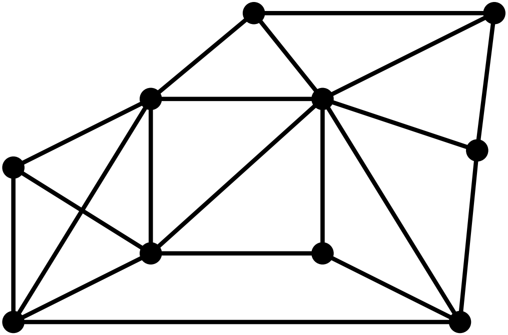
pick a vertex v uniformly at random

remove v and all its incident edges

clean: remove recursively vertices of degree less than d

update G

return the last non-empty graph



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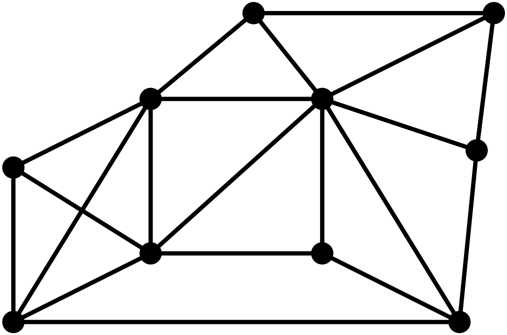

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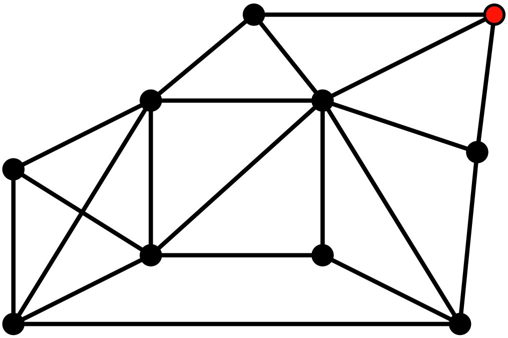

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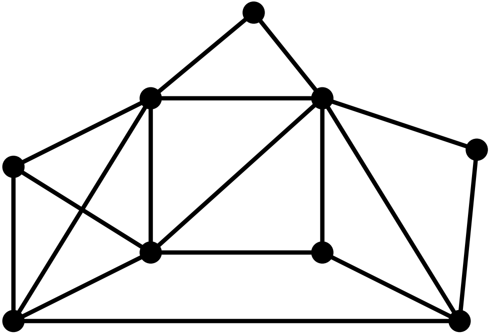

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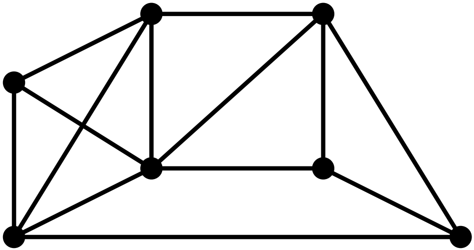

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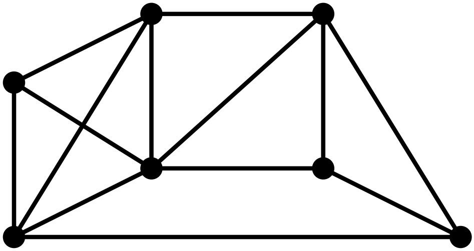

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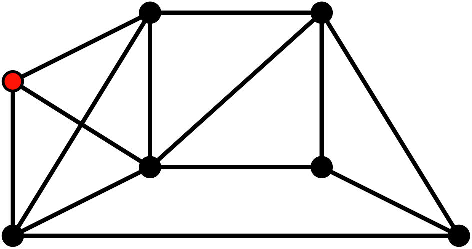

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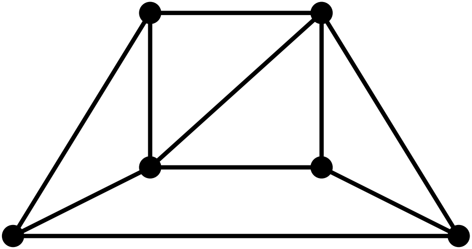

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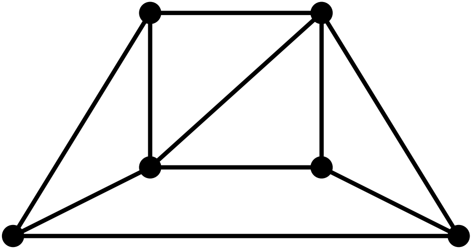

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Theorem

For any $d \geq 3$, the above procedure is w.h.p. a poly-time randomized approximation algorithm with ratio $n/\log n$ for the d-GIRTH problem.

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- As any planar graph has a vertex of degree at most 5, the d-GIRTH problem only makes sense for $d \in \{3, 4, 5\}$.

Theorem

For $d \in \{3, 4, 5\}$, the d-GIRTH problem is NP-hard in planar graphs with maximum degree at most $3d$.

- Reduction from MINIMUM VERTEX COVER in planar graphs with $\Delta \leq 3$, which is NP-hard by [Garey, Johnson. SIDMA'77].
- Given an instance H of VC, we build and instance G of d-GIRTH:

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- As any planar graph has a vertex of degree at most 5, the d-GIRTH problem only makes sense for $d \in \{3, 4, 5\}$.

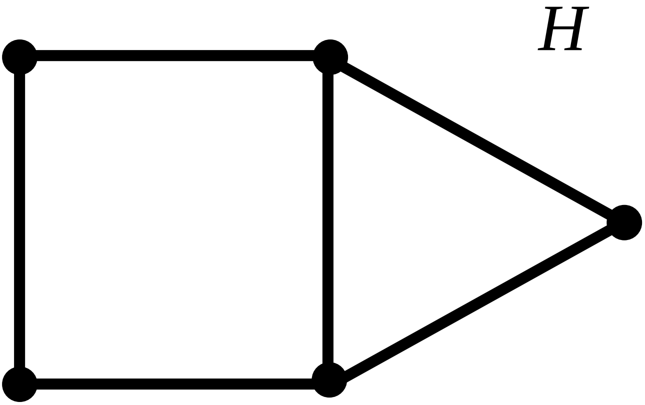
Theorem

For $d \in \{3, 4, 5\}$, the d-GIRTH problem is NP-hard in planar graphs with maximum degree at most $3d$.

- Reduction from MINIMUM VERTEX COVER in planar graphs with $\Delta \leq 3$, which is NP-hard by [Garey, Johnson. SIDMA'77].

- Given an instance H of VC, we build and instance G of d-GIRTH:

The reduction for $d = 3$



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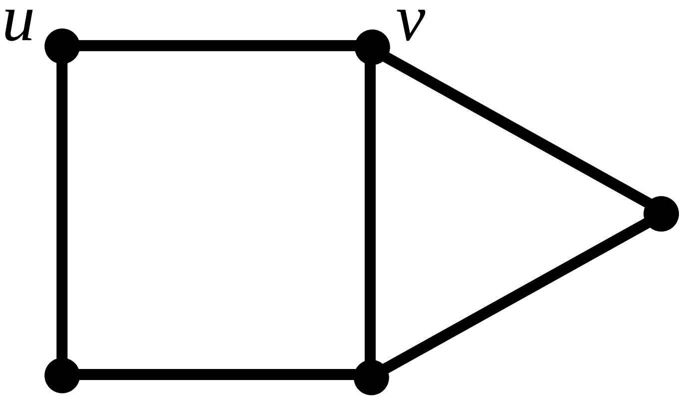

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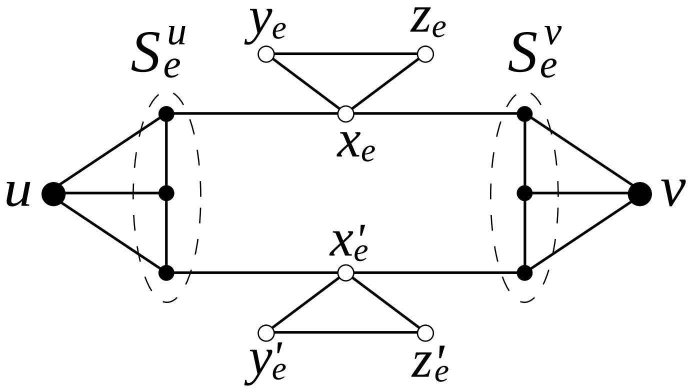

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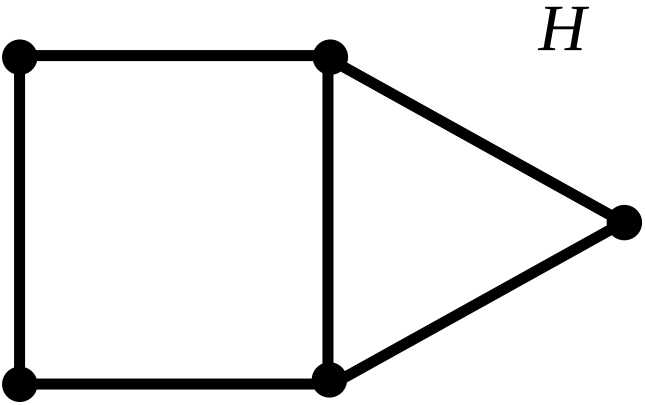

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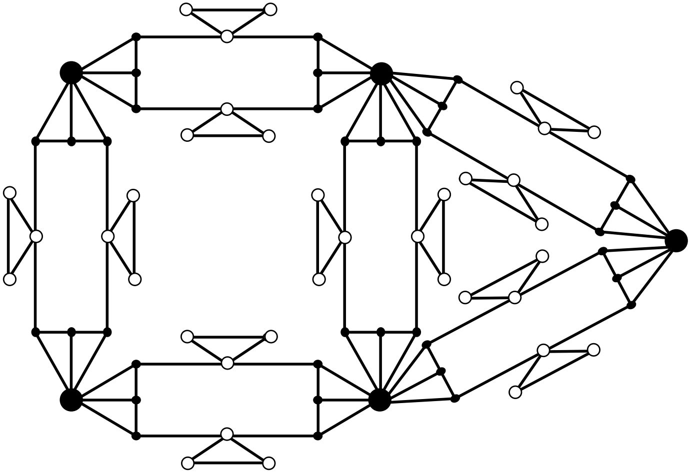

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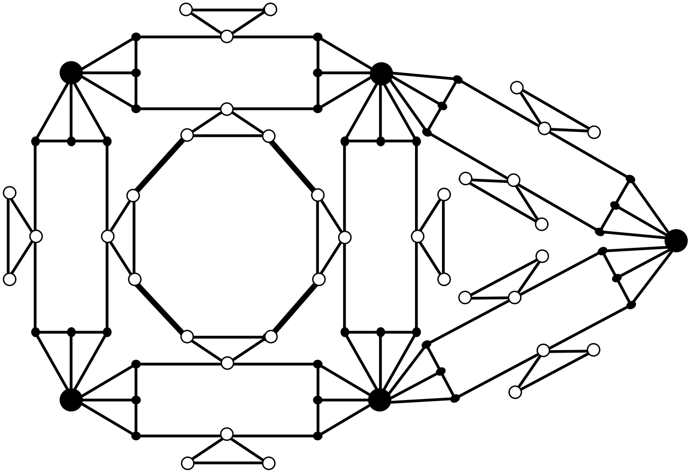

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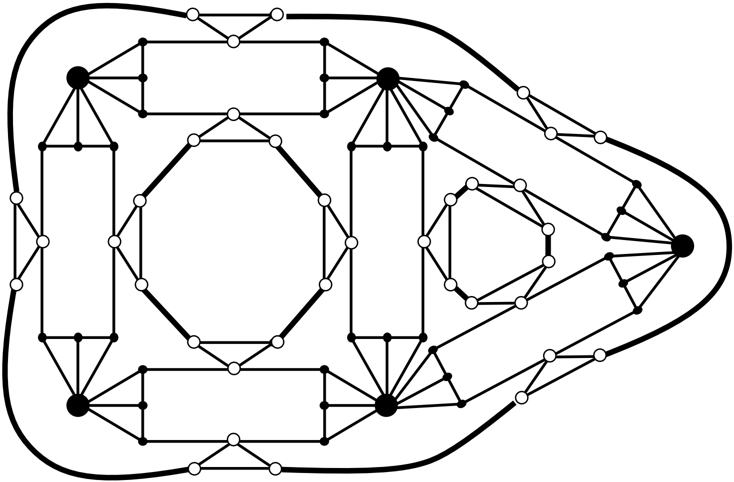

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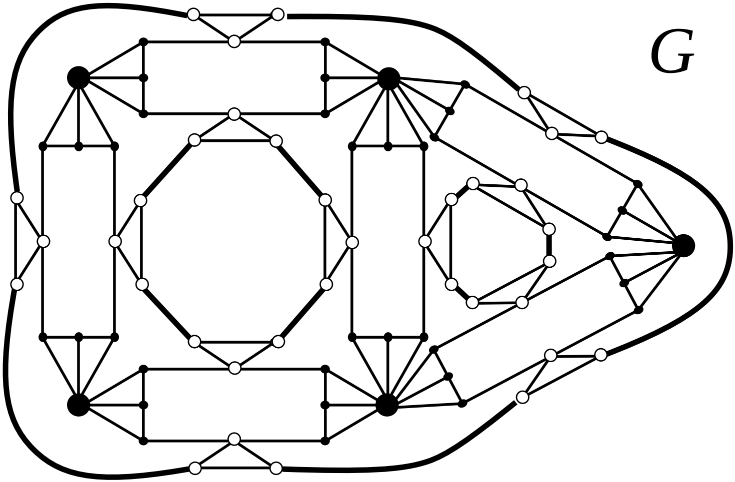

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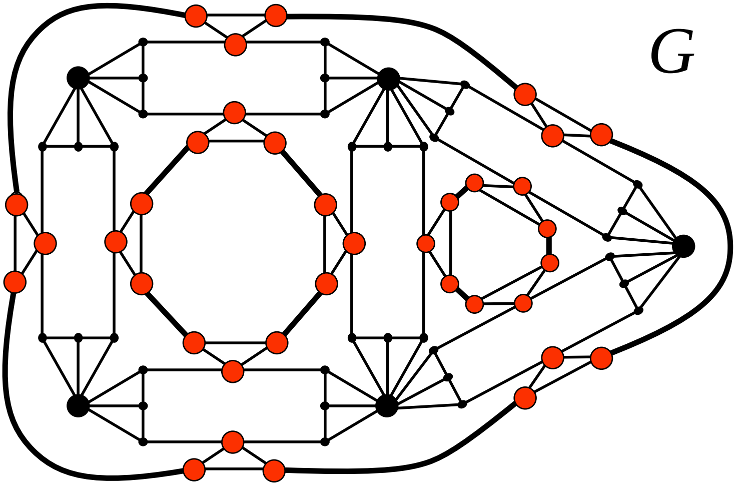

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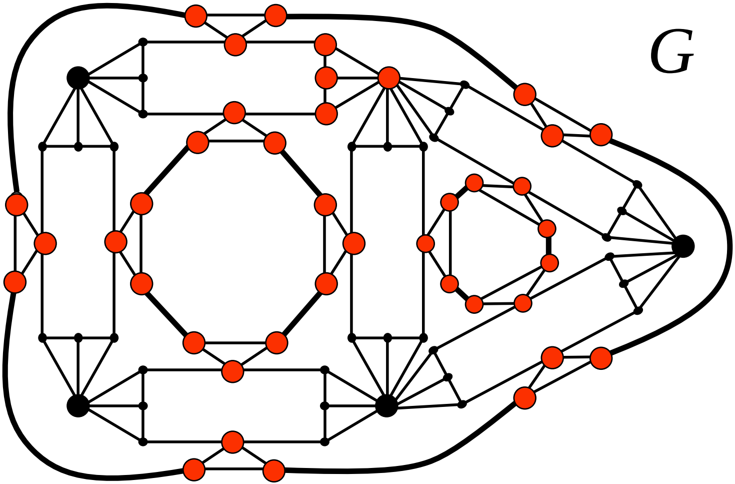

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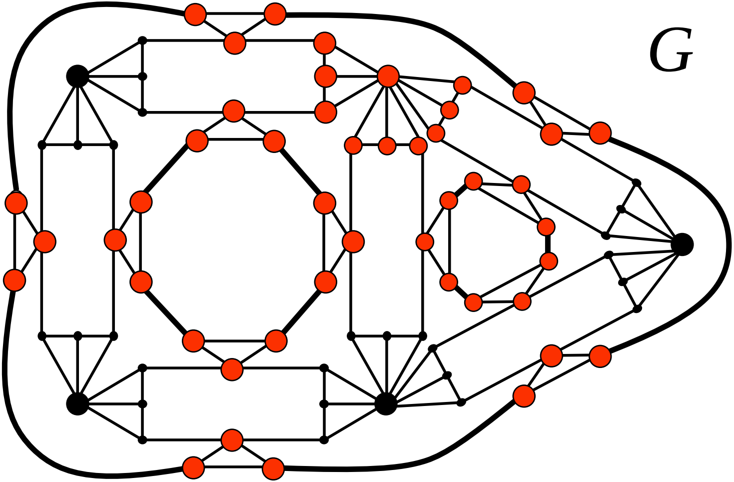

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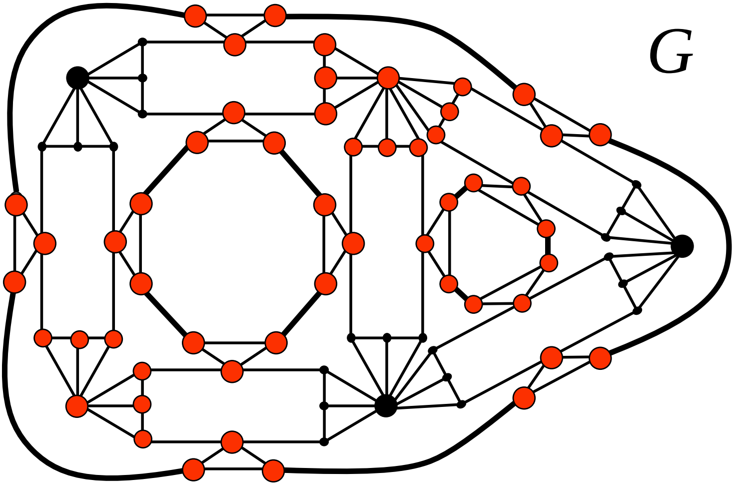

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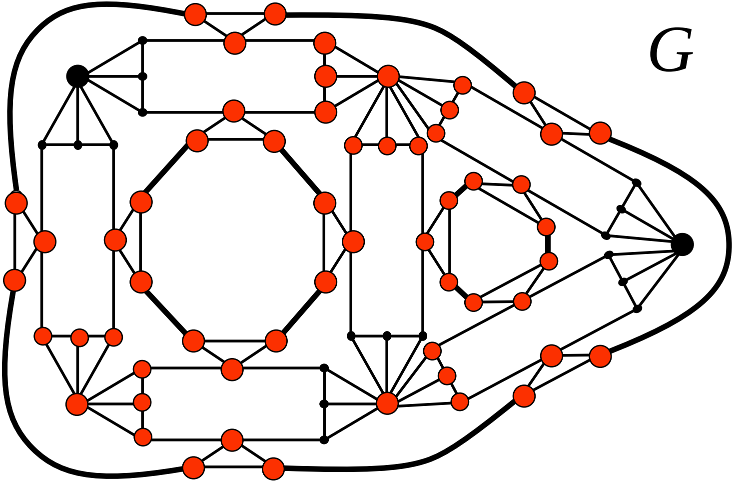

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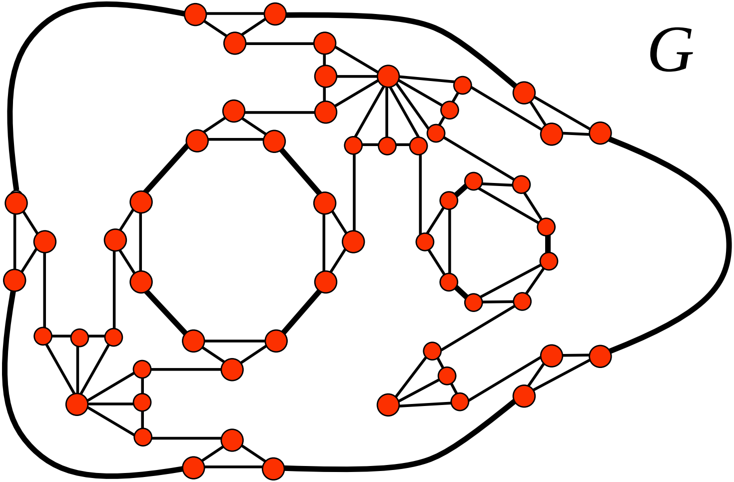

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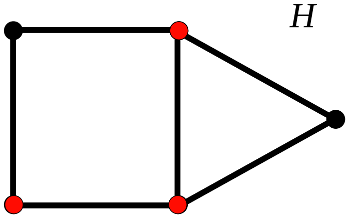
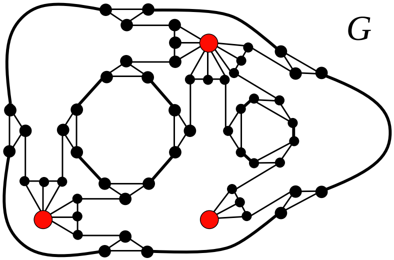
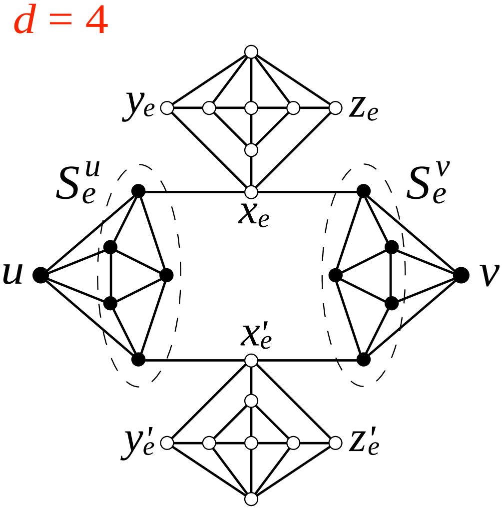




- We conclude that there is a bijection between vertex covers of H and feasible solutions to the 3-GIRTH in G.
- Therefore, $\mathrm{OPT}_{3-\mathrm{GIRTH}}(G) = 9 \cdot |E(H)| + \mathrm{OPT}_{\mathrm{VC}}(H)$.

Edge gadgets for $d = 4, 5$



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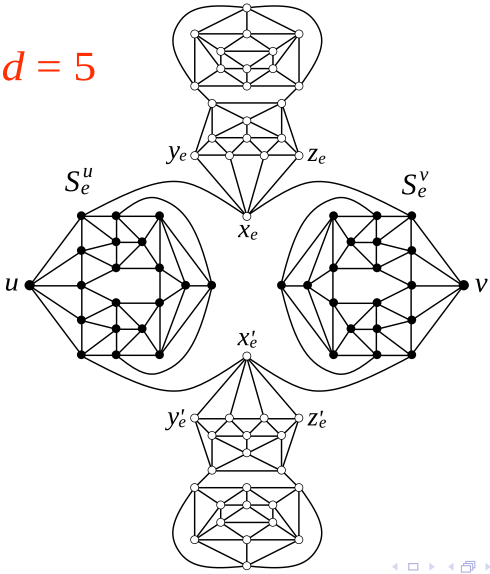

Outline

1 Introduction

2 Summary of results

3 A simple approximation algorithm

4 A hardness result for planar graphs

5 Conclusions and further research

Conclusions and further research

- ⋆ We proved that for any $d \geq 3$ and $\varepsilon > 0$, there is no poly-time algorithm for the d-GIRTH problem with ratio $2^{\mathcal{O}(\log^{1-\varepsilon} n)}$ unless $\mathrm{NP} \subseteq \mathrm{DTIME}\left(2^{\mathcal{O}(\log^{1/\varepsilon} n)}\right)$.

Conjecture

Unless $\mathrm{P} = \mathrm{NP}$, for every fixed $d \geq 3$ there is no poly-time approx. algorithm for d-GIRTH with ratio $n^{1-\delta}$, for some constant $\delta > 0$.

- ⋆ We provided the first approximation algorithms for the d-GIRTH problem in general graphs.

Improving the approximation ratio seems a challenging task.

- ⋆ We proved that the d-GIRTH problem is NP-hard in planar graphs for $d \in \{3, 4, 5\}$.

Does the problem admit a PTAS in planar graphs??

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Gràcies!!