On the complexity of computing the k-restricted edge-connectivity of a graph

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Outline of the talk

1. Introduction
2. Our results
3. Ideas of some of the proofs
4. Further research

Next section is...

1. Introduction
2. Our results
3. Ideas of some of the proofs
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Edge-connectivity

- We consider undirected simple graphs without loops or multiple edges.
- A set $S \subseteq E(G)$ of a graph G is an edge-cut if $G - S$ is disconnected.
- The edge-connectivity $\lambda(G)$ is defined as

$$\lambda(G) = \min\{|S| : S \subseteq E(G) \text{ is an edge-cut}\}.$$

- $\lambda(G)$ can be computed in poly time by a Max Flow algorithm.

Edge-connectivity and minimum degree

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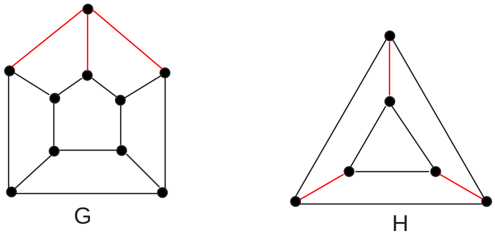

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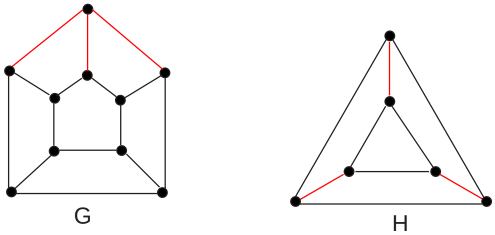

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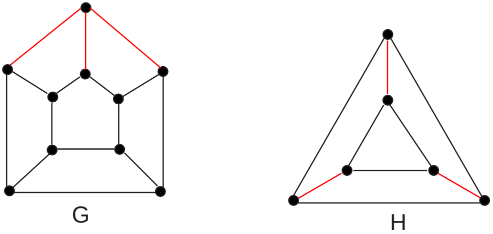


G is superconnected while H is not.

- A graph G is superconnected if every minimum edge-cut consists of the edges adjacent to one vertex.

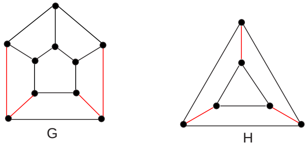
Restricted edge-connectivity

Definition [Esfahanian and Hakimi '88]

An edge-cut S is a restricted edge-cut if every component of $G - S$ has at least 2 vertices.

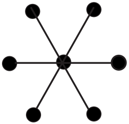
The restricted edge-connectivity $\lambda_2(G)$ of a graph G is defined as

$$\lambda_2(G) = \min\{|S| : S \subseteq E(G) \text{ is a restricted edge-cut}\}.$$



$\lambda_2(G) = 4$ and $\lambda_2(H) = 3$.

Restricted edge-connectivity



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A connected graph G is called λ_2-connected if $\lambda_2(G)$ exists.

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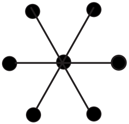

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A connected graph G is called λ_2-connected if $\lambda_2(G)$ exists.

Theorem [Esfahanian and Hakimi '88]

Every connected graph G that is not a star is λ_2-connected.

k-restricted edge-connectivity

In 1994, Fàbrega and Fiol proposed the concept of k-restricted edge-connectivity, where k is a positive integer.

Definition [Fàbrega and Fiol '94]

An edge cut S is a k-restricted edge cut if every component of $G - S$ has at least k vertices.

The k-restricted edge-connectivity $\lambda_k(G)$ of a graph G is defined as

$$\lambda_k(G) = \min\{|S| : S \subseteq E(G) \text{ is a k-restricted edge-cut}\}.$$

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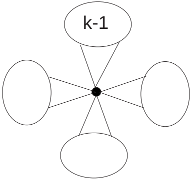
A connected graph G is called λ_k-connected if $\lambda_k(G)$ exists.

For any k-restricted cut S of size $\lambda_k(G)$, the graph $G - S$ has exactly two connected components.

k-restricted edge-connectivity

A k-flower is a graph containing a cut vertex u such that every component of $G - u$ has at most $k - 1$ vertices.



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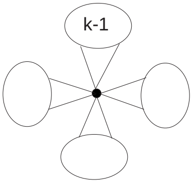


λ_k is not defined for k-flowers.

Theorem [Zhang and Yuan '05]

Every connected graph G that is not a k-flower with $k - 1 \leq \delta(G)$ is λ_k-connected.

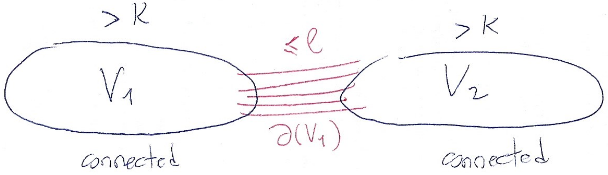
A lot of combinatorial results about λ_k (not exhaustive)

- Introduction of λ_2: [Esfahanian, Hakimi '88]
- Introduction of λ_k: [Fàbrega and Fiol '94]
- Case $k = 3$: [Bonsma, Ueffing, Volkmann. '02]
- General bounds on λ_k: [Zhang, Yuan '05]
- λ_k in graphs of large girth: [Balbuena, Carmona, Fàbrega, Fiol '97]
- λ_k in triangle-free graphs: [Yuan, Liu '10] [Holtkamp, Meierling, Montejano '12]

Meanwhile, in the parameterized complexity community...

Chitnis, Cygan, Hajiaghayi, and Pilipczuk[2] defined in 2012 this notion:

- Let G be a connected graph. A partition (V_1, V_2) of $V(G)$ is a (k, ℓ)-separation if $|V_1|, |V_2| > k$, $|\partial(V_1)| \leqslant \ell$, and $G[V_1]$ and $G[V_2]$ are both connected.



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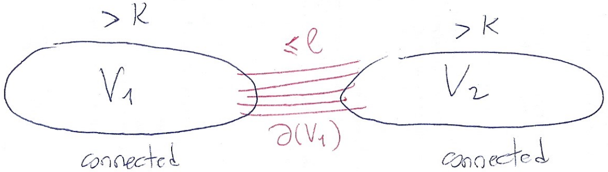

- A graph is (k, ℓ)-connected if it does not have a $(k, \ell - 1)$-separation.

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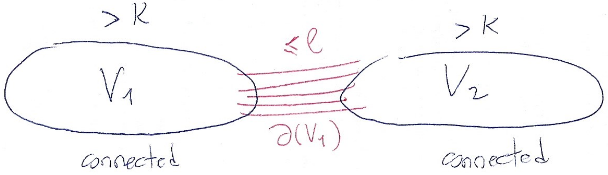

- A graph is (k,ℓ)-connected if it does not have a $(k,\ell-1)$-separation.

★ Both notions are essentially the same!

$\lambda_k(G) \leqslant \ell$ if and only if G admits a $(k-1,\ell)$-separation.

(k, ℓ)-separations are useful for FPT algorithms

Used in a technique known as recursive understanding:

- FPT algorithms for cut problems. [Chitnis, Cygan, Hajiaghayi, Pilipczuk^2 '12]
- A similar notion existed for vertex-cuts. [Kawarabayashi, Thorup '11]
- This technique has proved very useful. [Cygan, Lokshtanov, Pilipczuk^2, Saurabh '14] [Kim, Oum, Paul, S., Thilikos '15]

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⋆ Only one known algorithmic result about (k,ℓ)-separations:

Lemma (Chitnis, Cygan, Hajiaghayi, Pilipczuk[2] '12)

There exists an algorithm that given a n-vertex connected graph G and two integers k,ℓ, either finds a (k,ℓ)-separation, or reports that no such separation exists, in time $(k+\ell)^{O(\min\{k,\ell\})} n^3 \log n$.

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⋆ We initiate a systematic study of the complexity of computing the k-restricted edge-connectivity of a graph.

Next section is...

Summary of our results

Problem	Classical complexity	Parameterized complexity with parameter				
		$k+\ell$	k	ℓ	$k+\Delta$	$\ell+\Delta$
Is G λ_k-conn. ?	NPc, even if $\Delta \leqslant 5$	$\star$	FPT	$\star$	FPT	$\star$
$\lambda_k(G) \leqslant \ell$?	NPh, even if G is λ_k-conn.	FPT (known)	W[1]-hard	No poly kernels	FPT	?

Table : Summary of our results, where Δ denotes the maximum degree of the input graph G, and NPc (resp. NPh) stands for NP-complete (resp. NP-hard). The symbol '$\star$' denotes that the problem is not defined for that parameter.

Next section is...

Some words on parameterized complexity

- Idea given an NP-hard problem with input size n, fix one parameter k of the input to see whether the problem gets more "tractable".

  Example: the size of a Vertex Cover.

- Given a (NP-hard) problem with input of size n and a parameter k, a fixed-parameter tractable (FPT) algorithm runs in time

$$f(k) \cdot n^{O(1)}, \quad \text{for some function } f.$$

  Examples: k-Vertex Cover, k-Longest Path.

Determining whether a graph is λ_k-connected is hard

- Given a graph G, if n is even and $k = n/2$, it is NP-complete to determine whether G contains two vertex-disjoint connected subgraphs of order $n/2$ each. [Dyer, Frieze '85]

- This implies that the following problem is NP-hard:

RESTRICTED EDGE-CONNECTIVITY (REC)	
Instance:	A connected graph $G = (V, E)$ and an integer k.
Output:	$\lambda_k(G)$, or a report that G is not λ_k-connected.

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- Even if the input graph G is guaranteed to be λ_k-connected, computing $\lambda_k(G)$ remains hard:

Theorem

The REC problem is NP-hard restricted to λ_k-connected graphs.

Proof of the Theorem

- Proof for n even and $k = n/2$. Reduction from MINIMUM BISECTION in connected 3-regular graphs, which is NP-hard. [Berman, Karpinski '02]

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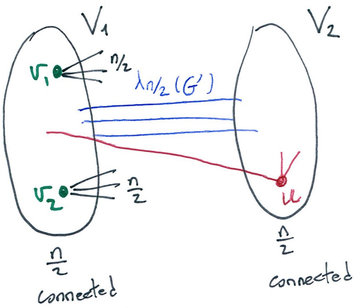

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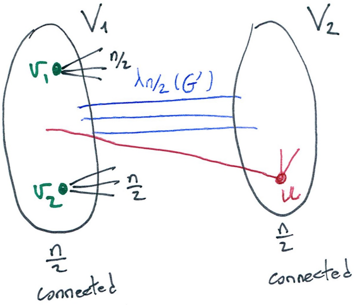

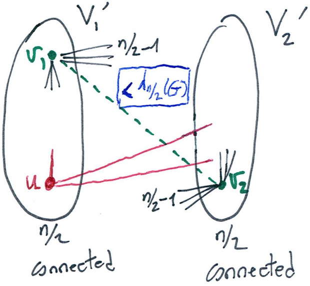

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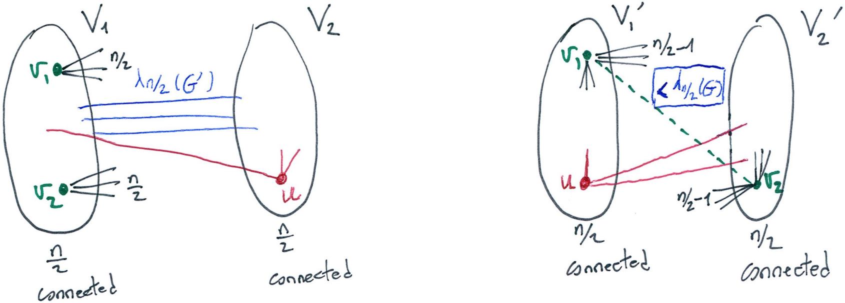

- Thus, REC problem in $G' \equiv$ MINIMUM BISECTION problem in G.

A parameterized analysis of the REC problem

Since the REC problem is NP-hard, we parameterize it:

PARAMETERIZED RESTRICTED EDGE-CONNECTIVITY (**p**-REC)

Instance:	A connected graph G and two integers k and ℓ.
Question:	$\lambda_k(G) \leqslant \ell$?
Parameter 1:	The integers k and ℓ.
Parameter 2:	The integer k.
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The **p**-REC problem is FPT when parameterized by both k and ℓ:

Theorem (Chitnis, Cygan, Hajiaghayi, Pilipczuk^2 '12)

There exists an algorithm that given a n-vertex connected graph G and two integers k, ℓ, either finds a (k, ℓ)-separation, or reports that no such separation exists, in time $(k + \ell)^{O(\min\{k,\ell\})} n^3 \log n$.

W[1]-hardness with parameter k only

Theorem

The $\mathbf{p}\text{-}\mathrm{REC}$ *problem is W[1]-hard when parameterized by* k*.*

It is easy to see that the problem is in XP: solvable in time $n^{O(k)}$.

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- Reduction from k-CLIQUE: the same as the one for CUTTING k VERTICES FROM A GRAPH, only the analysis changes. [Downey *et al.* '03]

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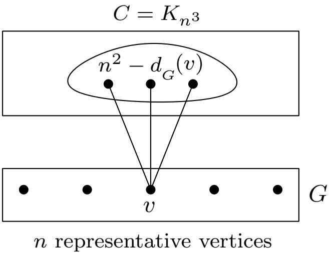
- Reduction from k-Clique: the same as the one for Cutting k Vertices from a Graph, only the analysis changes. [Downey *et al.* '03]
- Given $G \to G'$:



- Consider $\ell = kn^2 - 2\binom{k}{2}$ and take $k \leq n/2$.

Let's recap...

Theorem

Given a graph G and a positive integer k, determining whether G is λ_k-connected is NP-*hard.*

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The proof is based on a simple application of the technique of splitters.

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★ Parameterized complexity of $\boxed{\lambda_k(G) \leqslant \ell?}$ with parameter ℓ?

Parameterized complexity with parameter ℓ

Theorem

The **p**-REC *problem does not admit polynomial kernels when parameterized by* ℓ*, unless coNP* $\subseteq$ *NP/poly.*

Parameterized complexity with parameter ℓ

Theorem

The **p**-REC *problem does not admit polynomial kernels when parameterized by ℓ, unless $coNP \subseteq NP/poly$.*

- A kernel for a parameterized problem Π is an algorithm that given (x, k) outputs, in time polynomial in $|x| + k$, an instance (x', k') s.t.:
  - ⋆ $(x, k) \in \Pi$ if and only if $(x', k') \in \Pi$, and
  - ⋆ Both $\boxed{|x'|, k' \leqslant g(k)}$, where g is some computable function.
- If $g(k) = k^{O(1)}$: we say that Π admits a polynomial kernel.
- Folklore result: $\boxed{\Pi \text{ is FPT} \;\Leftrightarrow\; \Pi \text{ admits a kernel}}$
- Question: which FPT problems admit polynomial kernels?

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- If $g(k) = k^{O(1)}$: we say that Π admits a polynomial kernel.
- Folklore result: Π is FPT $\Leftrightarrow$ Π admits a kernel
- Question: which FPT problems admit polynomial kernels?
- It is possible to prove that polynomial kernels are unlikely to exist.

[Bodlaender, Downey, Fellows, Hermelin '08] [Bodlaender, Thomassé, Yeo '09] [Bodlaender, Jansen, Kratsch '11]

Non-existence of polynomial kernels with parameter ℓ

- The proof is inspired by the one to prove that the MIN BISECTION does not admit polynomial kernels. [van Bevern *et al.* '13]
- Main difference: both parts left out by the edge-cut are connected.

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- Main difference: both parts left out by the edge-cut are connected.

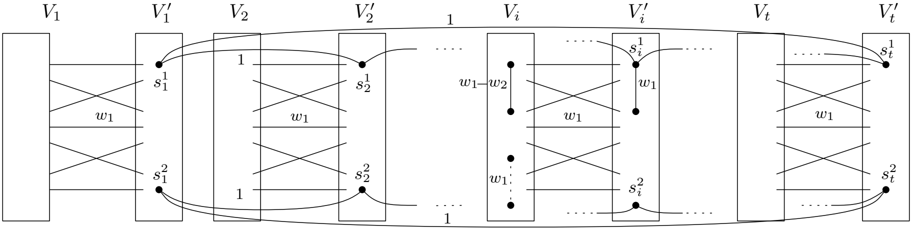
- We use the technique of cross-composition [Bodlaender, Jansen, Kratsch '11]

  Cross-composition from Max Cut (which is NP-hard) to Edge-Weighted **p**-REC parameterized by ℓ is a poly-time algorithm that, given t instances $(G_1, p_1), \ldots, (G_t, p_t)$ of Max Cut, constructs one instance (G^*, k, ℓ) of Edge-Weighted **p**-REC such that:

  1. (G^*, k, ℓ) is Yes iff one of the t instances of Max Cut is Yes, and
  2. ℓ is polynomially bounded as a function of $\max_{1 \leqslant i \leqslant t} |V(G_i)|$.

Idea of the proof

Given $(G_1, p), \ldots, (G_t, p)$, we create G^* as follows:



- We define $w_1 := 5n^2$ and $w_2 := 5$.
- And we set $k := |V(G^*)|/2$ and $\ell := w_1 n^2 - w_2 p + 4$.
- k is not polynomially bounded in terms of n, but this is not a problem since the parameter is ℓ, which is bounded by $5n^4$.
- This construction can be performed in polynomial time in $t \cdot n$.

Claim (G^*, k, ℓ) is a YES-instance of EDGE-WEIGHTED **p**-REC iff there exists $i \in \{1, \ldots, t\}$ such that (G_i, p) is a YES-instance of MAX CUT.

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Determining whether a connected graph G is λ_k-connected is NP-complete when k is part of the input, even if $\Delta(G) \leqslant 5$.

Appropriate modification of the reduction from 3DM given in [Dyer, Frieze '85]

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Theorem

The **p**-REC *problem is FPT when parameterized by k and the maximum degree Δ of the input graph.*

Algorithm based on a simple exhaustive search + MIN CUT algorithm.

Next section is...

Conclusions and further research

Problem	Classical complexity	Parameterized complexity with parameter				
		$k+\ell$	k	ℓ	$k+\Delta$	$\ell+\Delta$
Is G λ_k-conn. ?	NPc, even if $\Delta \leqslant 5$	$\star$	FPT	$\star$	FPT	$\star$
$\lambda_k(G) \leqslant \ell$?	NPh, even if G is λ_k-conn.	FPT (known)	W[1]-hard	No poly kernels	FPT	?

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- Polynomial kernels with parameter $k + \ell$?

Gràcies!