FPT algorithm for a generalized cut problem and some applications

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Outline of the talk

1. Introduction
2. Sketch of the FPT algorithm
3. Some applications
4. Conclusions

Next section is...

Some words on parameterized complexity

- **Idea** given an NP-hard problem with input size n, fix one parameter k of the input to see whether the problem gets more "tractable".

  Example: the size of a Vertex Cover.

- Given a (NP-hard) problem with input of size n and a parameter k, a fixed-parameter tractable (FPT) algorithm runs in time

$$f(k) \cdot n^{O(1)}, \quad \text{for some function } f.$$

  Examples: k-Vertex Cover, k-Longest Path.

Many cut problems have been proved to be FPT

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- Min Cut: polynomial by classical max-flow min-cut theorem.
- Multiway Cut: FPT by using important separators. [Marx '06]
- Multicut: Finally, FPT. [Marx, Razgon + Bousquet, Daligault, Thomassé '10]
- Steiner Cut: Improved FPT algorithm by using randomized contractions. [Chitnis, Cygan, Hajiaghayi, Pilipczuk^2 '12]
- Min Bisection: Finally, FPT. [Cygan, Lokshtanov, Pilipczuk^2, Saurabh '13]

We introduce a new cut problem

- A new cut problem: LIST ALLOCATION (to be defined in two slides).

Theorem

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The LIST ALLOCATION *problem is* FPT.

- LIST ALLOCATION generalizes, in particular, MULTIWAY CUT.
- General enough so that several other problems can be reduced to it:
  - ⋆ FPT algorithm for a parameterization of DIGRAPH HOMOMORPHISM.
  - ⋆ FPT algorithm for the MIN-MAX GRAPH PARTITIONING problem.
  - ⋆ FPT 2-approximation for TREE-CUT WIDTH.

Before defining the problem: allocations

- An r-allocation of a set S is an r-tuple $\mathcal{V} = (V_1, \ldots, V_r)$ of possibly empty pairwise disjoint subsets of S whose union is S.
- Elements of $\mathcal{V}$: parts of $\mathcal{V}$.
- We denote by $\mathcal{V}^{(i)}$ the i-th part of $\mathcal{V}$, i.e., $\mathcal{V}^{(i)} = V_i$.

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- We denote by $\mathcal{V}^{(i)}$ the i-th part of $\mathcal{V}$, i.e., $\mathcal{V}^{(i)} = V_i$.
- Let $G = (V, E)$ be a graph and let $\mathcal{V}$ be an r-allocation of V:

  $|\delta(\mathcal{V}^{(i)}, \mathcal{V}^{(j)})|$: #edges in G with one endpoint in $\mathcal{V}^{(i)}$ and one in $\mathcal{V}^{(j)}$.

Definition of the problem: List Allocation

List Allocation

Input: A tuple $I = (G, r, \lambda, \alpha)$, where G is an n-vertex graph, $r \in \mathbb{Z}_{\geqslant 1}$, $\lambda : V(G) \to 2^{[r]}$, and $\alpha : \binom{[r]}{2} \to \mathbb{Z}_{\geqslant 0}$.

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Parameter: $k = \sum \alpha$.

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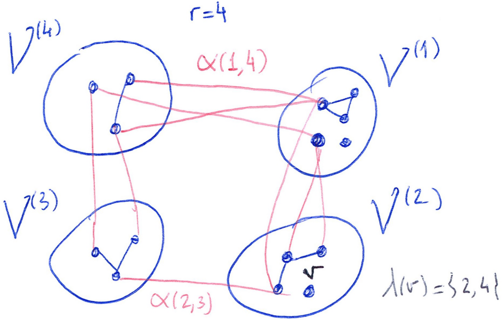
Parameter: $k = \sum \alpha$.

Question: Decide whether there exists an r-allocation $\mathcal{V}$ of $V(G)$ s.t.

- $\forall \{i, j\} \in \binom{[r]}{2}$, $|\delta(\mathcal{V}^{(i)}, \mathcal{V}^{(j)})| = \alpha(i, j)$ and
- $\forall v \in V(G)$, if $v \in \mathcal{V}^{(i)}$ then $i \in \lambda(v)$.



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High-level ideas of the FPT algorithm

- We use a series of FPT reductions:

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Problem $A \xrightarrow{\text{FPT}}$ Problem B: If problem B is FPT, then problem A is FPT.

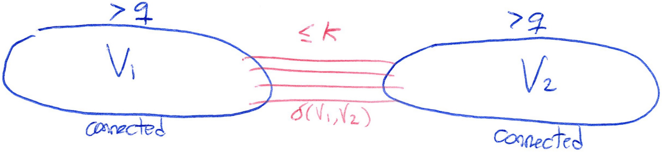
- At some steps, we obtain instances whose size is bounded by some function $f(k)$.
- Then we will use that the List Allocation problem is in XP:

Lemma

There exists an algorithm that, given an instance $I = (G, r, \lambda, \alpha)$ of List Allocation*, computes all possible solutions in time $n^{O(k)} \cdot r^{O(k+\ell)}$, where ℓ is the number of connected components of G.*

Some preliminaries

- Let G be a connected graph. A partition (V_1, V_2) of $V(G)$ is a (q,k)-separation if $|V_1|, |V_2| > q$, $|\delta(V_1, V_2)| \leqslant k$, and $G[V_1]$ and $G[V_2]$ are both connected.



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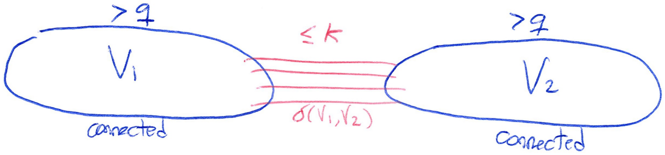

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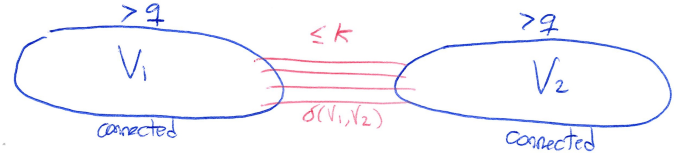

- A graph G is (q,k)-connected if it does not contain any $(q, k-1)$-separation.

Lemma (Chitnis, Cygan, Hajiaghayi, Pilipczuk^2 '12)

There exists an algorithm that given a n-vertex connected graph G and two integers q, k, either finds a (q,k)-separation, or reports that no such separation exists, in time $(q+k)^{O(\min\{q,k\})} n^3 \log n$.

Series of FPT reductions

List Allocation (LA)

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$\downarrow$ FPT

Connected List Allocation (CLA)

Same input + graph G is connected and $r \leqslant 2k$

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Claim (Unique big part)

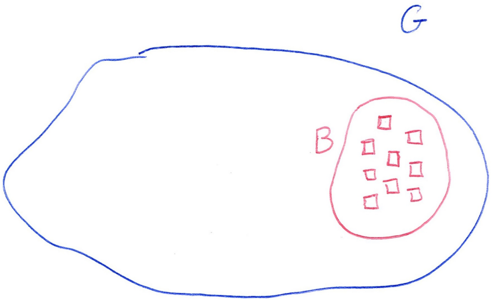
For any solution $\mathcal{V}$ of HCLA there exists a unique index $j \in [r]$ such that

$$\sum_{i \in [r] \setminus j} |\mathcal{V}^{(i)}| \leqslant k \cdot f_1(k).$$

- Part $\mathcal{V}^{(j)}$ is called the big part.

Reduction from CLA to HCLA: we shrink the graph

- We apply to G the following recursive algorithm `shrink`, which receives a graph G and a boundary set B with $|B| \leqslant 2k$ (start with $B = \emptyset$):



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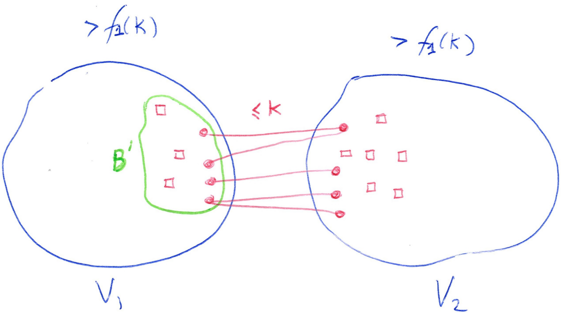

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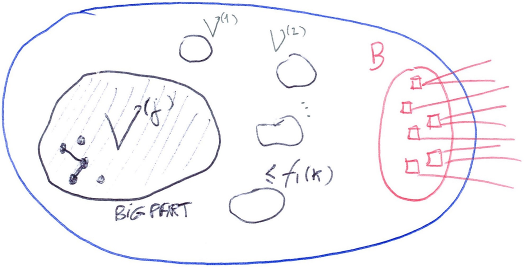
  2. Otherwise, find a set of "indistinguishable"' vertices, and identify them.

  Idea We generate all partial solutions in the boundary, and for each of them we compute a solution of HCLA, using our "black box".



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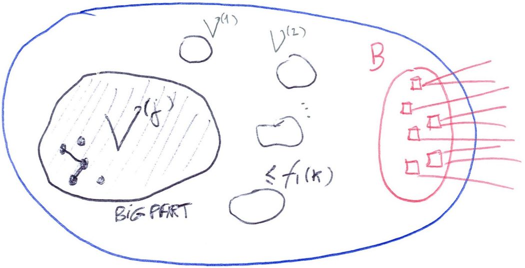
Idea By the high connectivity (Claim), each such solution has a unique big part $\mathcal{V}^{(j)}$: indistinguishable vertices for this behavior.



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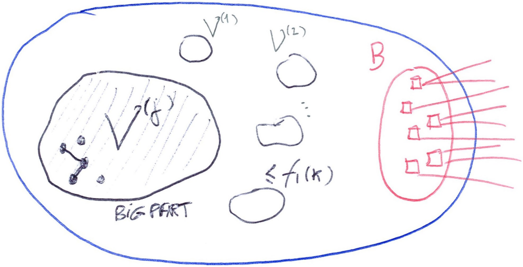
     Idea: If the graph is big enough, there are vertices that are indistinguishable for all behaviors $\Rightarrow$ identify them. Return the graph.



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Lemma

The above algorithm returns in FPT time an equivalent instance of CLA of size at most $f_2(k) := k \cdot (f_1(k))^2 + 2k + 2$. (Then we apply the XP algorithm.)

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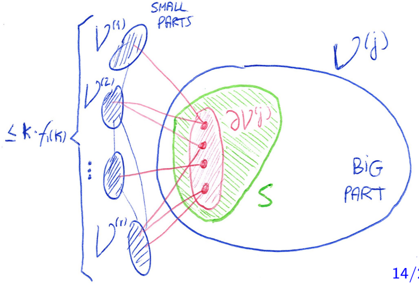
Split Highly Connected List Allocation (SHCLA)

Same input + set $S \subseteq V(G)$ and a solution $\mathcal{V}$ additionally needs to satisfy that if $j \in [r]$ is such that $\mathcal{V}^{(j)}$ is the big part of $\mathcal{V}$, then

$$\partial \mathcal{V}^{(j)} \subseteq S \subseteq \mathcal{V}^{(j)}.$$

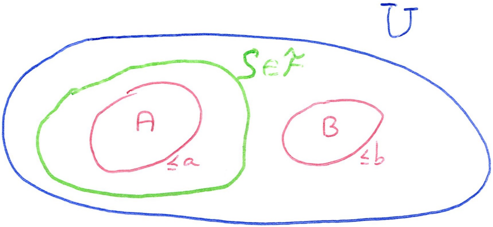


Crucial ingredient: Splitter Lemma

- Splitters were first introduced by [Naor, Schulman, Srinivasan '95]
- We use the following deterministic version:

Lemma (Chitnis, Cygan, Hajiaghayi, Pilipczuk[2] '12)

There exists an algorithm that given a set U of size n and two integers $a, b \in [0, n]$, outputs a set $\mathcal{F} \subseteq 2^U$ where $|\mathcal{F}| = (a+b)^{O(\min\{a,b\})} \cdot \log n$ such that for every two sets $A, B \subseteq U$, where $A \cap B = \emptyset$, $|A| \leqslant a$, $|B| \leqslant b$, there exists a set $S \in \mathcal{F}$ where $A \subseteq S$ and $B \cap S = \emptyset$, in $(a+b)^{O(\min\{a,b\})} \cdot n \log n$ steps.



Reduction from HCLA to SHCLA: we use splitters

- We use the Splitter Lemma with universe $U = V(G)$, $a = k$, and $b = k \cdot f_1(k)$, obtaining a family $\mathcal{F}$ of subsets of $V(G)$.

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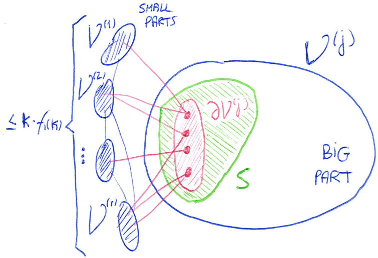
- Idea We want a set $S \subseteq V(G)$ that "splits" these two sets:

$$A = \partial\mathcal{V}^{(j)} \quad \text{and} \quad B = \bigcup_{i \in [r] \setminus \{j\}} \mathcal{V}^{(i)}.$$

For some $j \in [r]$: $|A| \leqslant k$ and $|B| \leqslant k \cdot f_1(k)$ (by the Claim).



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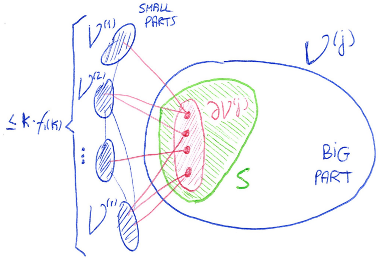

- It holds that I is a YES-instance of HCLA if and only if for some $S \in \mathcal{F}$, (I, S) is a YES-instance of SHCLA.

An algorithm to solve SHCLA

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  - $\mathcal{Y}$: those that cannot go entirely in $\mathcal{V}^{(j)}$.
  - $\mathcal{Z}$: those that are big ($> k \cdot f_1(k)$) and that can go entirely in $\mathcal{V}^{(j)}$.
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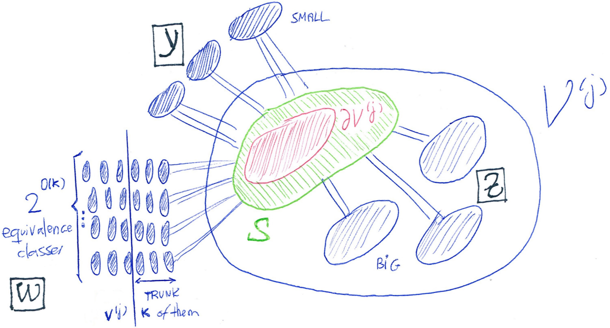

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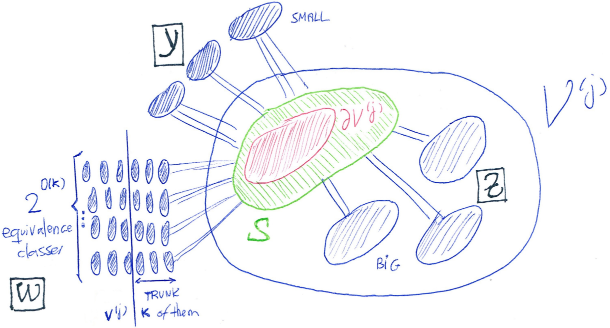

Lemma

The SHCLA problem can be solved in time $2^{O(k^2 \cdot \log k)} \cdot n$.

Piecing everything together

List Allocation (LA)

↓ FPT reduction

Connected List Allocation (CLA)

↓ FPT reduction

Highly Connected List Allocation (HCLA)

↓ FPT reduction

Split Highly Connected List Allocation (SHCLA)

↓ FPT algorithm to solve SHCLA

Theorem

List Allocation *can be solved in time* $2^{O(k^2 \log k)} \cdot n^4 \cdot \log n$.

Next section is...

Generalization of Digraph Homomorphism

Arc-Bounded List Digraph Homomorphism

Input: Two digraphs G and H, a list $\lambda : V(G) \to 2^{V(H)}$ of allowed images for every vertex in G, and a function α prescribing the number of non-loop arcs in G mapped to each arc of H.

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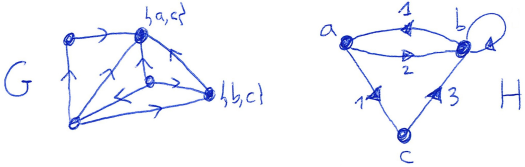

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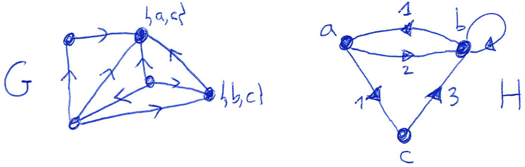

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Corollary

The Arc-Bounded List Digraph Homomorphism *problem is* FPT.

Graph partitioning problem

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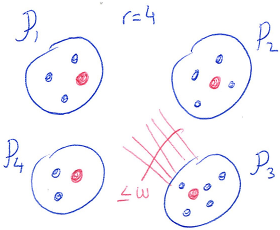

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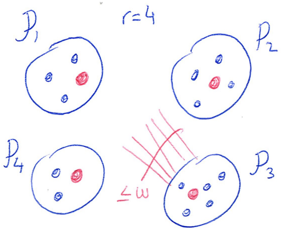

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- Tree-cut width is a graph invariant fundamental in the structure of graphs not admitting a fixed graph as an immersion. [Wollan '14]
- Tree-cut decompositions are a variation of tree decompositions based on edge cuts instead of vertex cuts.
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Corollary

There exists an algorithm that, given a graph G and a $k \in \mathbb{Z}_{\geqslant 0}$, in time $2^{O(k^2 \cdot \log k)} \cdot n^5 \cdot \log n$ either outputs a tree-cut decomposition of G with width at most $2k$, or correctly reports that the tree-cut width of G is strictly larger than k.

Next section is...

Conclusions and further research

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Gràcies!