Decomposition of a 3D triangular mesh into quadrangulated patches

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LSIS, Arles, France

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GRAPP 2010

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A patch:

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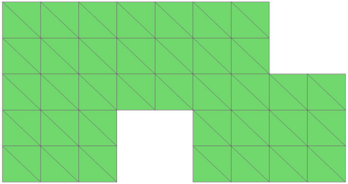

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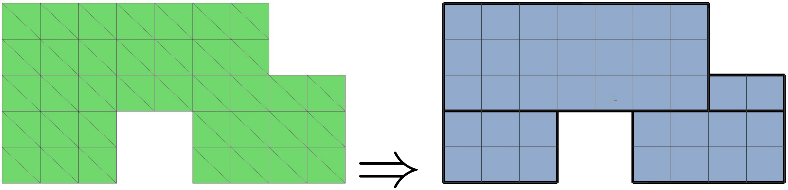


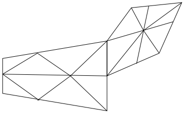
Motivation

Patches can be used for:

- interpolating or approximating a surface by a continuous representation,
- making reverse engineering to recognize the grid of the control points,
- compressing 3D mesh geometry without describing the topology,
- applying subdivision schemes,
- doing numerical simulation based on finite elements.

Constraints

Assumption: the mesh coordinates are exact ⇒ do not change the shape:

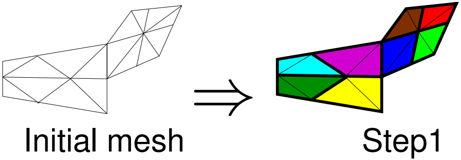


Initial mesh

Constraints

Assumption: the mesh coordinates are exact ⇒ do not change the shape:

- the vertices must be preserved,
- the edges are derived from the original triangular mesh.



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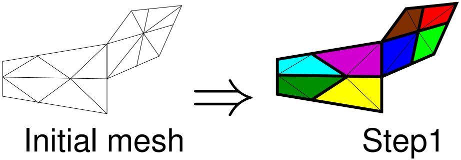
To create patches ⇒ last constraint:



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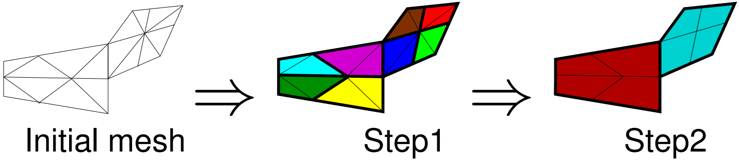
To create patches ⇒ last constraint:

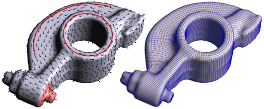
- the quadrangulated meshes are decomposed into quad rectangular grids.



State of the art: Triangular to quadrangular mesh

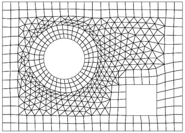
Remeshing algorithms

- Huang *et al.*
  Spectral quadrangulation with orientation and alignment control
  ACM trans. Graph. 27(5):1-9 2008



Advancing front algorithms

- Owen *et al.*
  Advancing front quadrilateral meshing using triangle transformations.
  7th International Meshing Roundtable:409-428 1998.



Merging algorithms

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  Adaptive Triangular-Quadrilateral Mesh Generation.
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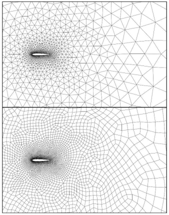

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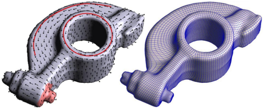

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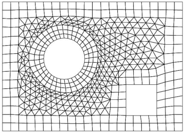

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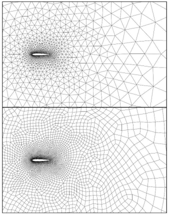
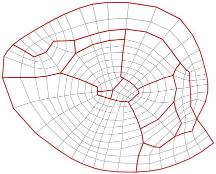


State of the art: Quadrangular meshes to patches

Decomposition into patches

- Eppstein *et al*.
  Motorcycle graphs: Canonical mesh partitioning.
  Comput. Graph. forum, 27(5):1477-1486 2008.



Outline

1. Presentation of the Decomposition Method
   - Computation of a Quality Coefficient
   - Construction of Quadrangulated Areas
   - Decomposition into Quadrangulated Patches
2. Experimental Results
   - First Results
   - Threshold Variations
   - CAD Objects
3. Conclusion and Future Work
   - Conclusion
   - Future Work

Our method

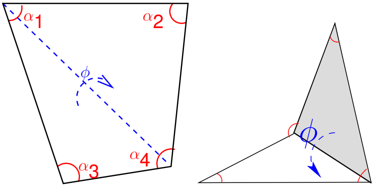
3 steps:

1. Computation of a quality coefficient for each pair of adjacent triangles
2. Construction of quadrangulated areas, using the quality coefficients
3. Decomposition into quadrangulated patches from quadrangulated areas

1) Computation of a Quality Coefficient

Computation of the quality coefficient Q based on:

- dihedral angle (ϕ)
- angles between connected edges (α_i)



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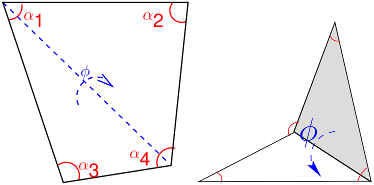

$$Q = \begin{cases} 2\pi & \text{if } \phi < \phi_{min} \\ \frac{1}{4}\sum_{i=1}^{4} |\frac{\pi}{2} - \alpha_i| & \text{elsewhere.} \end{cases}$$

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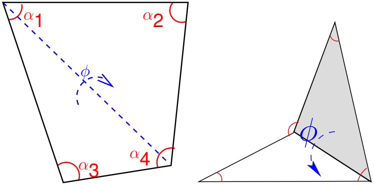

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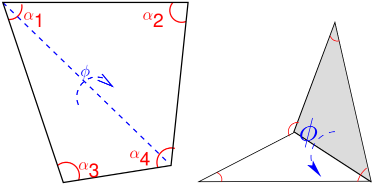

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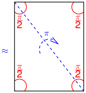
if $Q \approx 0 \Rightarrow$ quad $\approx$ planar rectangle 

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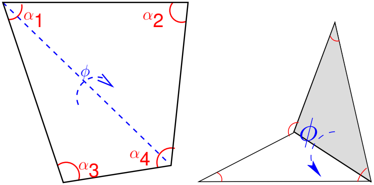

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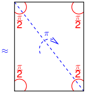
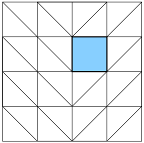


if $Q \approx 0 \Rightarrow$ quad $\approx$ planar rectangle
$\Rightarrow$ quad with good quality

2) Construction of Quadrangulated Areas

Iterative construction of quadrangulated areas:

Start with the best Q.

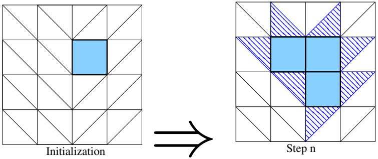


Initialization

2) Construction of Quadrangulated Areas

Iterative construction of quadrangulated areas:

Find the quad with the best Q in the neighborhood



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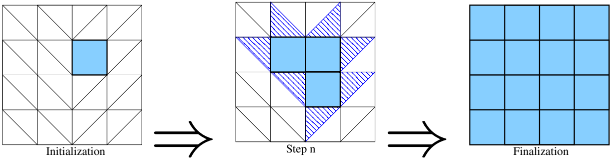
Iterative construction of quadrangulated areas:

No new quad can be created.



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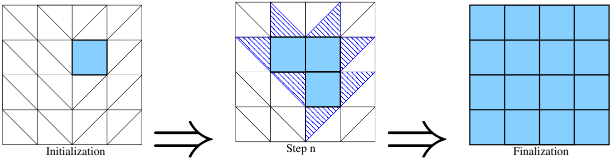
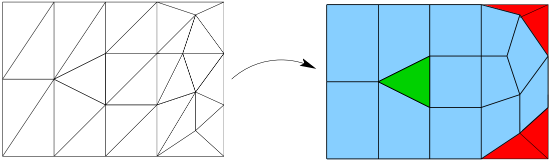


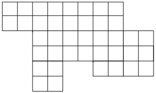


Left triangles:

- isolated triangles 🟩
- triangles of quads with $Q > Q_{max}$ 🟥

3) Decomposition into Quadrangulated Patches

3.1) The quads are arranged into “rectilinear polygons”



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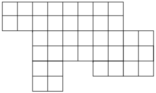
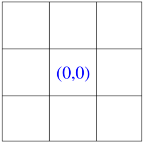


Each quad is labeled with a position using neighbors



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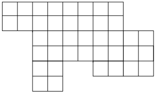

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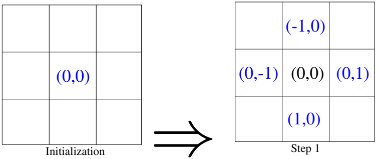

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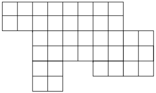

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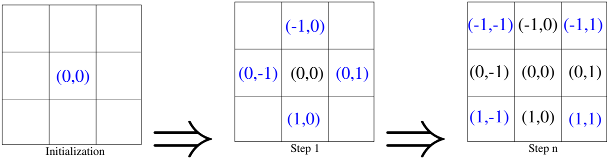

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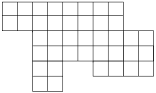

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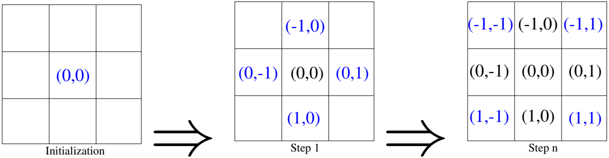

	(0,0)	

Initialization

⇒

	(-1,0)	
(0,-1)	(0,0)	(0,1)
	(1,0)	

Step 1

⇒

(-1,-1)	(-1,0)	(-1,1)
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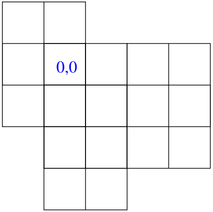
Step n

Example:



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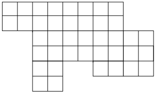

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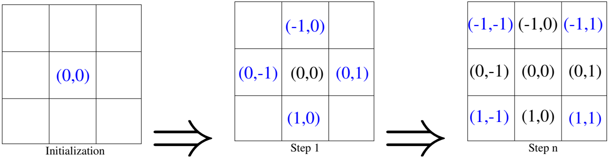

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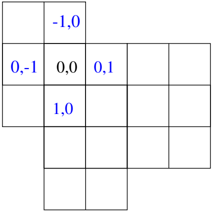

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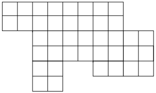

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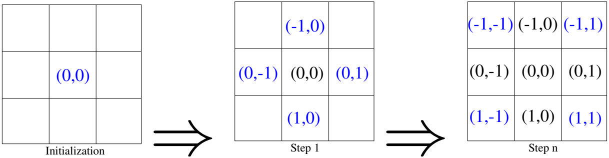

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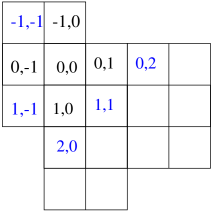

-1,-1	-1,0			
0,-1	0,0	0,1	0,2	
1,-1	1,0	1,1		
	2,0			

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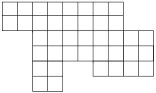

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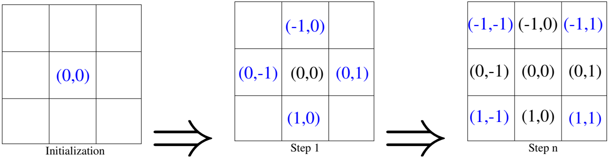

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Example:

-1,-1	-1,0			
0,-1	0,0	0,1	0,2	0,3
1,-1	1,0	1,1	1,2	
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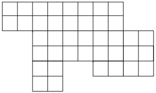

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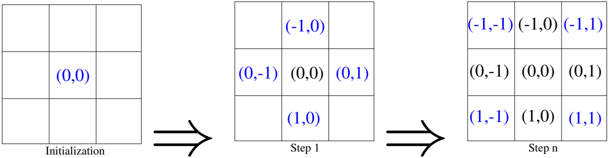

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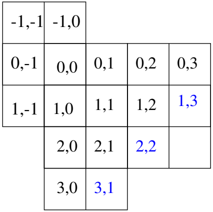

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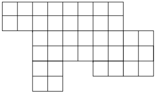

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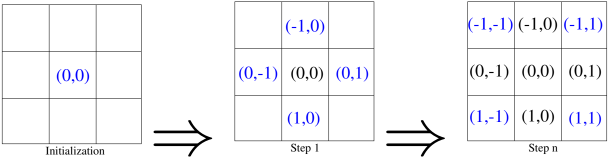

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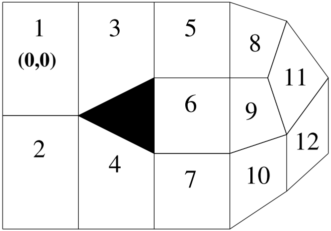
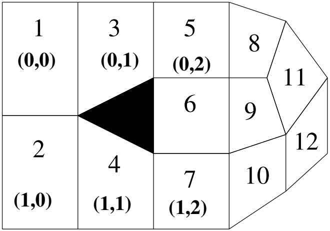
Problems:



3) Decomposition into Quadrangulated Patches

Problems:



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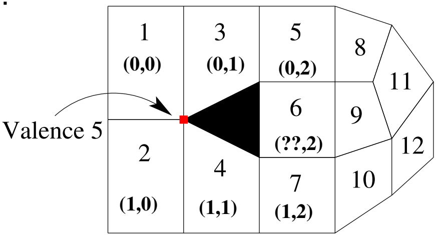

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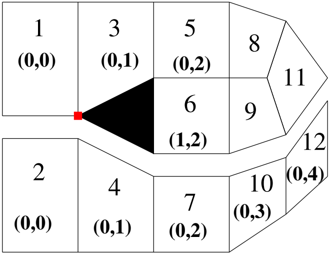

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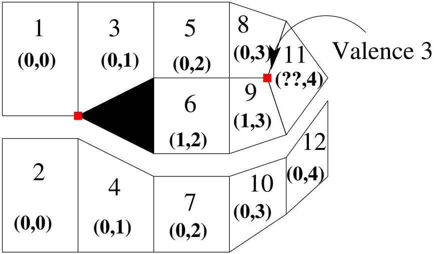

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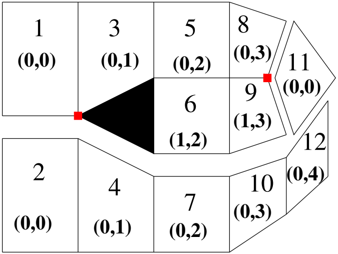
Problems:



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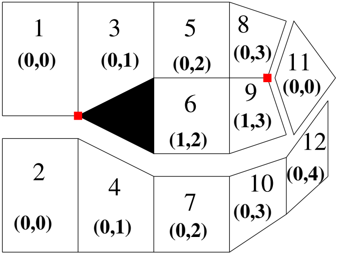
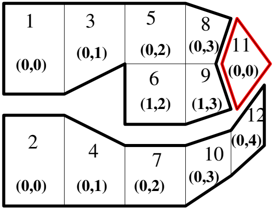


Decomposition into rectilinear polygons ⟹



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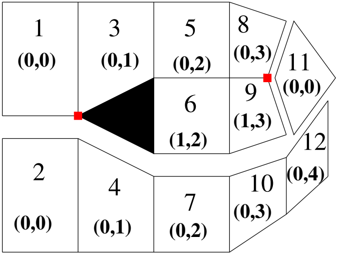
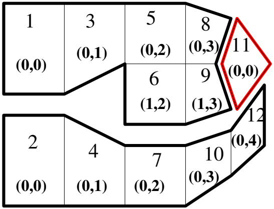


Decomposition into rectilinear polygons ⇒



Rectilinear polygons constituted by only one quad are not kept.

3) Decomposition into Quadrangulated Patches

3.2) The rectilinear polygons are decomposed into patches:
⇒ same number of rows for each column.

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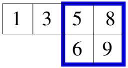
Iterative computation of the patches:



1	3	5	8
		6	9

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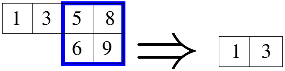

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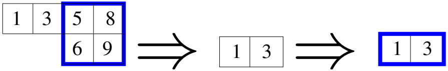

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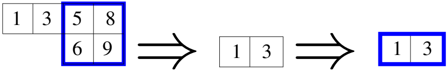
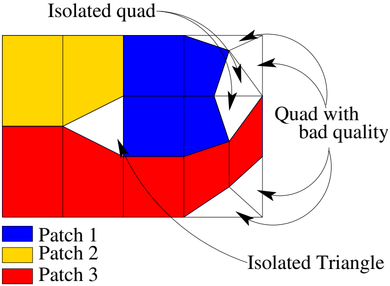
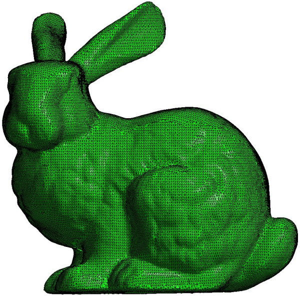


Final result:

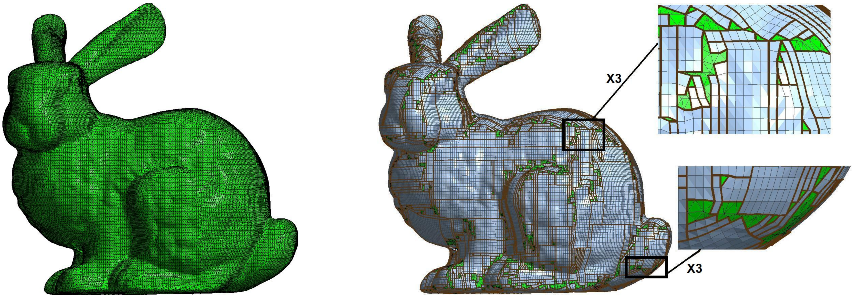


First Results



Stanford Bunny mesh: 69,451 triangles.

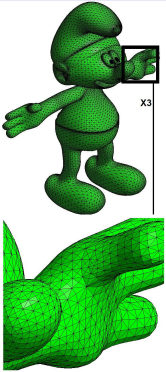
First Results



Stanford Bunny mesh: 69,451 triangles.

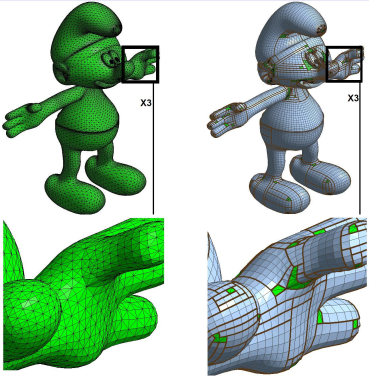
Q_{max}	ϕ_{min}	# patches	Covering	Time
$\frac{\pi}{2}$	$\frac{5\pi}{6}$	1,932	89.98%	4 min

Threshold Variations



Smurf mesh: 64,320 triangles.

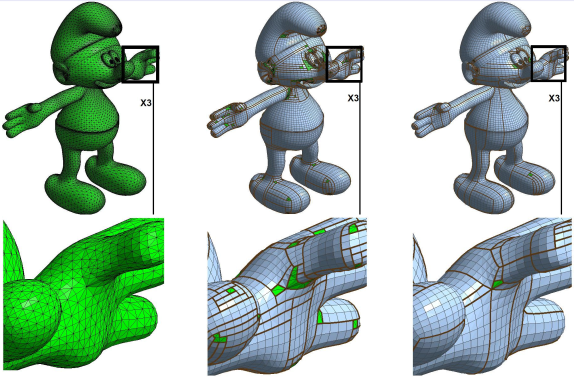
Threshold Variations



Smurf mesh: 64,320 triangles.

Q_{max}	ϕ_{min}	# patches	Covering	Time
$\frac{\pi}{2}$	$\frac{5\pi}{6}$	931	91.39%	2 min

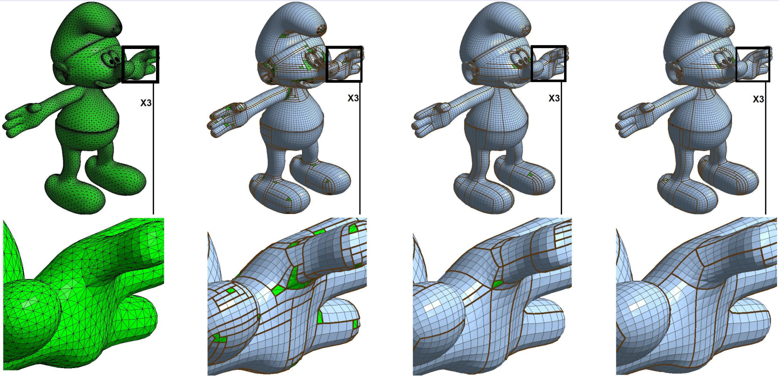
Threshold Variations



Smurf mesh: 64,320 triangles.

Q_{max}	ϕ_{min}	# patches	Covering	Time
$\frac{\pi}{2}$	$\frac{5\pi}{6}$	931	91.39%	2 min
π ↗	$\frac{5\pi}{6}$	519 ↘	98.52% ↗	4 min

Threshold Variations

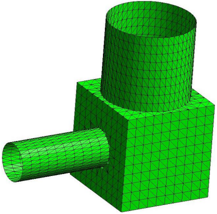
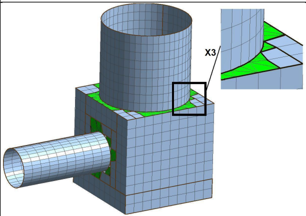


Smurf mesh: 64,320 triangles.

Q_{max}	ϕ_{min}	# patches	Covering	Time
$\frac{\pi}{2}$	$\frac{5\pi}{6}$	931	91.39%	2 min
π ↗	$\frac{5\pi}{6}$	519 ↘	98.52% ↗	4 min
2π ↗	2π ↗	502 ↘	98.56% ↗	5 min 30 sec

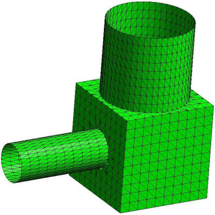
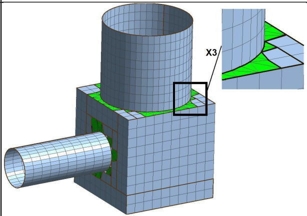
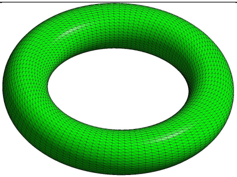
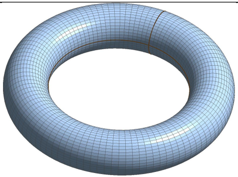
CAD Objects

$$Q_{max} = \frac{\pi}{2} \; / \; \phi_{min} = \frac{5\pi}{6}$$

	Initial mesh	Result	#triangles	#patches	Covering
CubeCylinders			2,608	21	95.47%

CAD Objects

$$Q_{max} = \frac{\pi}{2} \ / \ \phi_{min} = \frac{5\pi}{6}$$

	Initial mesh	Result	#triangles	#patches	Covering
CubeCylinders			2,608	21	95.47%
Torus			9,384	1	100%

Conclusion

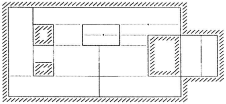
Our method:

- decomposes a triangular mesh into quadrangulated patches,
- has the particularity to use only the vertices and the edges of the triangular mesh,
- is implemented in the C4W framework.

Future Work

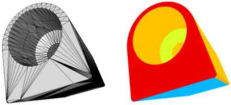
- Define other quality coefficients,
- Improve the quad propagation to minimize the number of isolated triangles,
- Optimize the rectilinear polygon search,

Soltan et al.
Minimum Dissection of a Rectilinear Polygon with Arbitrary Holes into Rectangles
Discrete and Computational Geometry 9(1):57-59 1993



- Use feature lines to guide the patch boundaries.

Lavoué et al.
A new CAD mesh segmentation method, based on curvature tensor analysis
Computer-Aided Design 37(10):975-987 2005



Thanks for your attention

QUESTIONS?

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Mail: roseline.beniere@lirmm.fr
C4W site: www.c4w.com

Roseline Bénière, G. Subsol, G. Gesquière, F. Le Breton and W. Puech,
Decomposition of a 3D triangular mesh into quadrangulated patches,
GRAPP, *Angers*, 2010