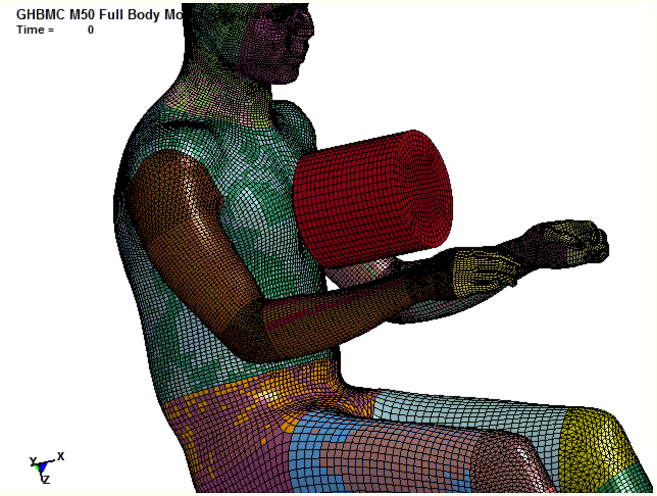
A NEW METRIC FOR STATISTICAL ANALYSIS OF RIGID TRANSFORMATIONS: APPLICATION TO THE RIB CAGE

Baptiste Moreau (1,2), Benjamin Gilles (1,4), Erwan Jolivet (3), Philippe Petit (2), Gérard Subsc (1)

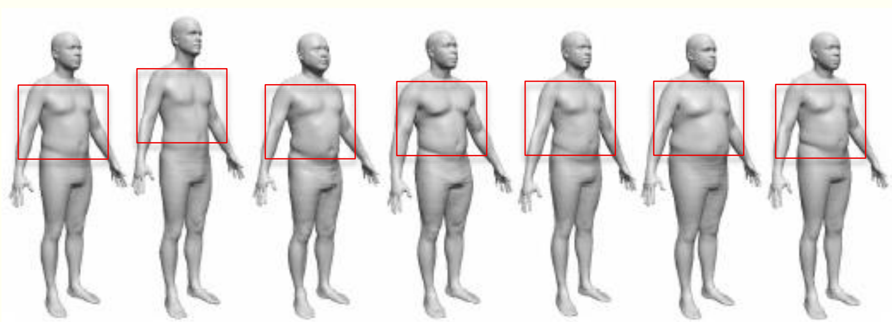
1. *Research-team ICAR, LIRMM, CNRS / University of Montpellier, France;*
2. *LAB PSA Renault, France;*
3. *CEESAR, Nanterre, France*
4. *AnatoScope, France*

INTRODUCTION

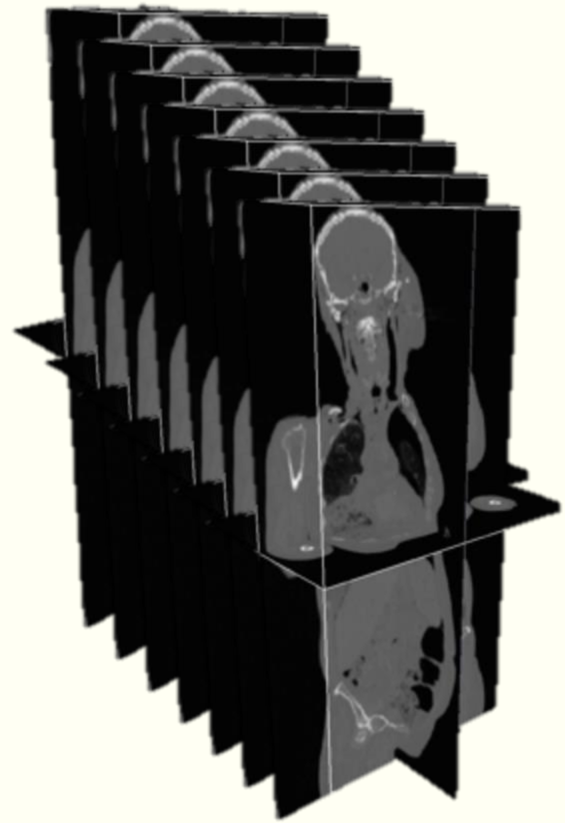
- Context



Male 50th percentile Human Body Model for car crash simulation (www.ghbmc.com)
≈ 2 millions deformable elements



Human body shape variations *(Allen et al., 2003)*



3D medical images – CT-scan

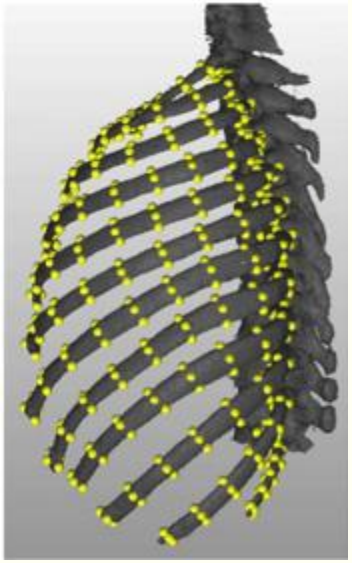
Statistical Analysis of the Morphology

CONTENTS

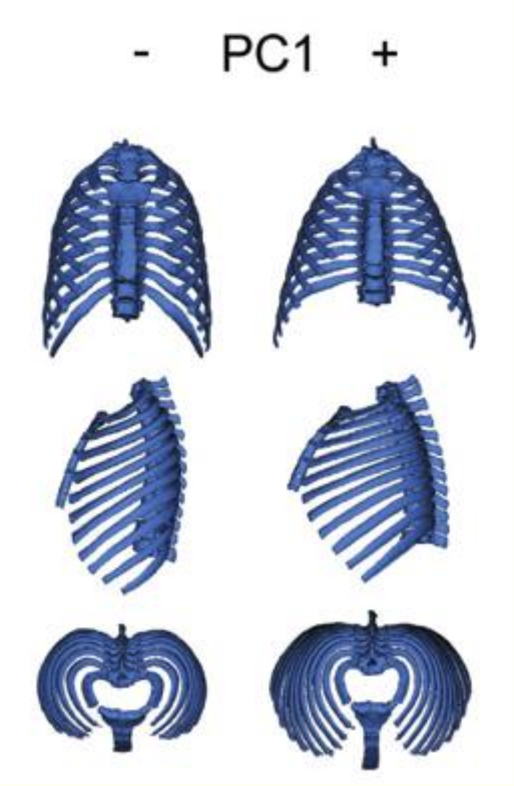
OVERVIEW

❖ Statistical Shape Models

- Point Distribution Model *(Cootes et al., 1995)*
  - Landmarks (point correspondence)
  - Mean shape
  - Shape variability (covariance)
    - PCA: orthogonal modes



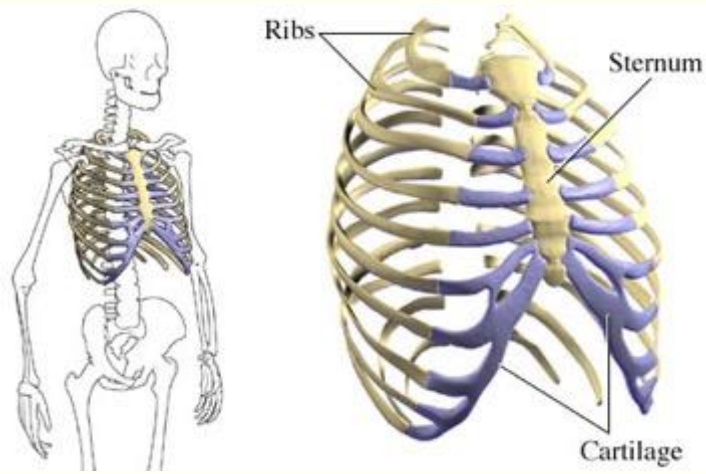
Landmarks *(Shi et al., 2014)*



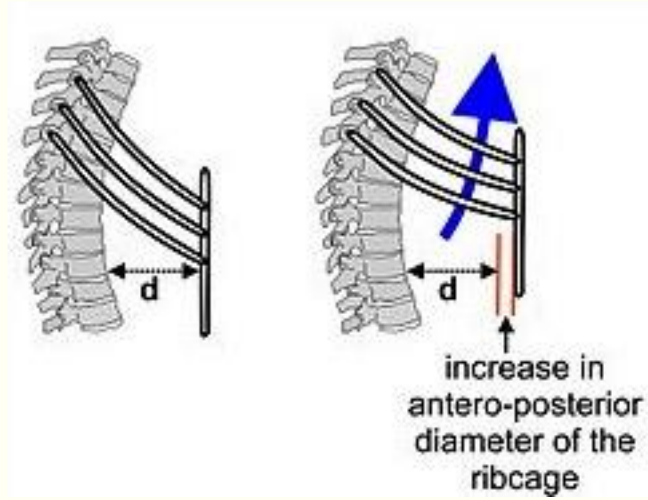
First PCA mode
(Bastir et al., 2017)

OVERVIEW

❖ Limitation for the Rib Cage



Articulated structure



Intrinsic shape of each bone

+

Relative position between different bones

→

Rib cage shape

OVERVIEW

- Multi-object Statistical Shape Model

For a bone from a subject *i*: $$\underset{\text{points coordinates}}{S_i} = (\underset{\text{mean shape}}{\overline{S}} + \underset{\text{intrinsic shape variation}}{\color{blue}{\delta_i}}).\overbrace{\color{red}{R_i + t_i}}^{\text{pose variation}}$$

Rib Cages in a common frame

Each bone

Intrinsic shape variations
(points coordinates)

→ Principal Component Analysis

Pose variations
(rigid transformations)

→ ~~PCA~~

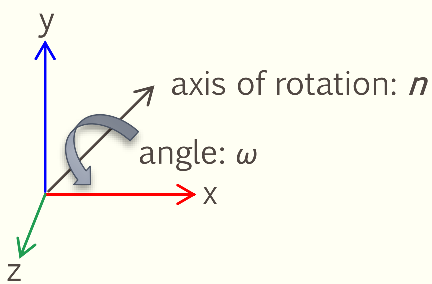
The space of rigid transformations is not an Euclidean space

❖ The Set of Rigid Transformations *SE(3)*

- Structure of a manifold
  - Metric: distance between two 3D rigid transformations
  - Tangent space: locally resembles an Euclidean space

- Mean and Covariance for *SE(3)* *(Pennec, 2006)*

- tangent PCA (tPCA)
  - maximizes the explanation of the covariance matrix



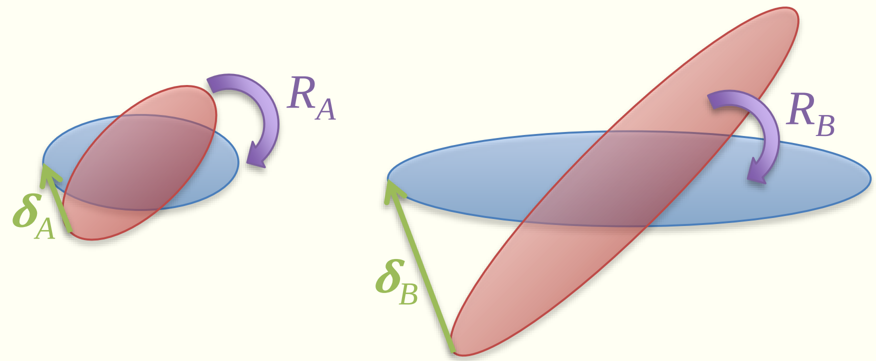
$$H \begin{cases} \vec{r} = \omega.(n_x, n_y, n_z)^T \\ \vec{t} = (t_x, t_y, t_z)^T \end{cases}$$

- Distance Function: $$d(H_1, H_2) = N_{rotv}(H_2^{-1} \circ H_1)$$

- Normalized Norm Function: $$N_{rotv}(H)^2 = \vec{t}^T.\vec{t} + \lambda \vec{r}^T.\vec{r}$$

  (Boisvert et al., 2008)

- Limits of the Current Metric



$$R_A = R_B \Rightarrow N_{rotv}(R_A) = N_{rotv}(R_B)$$

$$\delta_A < \delta_B$$

AIMS

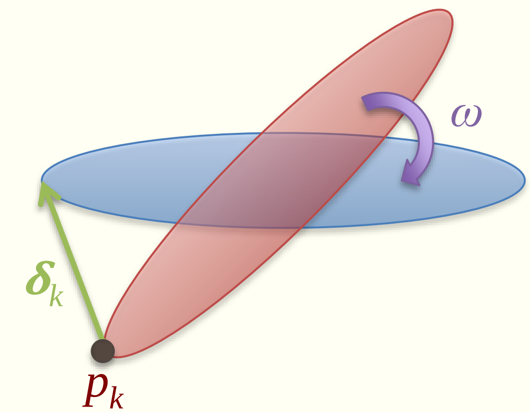
- Providing a distance function for SE(3), well adapted for elongated shapes, in order to apply a tPCA
- Applying this method to the construction of an articulated statistical shape model of the rib cage

STATISTICS – POSE VARIATIONS

❖ A New Metric for Elongated Objects

- New norm function: $$\boxed{N(H)^2 = \delta(H)^T.\delta(H).}$$

$\delta(H)$: displacement field



New norm

$$N(H)^2 = \sum \vec{\delta_k}^T.\vec{\delta_k},$$
$$= \omega^2 \vec{n}^T.\overline{\overline{I}}.\vec{n} + \vec{t}^T.\vec{t}$$

- Object shape

Usual norm

$$N_{rotv}(H)^2 = \lambda \vec{r}^T.\vec{r} + \vec{t}^T.\vec{t},$$
$$= \omega^2 \vec{n}^T.\lambda Id.\vec{n} + \vec{t}^T.\vec{t}$$

Inertia tensor analogy: $\overline{\overline{I}} = \lambda Id$

- Spherical shape

STATISTICS – POSE VARIATIONS

- Tangent Description $$H_i = \underset{H \in SE(3)}{\arg\min} \| H(\vec{F_i}) - \vec{\bar{F}} \|.$$

  - Displacements: $$\vec{\delta}(H_i) = H_i(\vec{\bar{F}}) - \vec{\bar{F}}$$
  - Norm function: $$N(H_i)^2 = \vec{\delta}(H_i)^T . \vec{\delta}(H_i)$$
  - Left-invariant distance: $$d(H_{i_1}, H_{i_2}) = N(H_{i_2}^{-1} \circ H_{i_1}).$$
  - Exp and Log maps:

$$\vec{Log}_{Id}(H_i) = \vec{\delta}(H_i),$$

$$\vec{Log}_{H_{i_1}}(H_{i_2}) = \vec{Log}_{Id}(H_{i_2}^{-1} \circ H_{i_1}).$$

$$Exp_{Id}(\vec{\delta_i}) = \underset{H \in SE(3)}{\arg\min} (N(H)^2),$$

$$Exp_{H_{i_1}}(\vec{\delta_{i_2}}) = H_{i_1} \circ Exp_{Id}(\vec{\delta_{i_2}}).$$

- Covariance Generalization *(Pennec, 2006)*

  - Fréchet mean: $$\mu_{n+1} = Exp_{\mu_n}\left(\frac{1}{N_i}\sum_{i=1}^{N_i}\vec{Log}_{\mu_n}(H_i)\right).$$

  - Covariance: $$Cov(set_{j_1}, set_{j_2}) = \frac{1}{N_i - 1}\sum_{i=1}^{N_i}\vec{Log}_{\mu_{j_1}}(H_{i,j_1}).\vec{Log}_{\mu_{j_2}}(H_{i,j_2})^T.$$

  - tPCA – reconstruct a bone *j* with a reduced number of modes *r*:

asssociated score

principal displacement

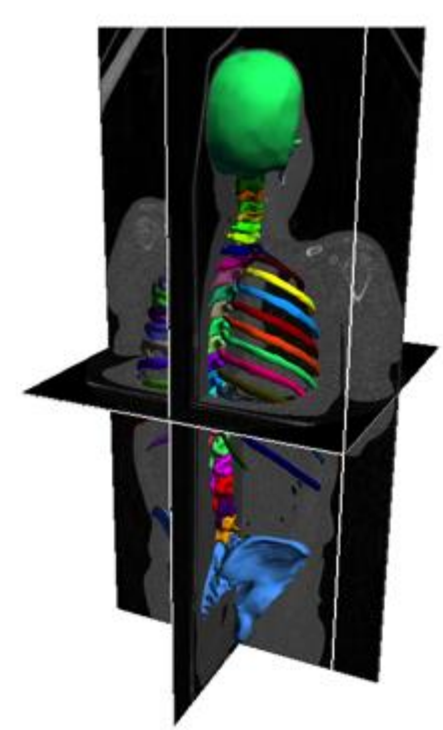
$$H_{tPCA,i,j} = Exp_{\mu_j}\left(\sum_{k=1}^{r}\alpha_{tPCA_k,i,j}.\vec{\delta_{tPCA_k,j}}\right),$$

$$\vec{F_{tPCA,i,j}} = H_{tPCA,i,j}(\overline{\vec{F_j}}).$$

mean shape

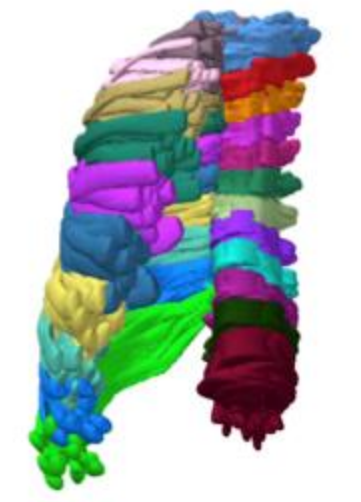
APPLICATION TO THE RIB CAGE

❖ The Dataset



26 male subjects

- ✓ 73.3 ± 11 years old
- ✓ 70 ± 9.8 kg
- ✓ 172 ± 5.5 cm



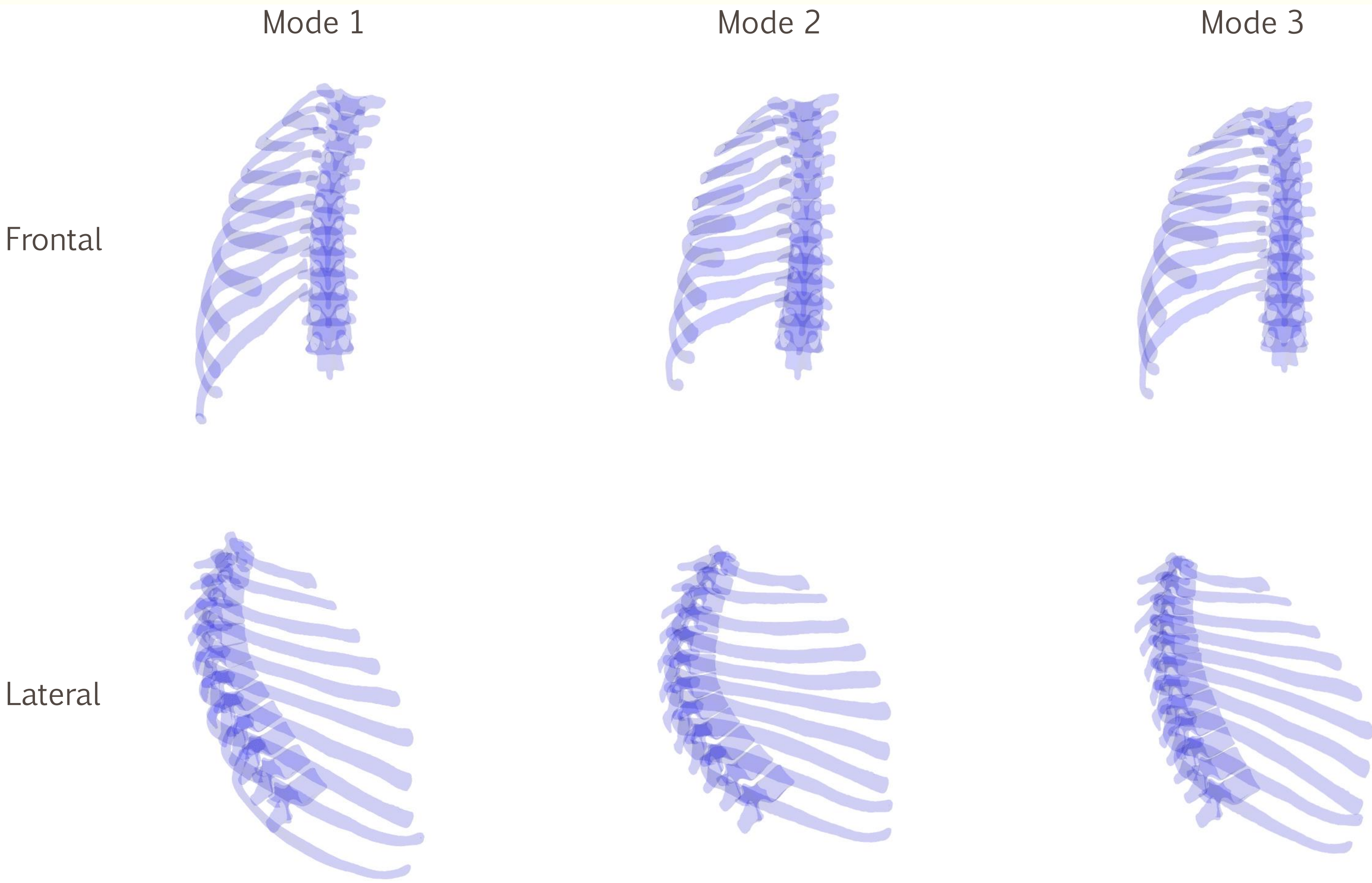
Mesh-to-Image registration

(Gilles et al. 2010)

- ✓ point-to-point correspondence between subjects
- ✓ ≈ 1570 nodes per bone
- ✓ 12 vertebrea, 24 ribs and a sternum

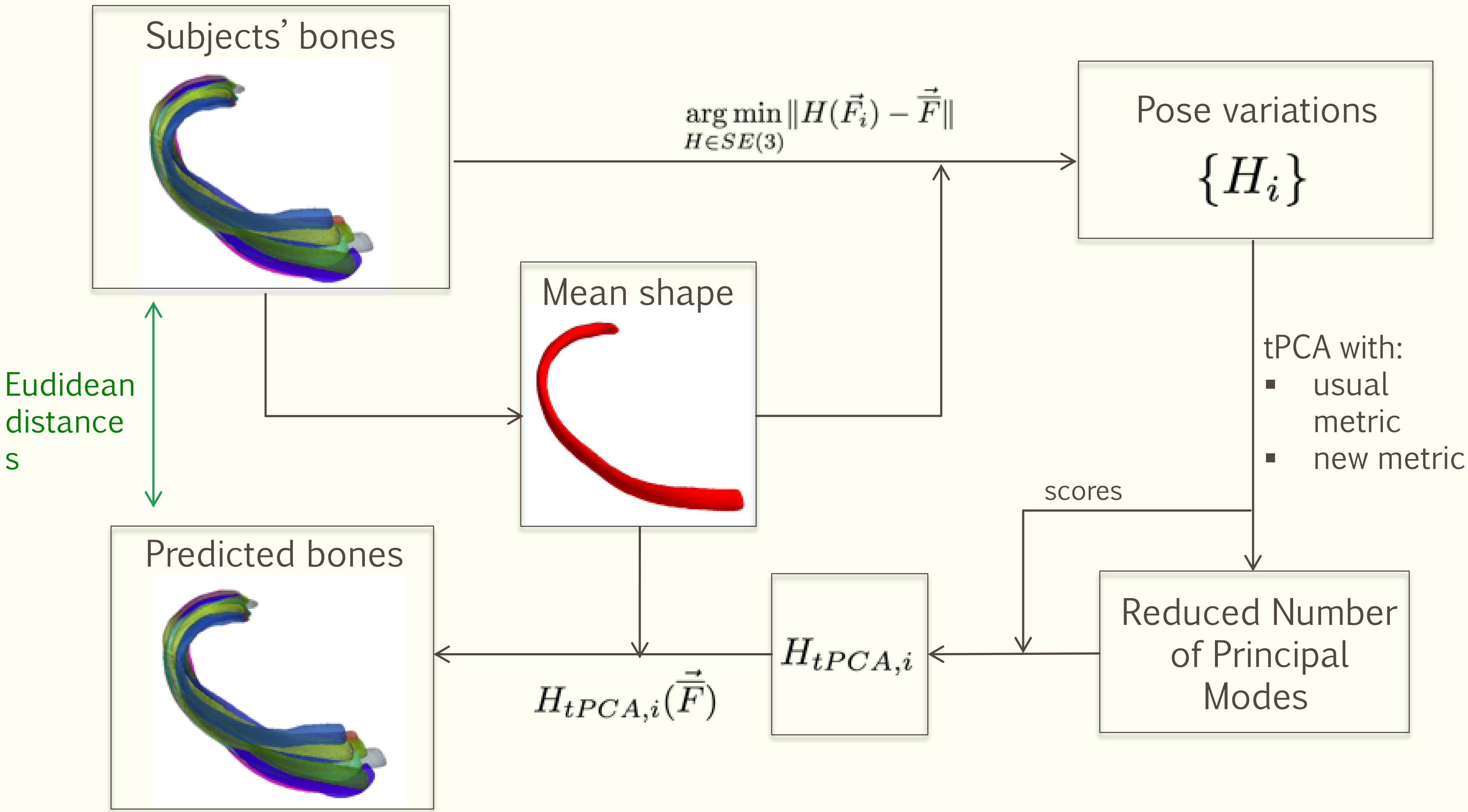
➢ Implemented by a Python script (Numpy)
  - ➢ Fréchet mean (≈ 5 iterations)
  - ➢ Dual PCA to accelerate computations
  - ➢ Fast and easy to use

- First Modes



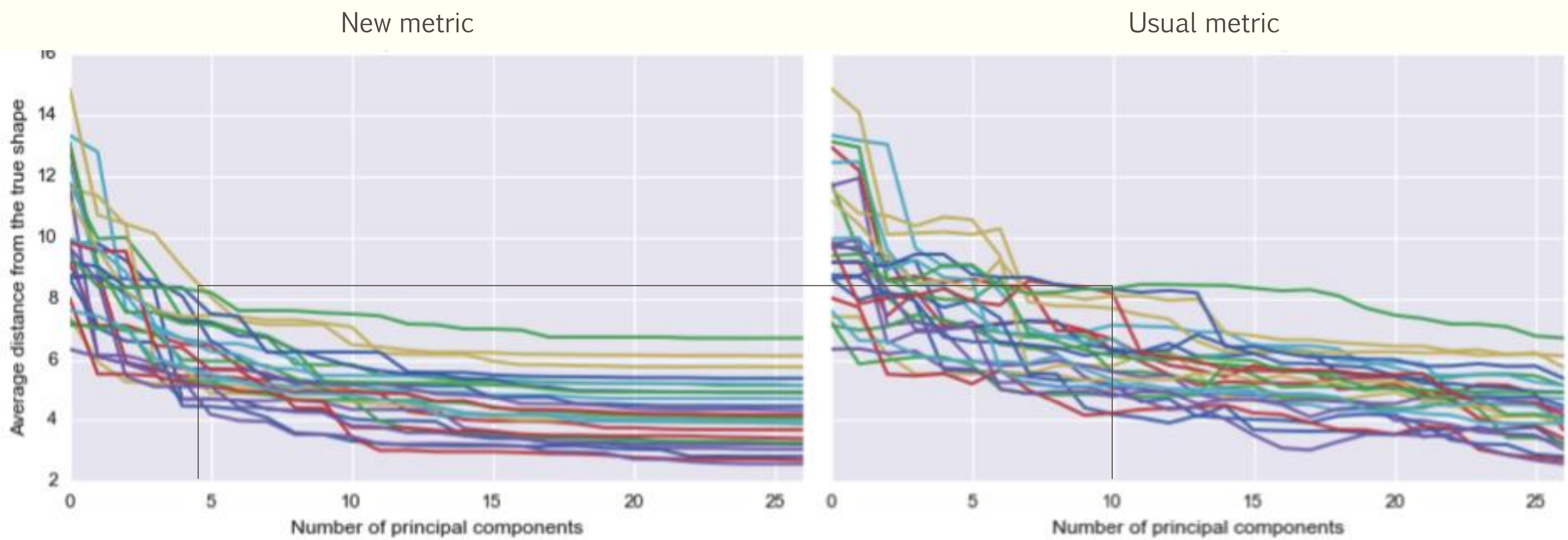
APPLICATION TO THE RIB CAGE

- Reconstruction Evaluation Method



APPLICATION TO THE RIB CAGE

- Reconstruction Errors – New Metric Improvement



CONCLUSION

- ✓ New metric taking into account the shape of the object by integrating the inertia tensor.
- ✓ No rotation/translation empirical normalization needed
- ✓ tPCA is easy to implement and fast.
- ✓ More suitable for elongated shapes – Ribs for instance

Perspectives:

- Looking for correlation with anthropometric parameters to characterize a population.
- Adding intrinsic shape variations to the pose variations model.
- Application to crash simulation with a more representative database (130 subjects)