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REDUCING THE PRECISION / UNCERTAINTY DUALITY IN THE HOUGH TRANSFORM

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ABSTRACT

The Hough transform is a popular method for detecting complex forms in digital images. However, the technique is not very robust since several parameters that determine the scope of the detection results, such as quantization thresholds and intervals, must first be defined. In the present paper, we propose to enhance shape detection with the Hough transform through fuzzy analysis. One chief drawback of the Hough transform, i.e. the uncertainty / precision duality, is thus reduced.

I. INTRODUCTION

Since the paper by Hough [HOU 62] presenting a transform that aimed to detect sets of linear points in noisy binary images, several improvement of the Hough transform (HT) have been proposed.

The method and its extensions have been reviewed in detail [MAI 85], [ILL 88] and an exhaustive bibliography is available [PIC 87]. In particular, many papers have focused on the effect of errors due to random noise and data quantization [HUN 90] [STE 91] [PRI 94].

Despite the large number of papers dedicated to the Hough Transform, relatively little attention has been paid to what we termed the *uncertainty/precision duality*. This duality could be set out as follows: as shape detection precision increases, the reliability of the detection decreases. This seems to be due to the binary aspect of the vote in the classical Hough transform (CHT).

Han and al. [HAN 93] proposed to use fuzzy subset theory to deal with the problem of approximate concepts in HT. They designed a Fuzzy Hough Transform (FHT) which generalizes the distributed voting principle described by Thrift and Dunn [THR 83]. However, this method makes no distinction between data uncertainty and expected or computationally–induced parameter uncertainty. In addition, there is no benefit from assessing the data in terms of confidence.

In this paper we present a new FHT approach that takes current knowledge on uncertainty into consideration to improve shape detection.

II. THRESHOLDING

II.1. EFFECT OF THRESHOLD

Because it was designed for binary images, using HT is not problem-free on grey-level images. Users first have to face the problem of defining the subset of image points that supposedly belong to the sought–after straight lines (generally by thresholding the image). An image that is either non-uniformly illuminated or ill-contrasted requires to over- or under-estimate the threshold. Which induces an uncontrolled bias on the HT. Statistically finding an "optimal" threshold [JOL 89] supposes to test each pixel for two hypotheses that are: "the pixel belongs to one of the sought-after lines" and "the pixel belongs to none of the sought-after lines". Those two hypotheses are used to define two thresholds. Those two thresholds are used in the FHT to define the fuzzy subset of the points in the image that are supposed to belong to one of the searched lines.

II.2. FUZZY THRESHOLDING

Image pixels can generally be separated into two subsets E and according to grey level, gradient and curvature properties. E is the subset of points belonging to one of the sought–after straight lines, and its complement is the subset of points that belong to no straight line.

Many solutions have been proposed to overcome this problem, including that of [O'GO 73], whereby grey–level values are used directly to weight HT votes.

We use a similar technique to define E as a fuzzy subset of the original image.

When searching for black lines on a white background, the membership $\mu_E(x_i,y_j)$ of a pixel (i,j) with a grey level $ng(i,j)$ to a fuzzy subset E could be defined from an $\mathcal{L}$ function by:

Erreur !

When I_{pq}, the crisp subset of I, is denoted by:

$$I_{pq} = \left\{(x,y)\in I \;/\; |x.\cos(\theta_q)+y.\sin(\theta_q)-\rho_p| < \partial\rho\right\}$$

then, in assessing the $P_p \times \Theta_q$ cell, the HT is equal to the fuzzy cardinal of the subset $E \cap I_{pq} = E_{pq}$.

$$\mathcal{h}(P_p \times \Theta_q) = \mathrm{CARD}(E_{pq}) = \textbf{Erreur !}$$

In practice, when the image grey–level distribution is unknown, it is better to use a linear function:

$$\mathcal{L}(u) = 1-u \ \text{ if } u \in [0,1]$$

III. QUANTIZATION

III.1. EFFECT OF QUANTIZATION

In practice, using HT implies subdividing the parameter space into a certain number of cells. Many papers have rightly focused on this problem of quantization. There are two objectives in adjusting quantization:

— enhanced computation performance (less storage memory required, reduced computation time),
— enhanced algorithm performance (precision, confidence).

The first objective reduces the cell number (increased quantization), while the status of the second is not as clearcut, and is the focus of the present study.

In order to increase line detection **precision**, quantization clearly has to be reduced. This reduction can also be motivated by a high density of lines in the image. In addition to a substantial increase in computation time, there will be a lower accumulator coefficient, and therefore a larger **uncertainty** in the detection of each straight line. When this phenomenon is exaggerated with an excessive quantization, the accumulator array will only contain a maximum of one point per cell.

Conversely, if we intend to tolerate a poor fit between the model and reality, then it would be better to increase quantization. This increases the values of the accumulators associated with each cell, thus increasing the confidence that straight lines are present in the image. However, enhancing the **certainty** in the detection of each line will increase the **imprecision** of this detection.

The main reason for this dichotomy between the expected line detection precision and the certainty that the line is present on the image is the binary aspect of the vote in CHT.

Fuzzy voting scheme is a good way of approaching the quantization difficulties in Hough transform. It consists of distributing the votes in the parameter space in an uncertainty zone defined by the fuzzy approach. This avoids the all-or-none aspect of the usual incrementation of the accumulator associated with each cell in parameter space. Then, the quantization no more affects precision but only the resolving power of the transform i.e., its ability to create two different peaks for two different close features.

III.2. DISTRIBUTED VOTE

In a classical Hough transform (CHT) [CHA 91], each pixel (i,j) is considered as an intersection point of coordinates (x_i,y_j) on the image $I \subset \mathbb{R}^2$. The parameter space is subdivided into $(2.\partial\rho, 2.\partial\theta)$–sized boxes $P_p \times \Theta_q$ centered on (ρ_p,θ_q).

Each pixel (x_i,y_j) gives rise to a sine curvature in Ω space. The Hough transform is assessed by incrementing, for each characteristic point (i.e. $(x_i,y_j) \in E$), accumulators associated with Ω boxes that intersect the sine curvature in a non–null manner (FIG. 1).

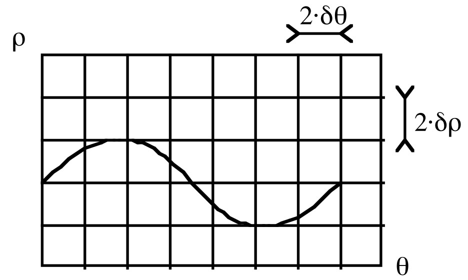


FIG 1 : Sine curvature fitted to an image point (x_i,y_j).

In practice, this means calculating the ρ values matching each θ_q value of θ: $\rho_p = (x_i.\cos\theta_q + y_j.\sin\theta_q)$ and incrementing the $P_p \times \Theta_q$ cell accumulator, such that: $\rho \in [\rho_p - \partial\rho, \rho_p + \partial\rho]$.

To account for uncertainty in detecting characteristic points, it is necessary to consider each pixel (i,j) as a $(2.\partial x, 2.\partial y)$–sized box $X_i \times Y_j$ of the image $I \subset \mathbb{R}^2$ centered on (x_i,y_j).

A box $X_i \times Y_j$ of I maps a sine curvature set in parameter space. It is thus essential to increment accumulators associated with all boxes that intersect at least one sine curvature of this set in a non–null manner. This is termed the distributed vote.

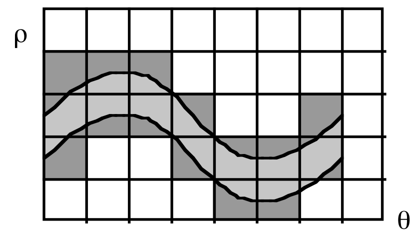


FIG 2 : Sine curves fitted to an image box $X_i \times Y_j$.

A first–order approximation can simplify this intersection calculation. Hence, for each θ_q value, we evaluate the interval $[\rho-\Delta\rho, \rho+\Delta\rho]$, such that:

$$\rho = (x_i.\cos\theta_q + y_j.\sin\theta_q)$$

$$\Delta\rho = |\partial x.\cos\theta_q| + |\partial y.\sin\theta_q| + |(y_j.\cos\theta_q - x_i.\sin\theta_q)\partial\theta|.$$

The $P_p \times \Theta_q$ cell accumulator is then incremented, for which :

$[\rho-\Delta\rho,\rho+\Delta\rho] \cap [\rho_p-\partial\rho,\rho_p+\partial\rho] \neq \emptyset$. (1)

Equation (1) states that $(x,y,\rho,\theta) \in X_i \times Y_j \times P_p \times \Theta_q$ exists, such that $\rho = (x.\cos\theta + y.\sin\theta)$, indicating that the relation $f(x,y,\rho,\theta) = 0$ is entirely **possible** in this box.

The all–or–none aspect of this incrementation can be astutely reduced by incrementing the accumulator associated with the $P_p \times \Theta_q$ cell using a proportional value, whereby the $[\rho-\Delta\rho,\rho+\Delta\rho]$ interval overlaps each P_p interval. The fuzzy Hough transform formalizes this heuristic process.

III.3. FUZZY VOTE

Let us consider that each pixel (i,j) is a fuzzy box, i.e. a product of two fuzzy intervals X_i and Y_j. X_i (rsp.Y_j) is a symmetrical fuzzy interval centered on x_i (rsp.y_j) with kernel $[x_i-\partial x, x_i+\partial x]$ (rsp.$[y_j-\partial y, y_j+\partial y]$) and spread σ_x (rsp.σ_y) (FIG. 3).

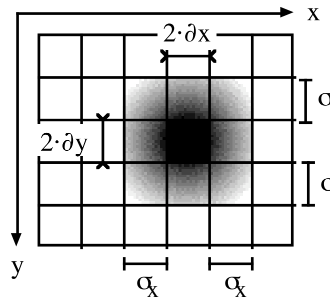


FIG 3 : Fuzzy pixel (fuzzy image box)

Therefore, the sine curve set produced by each pixel in Ω is an induced fuzzy set (FIG. 4).

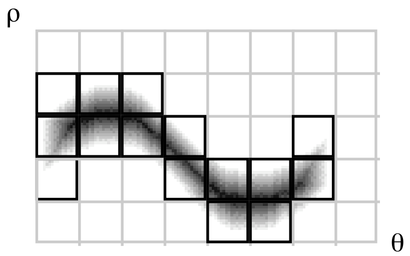


FIG 4 : Fuzzy sine curve set fitted to a fuzzy image box $X_i \times Y_j$.

Each Ω cell is stated in the same manner as the Cartesian (fuzzy) product of two fuzzy intervals P_p and Θ_q. P_p (rsp.Θ_q) is the fuzzy interval with a kernel $[\rho_p-\partial\rho,\rho_p+\partial\rho]$ (rsp. $[\theta_q-\partial\theta, \theta_q+\partial\theta]$), and spread σ_ρ (rsp.σ_θ).

We find that, for each $X_i \times Y_j$ box of E, the relation $f(x,y,\rho,\theta)=0$ is possible for $P_p \times \Theta_q$ cells with incremented accumulators. This potential is shown by the fact that the value 0 belongs to the variation domain of $\phi=f(x,y,\rho,\theta)$ when variables x,y,ρ and θ are restricted by their variation domain.

Variation domains in the FHT are fuzzy. Hence, when (x,y,ρ,θ) belongs to $X_i \times Y_j \times P_p \times \Theta_q$, the range of the variable ϕ is the fuzzy variation domain Φ_{ijpq}. According to the extension principle, when $\mu_{\Phi_{ijpq}}$ is the membership function of Φ_{ijpq}, we can state:

$$\mu_{\Phi_{ijpq}}(\phi) = \mathrm{SUP}_{x\in X_i,\ y\in Y_i,\ \rho\in P_p,\ \theta\in\Theta_q} \left\{ \begin{array}{l} \mathrm{MIN}\left(\mu_{X_i}(x),\mu_{Y_j}(y),\mu_{P_p}(\rho),\mu_{\Theta_q}(\theta)\right) \\ / \ f(x,y,\rho,\theta) = \phi \end{array} \right.$$

The possibility of membership of a pixel (i,j) to the fuzzy line represented by the cell (p,q) is clearly membership of the value 0 to the fuzzy set Φijpq:

$$\pi(i,j,p,q) = \mu_{\Phi_{ijpq}}(0)$$

FHT assessment of the fuzzy cell $P_p \times \Theta_q$ is then defined, for all fuzzy pixels of E, as the sum of this occurrence possibility $\pi(i,j,p,q)$. Replacing the fuzzy boxes by crisp boxes will clearly lead to a classical HT.

Now, to evaluate $\mu_{\Phi_{ijpq}}(\phi)$, we will use the properties of fuzzy intervals in an LR representation. The fuzzy intervals used here are for obvious reasons considered to be symmetrical. According to the properties of fuzzy intervals, the fuzzy interval Φ_{ijpq} can be defined by:

$$\Phi_{ijpq} = P_p - X_i.\cos(\Theta_q) - Y_j.\sin(\Theta_q).$$

In consideration of a few restrictions specific to limited developments, Φ_{ijpq} can be mapped with an LR–type interval, as defined by its center ϕ_{ijpq}, the half–width of its kernel $\partial\phi_{ijpq}$ and its spread σ **Erreur !**).

Erreur !;≈ ;$\sigma_\rho+|\cos\theta_q|.\sigma_x+|\sin\theta_q|.\sigma_y+$**Erreur !**$.\sigma_\theta$)

The FHT assesses the cell {p,q} as follows:

$\hbar(P_p \times \Theta_q)$ =**Erreur !** μ**Erreur !** = **Erreur !** $\pi(i,j,p,q)$

The latter equation still has to be corrected to account for the fact that E is a fuzzy set of I with a membership function $\mu_E(X_i \times Y_j)$:

$\hbar(P_p \times \Theta_q)$ =**Erreur !**

IV. DEFFUZZIFICATION

Defuzzification of the HT involves retrieving the distributed vote for a line of a cell from the neighboring cells by calculating the barycenter of the concerned fuzzy subsets [FOU 92]. This last procedure improves the accuracy. This accuracy is no more linked to the quantization in parameter space. Certainty is related to the relative height of the peaks.

V. EXPERIMENTS

We run the experiments on a simple image in order to highlight the different specificity of the FHT versus the CHT. This image has been chosen because it presents most of the flaw the FHT is coping with:

grainy and non-uniform background, poor contrasts, fuzzy unclear lines, ….

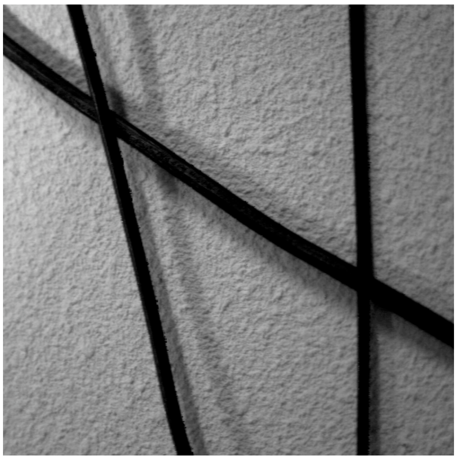


FIG. 5 : Original image of three cables.

The combined effect of fuzzy thresholding and fuzzy voting reduce the problems of false peaks due to noise in the image (FIG. 6). There are very few noise peaks on the accumulator array for the FHT, in contrast to that of the CHT. This could be explained by joint effects of a distributed vote and fuzzy weighting, thus “smoothing” the transform.

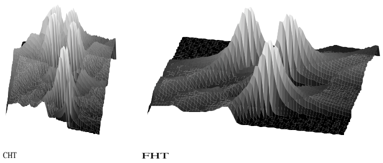
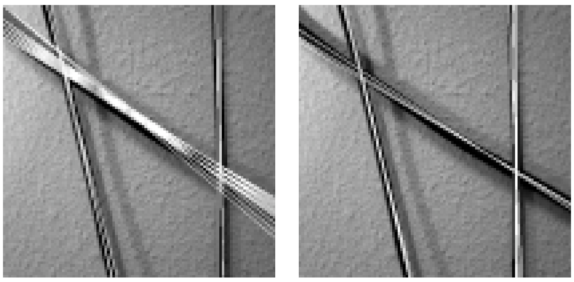
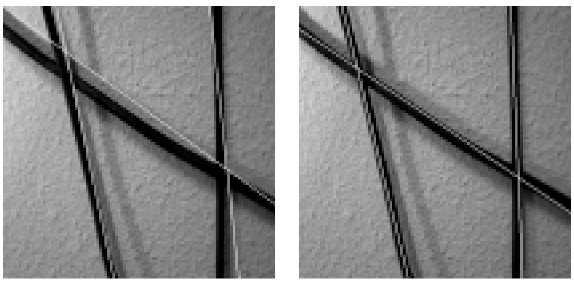


FIG. 6 Accumulator arrays
for the CHT and the FHT.



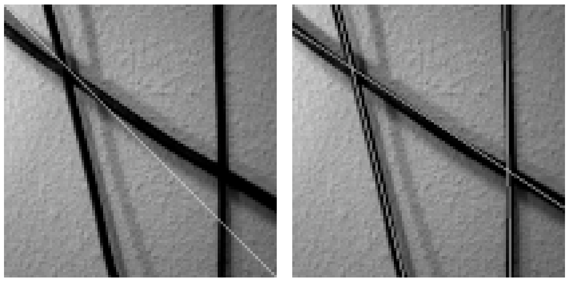
CHT FHT

FIG. 7 Detected straight lines superimposed on the original image



CHT FHT

FIG. 8 : Poor quantization of the parameter space



CHT FHT

FIG. 9 : Poor gray level threshold

V. CONCLUSION

Overall, the results of several tests showed that when the analyzed image is crisp, well contrasted, with little noise and quantization is adequate, then there is no visible improvement in the quality of straight line detection in the image when using the FHT. However, in the absence of any of these characteristics, there is a clear detection degradation with the CHT but not with the FHT. The FHT is therefore more robust than the CHT.

[CHA 91] J.M. CHASSERY et A. MONTANVERT – “Géométrie discrète en analyse d'images” – Hermes – 1991.

[FOU 92] L.FOULLOY et S. GALCHET “Contrôleur flous : représentation, équivalences et études comparatives” – Rapport LAMII 92-4 – juillet 1992.

[HAN 93] J.H. HAN, L.T KÓCZY, T. POSTON – “Fuzzy Hough Transform” – Second International Conference on Fuzzy Systems – San Francisco – march 1993.

[HOU 62] P.HOUGH – “Method and means for recognizing complex pattern” – U.S. Patent n°3,069,654 – december 1962.

[HUN 90] D. HUNT and al. – “Hough transform and signal detection theory performance for images with additive noise” – Computer Vision Graphics and Image Processing vol.52 – n° 3 (pp 386-401) – December 1990.

[ILL 88] J. ILLINGWORTH and J. KITTLER “A survey of the Hough transform” – Computer Vision Graphics and Image Processing n°44 (pp 87–116) – 1988.

[JOL 89] J. JOLION and A. ROZENFELD – “A O(log n) pyramid Hough transform” – Pattern Recognition Letters 9 (pp 343–349) – 1989.

[MAI 85] H.MAÎTRE – “Un panorama de la transformation de Hough” – Traitement du Signal – vol.2 – n°4 – 1985.

[PIC 87] P. PICTON – “Hough transform references” – International Journal of Pattern Recognition and Artificial Intelligence – vol.1 (pp 413-425) – 1987.

[PRI 94] J. PRINCEN and al. – “Hypothesis testing: a framework for analysing and optimizing Hough transform performance” – IEEE

Transaction on P.A.M.I. vol.16 – N°4 (pp 329–341) – april 1994.

[STE 91] R. STEPHENS – “Probabilistic approach to the Hough Transform” – Image and Vision Computing vol.9, (pp. 66-71) – 1991.

[THR 83] P. THRIFT and S. DUNN – “Approximating point-set images by line segments using a variation of the Hough transform” – Computer Vision Graphics and Image Processing vol. 21 (pp 383-394) –1983.