

Confirmation of Hypothesis on Cable Properties for Cable-driven Robots

Julien Alexandre dit Sandretto¹, Gilles Trombettoni^{1 2} and David Daney¹

¹*Coprin INRIA, France e-mail:*

{Julien.Alexandre_dit_Sandretto,David.Daney}@inria.fr

²*IRIT, I3S, University of Nice–Sophia, France e-mail: Gilles.Trombettoni@inria.fr*

Abstract. In the cable-driven robot studies, the mass and the elasticity of cables are often neglected, particularly for small-sized robots. Indeed, this assumption allows one to simplify the robot model and is used in control, design or calibration. We propose in this paper a method using interval analysis to judge the validity of this hypothesis in a given workspace, whatever are the cable characteristics, i.e., the applied tensions and the robot configuration.

Key words: Cable-driven robot, interval analysis

1 Introduction

Parallel cable-driven robots have several interesting properties such as a reduced mass of mobile parts (i.e., cables have a mass negligible compared to the load), and a potentially large workspace. They are used in several applications, including flying camera systems [13], heavy load transportation, contour crafting [6].

These robots are structurally similar to classical parallel manipulators, but are driven by cables instead of rigid links. The past studies performed have shown the kinematics complexity of parallel robots [12]. In addition, the unilateral driving property of cables leads to hardly solvable kinematics and dynamics behavior due to the flexibility, mass and elasticity of the cables.

Several studies have been achieved on cable-driven robot kinematics [15], workspace [7] or calibration [11]. However, most of the researches on cable-driven robots use a fundamental hypothesis: the cables driving the platform are mass-less and non-elastic. This hypothesis leads to a simplification which permits control, kinematics, calibration, design, as a classical redundant Gough platform for example. In this paper, we propose a way to verify the validity of this hypothesis in the whole workspace of a given robot.

In robotics, especially when the handled problem is complex, several pioneering researchers have used the capabilities of interval methods. For example, Jaulin et al. showed in their book how to identify parameters [4]; Merlet solved the forward kinematics of a Gough platform [14]; Chablat et al. designed 3 d.o.f parallel

machines [1], and Pott and Hiller performed an optimization process for parallel machines [2].

In these works, interval analysis is used because it can certify properties, which is sometimes crucial in robotics. Indeed, interval analysis [4, 9, 10] can handle the whole continuous space, contrarily to approaches based on discretization that may fail in checking some points.

2 Cable-driven robots

The goal of our national project, named CoGiRo, is to build a giant parallel cable-driven robot. This raises numerous issues: design, mechanical conception, modeling, vision-based control, etc. We will focus on the kinematics. Static is only used for the cable model, but not for the robot equilibrium. Also, we do not deal with the dynamic model.

2.1 Robot description

A parallel **cable-driven robot** is made of a mobile platform (end-effector) connected to a fixed base by m cables. These cables can vary in length by the actuation of m pulleys linked to m rotary engines. The variation in length and tension of cables generates a movement with n degrees of freedom (position and/or orientation). In the sketch presented in Fig. 1, the mobile platform (mobile reference frame Ω_C)

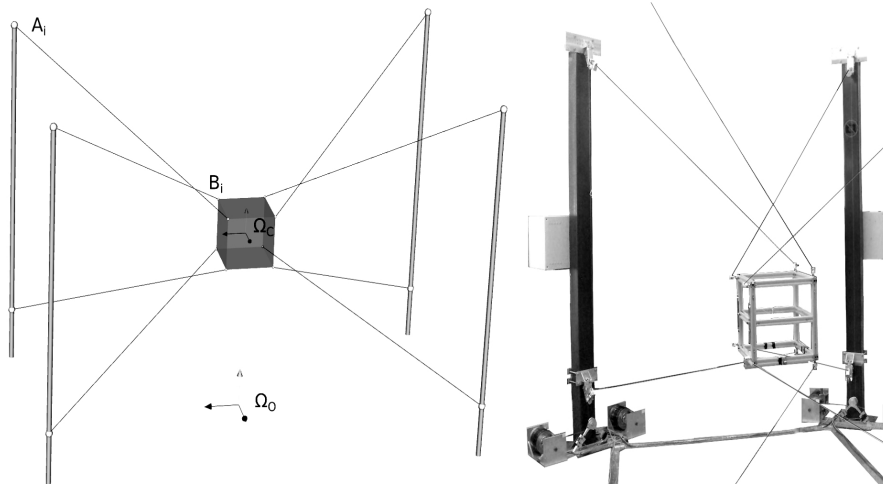


Fig. 1 A cable-driven robot sketch and a ReelAx8 picture

is connected to the base (fixed reference frame Ω_O) by $m = 8$ cables ($m > n$ to be fully controllable [15]). The i^{th} cable connects the point A_i of the base (coordinate

a_i in Ω_O) to the point B_i on the mobile platform (coordinate b_i in Ω_C). The pose of the mobile $X = (P, R)$ (defined by the position P and the orientation matrix R of Ω_C w.r.t. Ω_O) is directly controlled by the length and the tension in each cable. The workspace W_X is the set of all possible pairs (P, R) for the robot.

The prototype, named *ReelAx8* and shown in Fig. 1, was built by the TECNALIA company (www.tecnalia.com) in collaboration with the LIRMM laboratory (www.lirmm.fr). Eight cables, wound on winches, are attached by spherical joints to the eight corners of a cube shaped platform of about 40 centimeters large. Four pairs of winches are fixed on posts up to three meters high arranged at the four corners of a 3 meters by 4 meters rectangle. The prototype is given with a rectangular and centered workspace (see Fig. 3) of 2 by 1 meter on floor, 1 meter high and ± 5 degrees of rotation on each axis: $W_X = \{X = [P, R], P \in [1, 3] \times [1, 2] \times [1, 2], R \in [-5, 5] \times [-5, 5] \times [-5, 5]\}$.

2.2 Cable model

Cable-driven robots take advantage of the use of cables, providing a large workspace, light actuators in comparison to the possible load mass, and low cost. However, cable-driven robots suffer from the complex kinematics and dynamics of cables.

A well-known realistic model that is often used for the kinematics of cables is proposed by Irvine in [5]. In the **Irvine model**, the length of a cable depends on its tension. It is given for one cable and the equations are expressed in a plane made of the points \mathcal{A} and \mathcal{B} and the gravitational force.

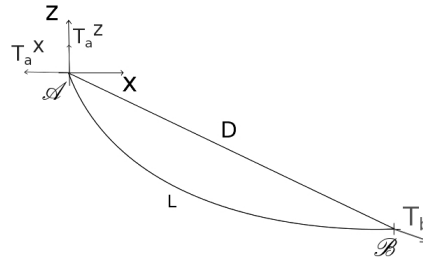


Fig. 2 A cable in a plane $\{\mathcal{A}, \mathcal{B}, \vec{P}\}$

The Irvine model considers the geometric and static parameters of the configuration and the cable properties: attachment points \mathcal{A} (on base) and \mathcal{B} (on platform); linear mass m , tightness k and length L of the cable; applied tensions T_b in \mathcal{B} and T_a in \mathcal{A} . The system of three equations to be solved in order to obtain the actual length of cable and the tension distribution on point $\mathcal{A} = [0, 0]$ is:

$$\begin{cases} \mathcal{B}_x = & \frac{T_a^x L}{k} + \frac{|T_a^x|}{mg} [\sinh^{-1}(\frac{T_a^z}{T_a^x}) - \sinh^{-1}(\frac{T_a^z - mgL}{T_a^x})] \\ \mathcal{B}_z = & \frac{mgL^2}{k} (\frac{T_a^z}{mgL} - \frac{1}{2}) + \frac{1}{mg} [\sqrt{T_a^{x^2} + T_a^{z^2}} - \sqrt{T_a^{x^2} + (T_a^z - mgL)^2}] \\ T_b = & \sqrt{T_a^{x^2} + (T_a^z - mgL)^2} \end{cases} \quad (1)$$

The fact that a robot is controlled with cables having a complex kinematics leads to some problems in the classical fields of robotics:

- complex modeling, control and design;
- unworkable existing methods for the workspace determination;
- unfeasible self-calibration.

The hypothesis of non-elastic and mass-less cables is very useful to simplify the modeling. Moreover, this hypothesis is often realistic and generates a negligible error in robot accuracy. Most of the papers dealing with these subjects use this hypothesis and replace the real length of cables L (depending on tensions) by the distances $D = AB$. Under this assumption, the model is highly simplified.

3 Checking of non-elasticity and mass-less hypothesis

The hypothesis of non-elasticity and mass-less done on cables properties has to be checked on one cable before any static or dynamic modeling (currently not mastered by the community). Our problematic is therefore to verify this hypothesis in the whole workspace of the robot to bring the guarantee that the simplification is valid.

For this purpose, we compute the errors $\sigma_i = |L_i - D_i|$ made between the length L_i given by Irvine's model -function of B_i , T_{B_i} and the cable parameters- and the distance D_i , only function of B_i . B_i itself is function of $X \in W_X$. We then verify that these errors all lie under an acceptable threshold ε (which could be selected in function of the expected articular accuracy): $\sigma_i \leq \varepsilon$, $i = 1..m$. The coordinates of B_i in Ω_O are $e_i = P + R.b_i$, b_i being the coordinates of B_i expressed in the platform reference Ω_C (defined by the platform geometry). The hypothesis is verified on a pose $X \in W_X$, if for each of the m points e_i : $\sigma_i \leq \varepsilon$.

Therefore, we can define the subset S_X of acceptable poses as follows:

$S_X = \{X \in W_X, \forall i \in 1..m : \sigma_i \leq \varepsilon\}$. The proposed verification consists in checking the hypothesis in all the poses of the workspace. A sufficient condition is based on the dual set $S_{!X} = \{X \in W_X, \exists i \in 1..m : \sigma_i > \varepsilon\}$. Interval methods can determine if $S_{!X} = \emptyset$, which implies the hypothesis holds on W_X .

Moreover, the m points B_i depending on X all belong to the same parallelepiped, whatever can be $X \in W_X$. Thus, it is sufficient to test the hypothesis for only one cable. In addition, the parallelepiped built with the e_i is entirely covered by the diagonal plane with a simple rotation around the z axis. Overall, the study of the workspace W_X can be reduced to the one of the diagonal plane W_B , as shown in Fig. 3. The point $B \in W_B$ could be expressed in the plane reference frame like in

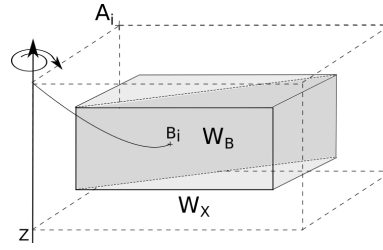


Fig. 3 Robot space frame in dashed lines, workspace W_X and diagonal plane W_B

Irvine's model definition: $B = [\mathcal{B}_x, \mathcal{B}_z]$. With this simplification, we define the subspace of point B where the hypothesis is valid: $S_B = \{B \in W_B : \sigma \leq \varepsilon\}$. And we will introduce the complementary set: $S_{!B} = \{B \in W_B : \sigma > \varepsilon\}$. We remark that $S_B \cup S_{!B} \equiv W_B$. By construction of the simplification, if a solution is found in $S_{!B}$, a solution exists in $S_{!X}$ and the hypothesis is not valid in the whole workspace. It is also easier to find zero solution in $S_{!B}$ than to prove that $S_B \equiv W_B$. Proving that $S_{!B}$ has no solution implies that the hypothesis $\sigma \leq \varepsilon$ is verified for every point in the workspace.

4 Interval methods for constraint satisfaction and optimization

This problematics demands a rigorous solver which could consider a whole space made of an infinity of points and give a reliable result. Interval analysis meets this requirement by using algorithmic principles exploiting constraints and sub-spaces containing an infinity of points, without risk of solution loss.

An interval $[x_i] = [\underline{x}_i, \overline{x}_i]$ defines the set of reals x_i s.t. $x_i \leq x_i \leq \overline{x}_i$. A **box** $[x]$ is the Cartesian product of intervals $[x_1] \times \dots \times [x_i] \times \dots \times [x_n]$. Its width is defined by $\max_i w([x_i])$.

Interval methods for solving a constraint system

Interval methods can accurately approximate, by boxes, the set of solutions of a constraint system. The solving process starts from an initial box representing the search space and builds a search tree, following a *Branch & Contract* scheme:

- *Branch*: the current box is **bisected** on one dimension (variable), generating two sub-boxes.
- *Contract*: filtering (also called *contraction*) algorithms reduce the bounds of the box with no loss of solution.

The process terminates with *atomic* boxes of size at most ε on every dimension. Contraction algorithms comprise interval Newton-like algorithms issued from the numerical interval analysis community [9] along with algorithms from constraint programming.

Constrained optimization

Interval methods can also deal with a more difficult problem, constrained optimization, in which a solution must be found that minimizes an *objective* function while satisfying a set of constraints. To do so, the strategy follows a branch and bound schema [8]. At each iteration, the algorithm selects in the list a box $[x]$. It chooses a branching variable $x_i \in x$ heuristically, bisects $[x_i]$ and applies the main Contract&Bound procedure on the two sub-boxes. In addition to the contraction phase mentioned above, the procedure *Contract & Bound* resorts to a *lower bounding* phase updating a cost lb (that the objective function will never reach for sure) and an *upper bounding* phase searching for a solution that improves the current best cost ub [8]. The search terminates when $ub - lb$ reaches a precision ε_{obj} .

5 Experimentation

The solving follows the principles briefly described in Section 4. It is implemented with an interval tool developed in the COPRIN team.

Case 1: existing prototype

The cables used have the following characteristics: $k = 137kN/m$, $m = 0.007kg/m$. With the workspace W_X introduced in the description of the robot, the plane W_B to be tested is the diagonal plane of the rectangular parallelepiped $[1, 3] \times [1, 2] \times [1, 2]$ and $W_B = [1, 3.7] \times [1, 2]$. We fix $\varepsilon = 0.005m \simeq$ expected accuracy of robot.

The sensors give, during our tests, a minimal tension of $20N$ and a maximal one of $120N$. So $20 \leq T_b \leq 120$.

Case 2: robot under construction

We consider the same architecture robot but with heavier cables and a larger workspace. The cables are made of the same steel, with a tightness $k = 137kN/m$ and a lineic mass $m = 0.092kg/m$. The workspace W_B is $[1, 8] * [1, 10]$ (in meters) and we also expect an accuracy of 1 cm. The tension should fall between $40N$ (without load) and $1000N$ (at maximal load).

5.1 Hypothesis confirmation

We use the constraint system (1) extended with the additional constraint $\sigma > \varepsilon$. Due to interval methods, we compute an overestimate of the subset S_{I_B} , noted $\square S_{I_B}$. If no solution is found in $\square S_{I_B}$, no solution exists in S_{I_B} , and the hypothesis is valid in the considered workspace.

Case 1: No solution is found by our tool, therefore the hypothesis is acceptable for the studied robot. The model using the simplification is thus sufficiently accurate. The solving process achieved in the whole workspace takes about 2 hours. For a reduced workspace, for example one by one meter, the resolution is performed in about 10 minutes.

Case 2: A solution for $\square S_{1B}$ is immediately found (≈ 1 second). The hypothesis seems therefore too strong, which suggests that a more complex model must be developed for the giant robot under construction. Otherwise, the robot model accuracy could be highly deteriorated.

5.2 Quantifying the error

In addition to the yes/no results obtained about the hypothesis validation, global optimization gives the opportunity to enrich the knowledge about the robot. Indeed, we could expect additional quantified information, such as:

- the minimal tension satisfying the hypothesis, defined by $S_{T_b} = \text{Min}_{T_b}, \forall B \in S_B$,
- the maximal error committed in the workspace, defined by $S_\sigma = \text{Max}_\sigma, \forall B \in W_B, \forall T_b \in [T_{min}, T_{max}]$.

Case 1: The analysis of the prototype model (for which the hypothesis has been proven acceptable by our verification method) provides useful information gathered in Table 1:

- minimal T_b to keep $|\sigma| < \varepsilon$, see Table 1, column 1;
- maximal $|\sigma|$ for $T_b = 20N$, see Table 1, column 2;
- maximal $|\sigma|$ for $T_b = 120N$, see Table 1, column 3;
- maximal T_b to keep $|\sigma| < \varepsilon$, see Table 1, column 4.

The values found confirm the hypothesis validation.

Case 2: The same model analysis protocol is followed for the cable-driven robot under construction and the results are presented in Table 1:

- maximal $|\sigma|$ for $T_b = 40N$, see Table 1, column 5;
- minimal T_b to keep $|\sigma| < \varepsilon$, see Table 1, column 6;
- maximal T_b to keep $|\sigma| < \varepsilon$, see Table 1, column 7;
- maximal $|\sigma|$ for $T_b = 1000N$, see Table 1, column 8.

The values found confirm the hypothesis rejection, even if the lower tension bound is close to the minimal tension for which the hypothesis is valid.

	1	2	3	4	5	6	7	8
Error σ (m)	0.005	0.0006	-0.0037	-0.005	0.01009	0.01	-0.01	-0.093
Tension T_b (N)	1.4	20	120	171.5	40	40.1	121	1000
Solv. time (s)	200	3	5	30	72	3	6000	5

Table 1 Results (in bold) obtained by optimization processes

6 Conclusion

In our research, we have done the hypothesis of mass-less and non-elasticity of cables for self-calibrating the robot presented in Section 2. Indeed, to self-calibrate a cable-driven robot, we must consider it as a redundantly actuated manipulator.

This redundancy is obtained with the simplified model under non-elasticity and mass-less assumption. If this hypothesis is validated using our method, the simplification is thus acceptable and the robot is self-calibratable. Concerning the robot under construction, the hypothesis is rejected. To self-calibrate this giant crane, we must define a slightly smaller sub-workspace where the hypothesis is acceptable or use a more complex model.

To conclude, we have designed an operational tool for analyzing the difference between a real cable model and a strong simplification of it. The method described in this paper has provided interesting and useful information for our study of cable-driven robots. Its implementation represents a first software version of a dedicated design tool which could be incorporated in an “Appropriate design” approach.

Finally, this dedicated tool can be useful for modeling, designing and optimizing in a reliable way robots, but also other mechanisms that make use of cables.

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