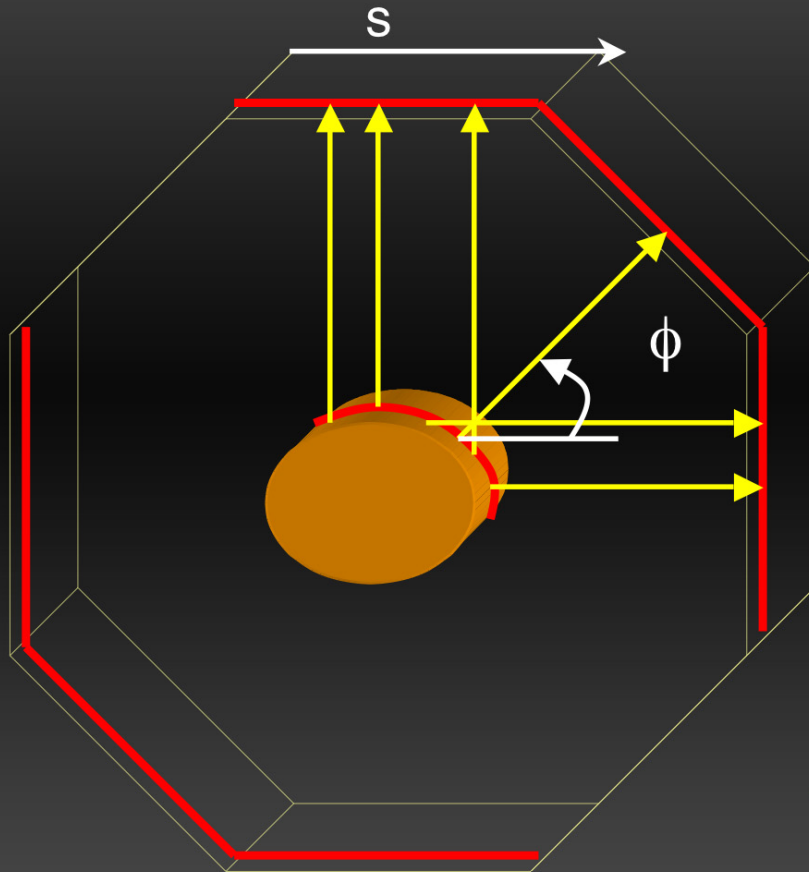
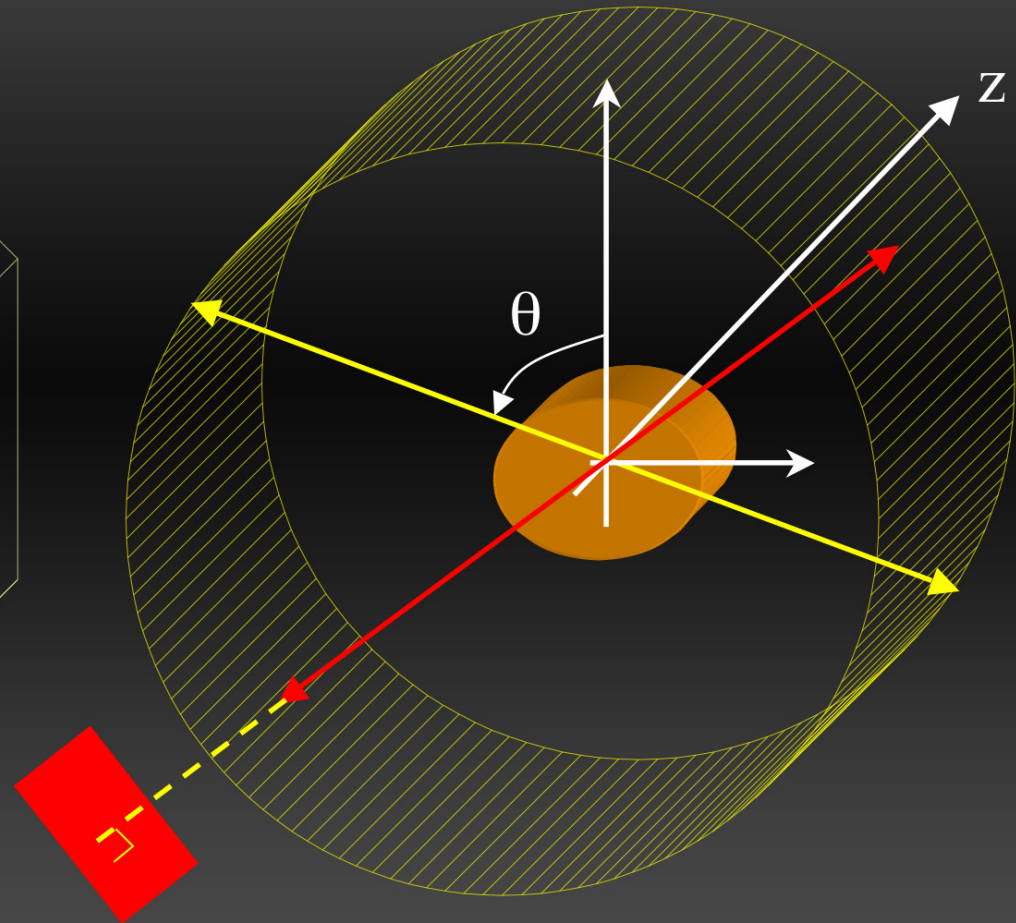


Reconstruction tomographique

Tomographie 2D et 3D



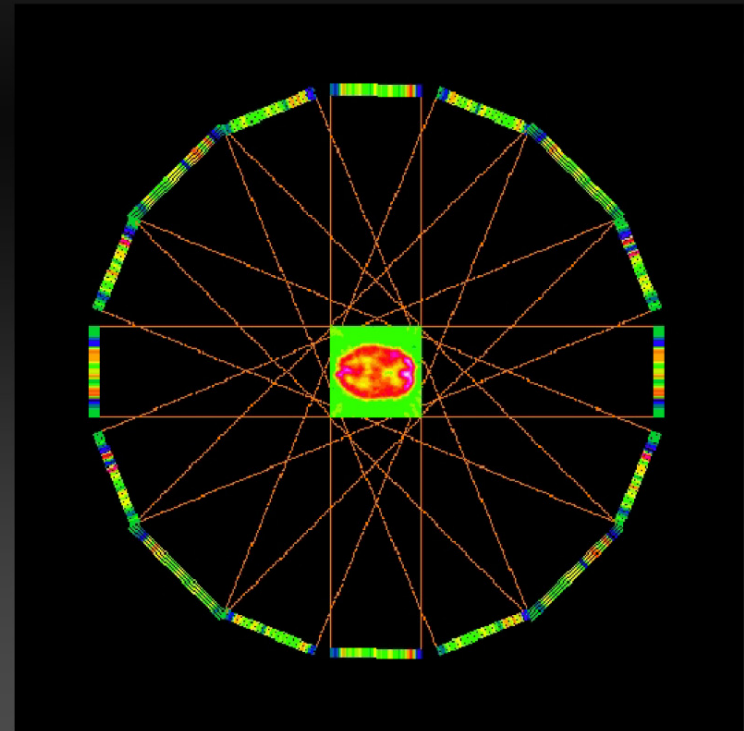
$p(s, \phi) : \text{lignes}$



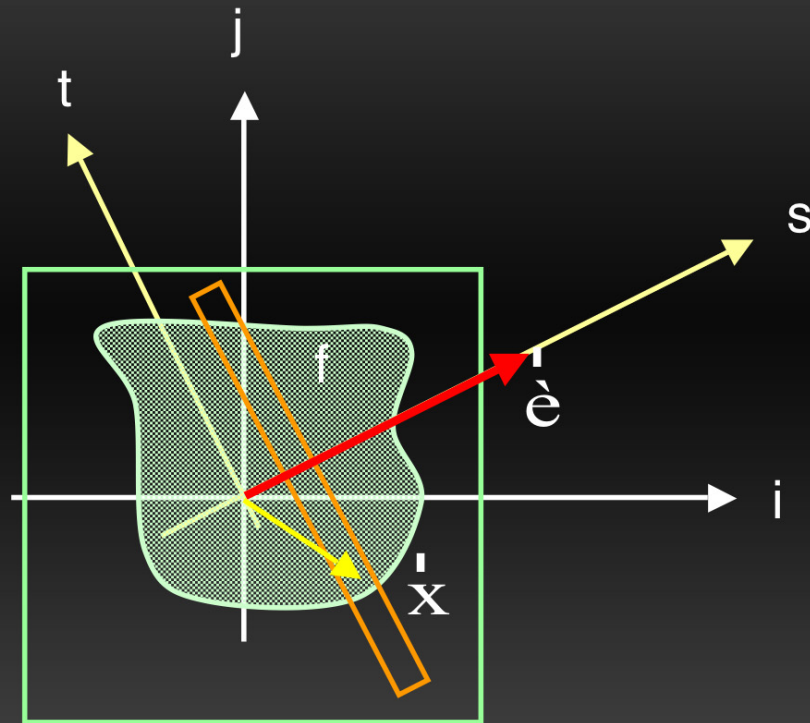
$p(s, \phi, z, \theta) : \text{plans}$

Tomographique 2D

- Modélisations analytique et algébrique
- Théorème de Radon
- Rétroprojection filtrée
- Algorithmes itératifs
- Régularisation



Modélisation analytique (I)



$$p_e^r(s) = \int f(\vec{x}) \cdot d[s - \langle \vec{x}, \vec{e}^r \rangle] \cdot d\vec{x}$$

$$p_e^r(s) = \int_{\langle \vec{x}, \vec{e}^r \rangle = s} f(\vec{x}) \cdot d\vec{x}^r$$

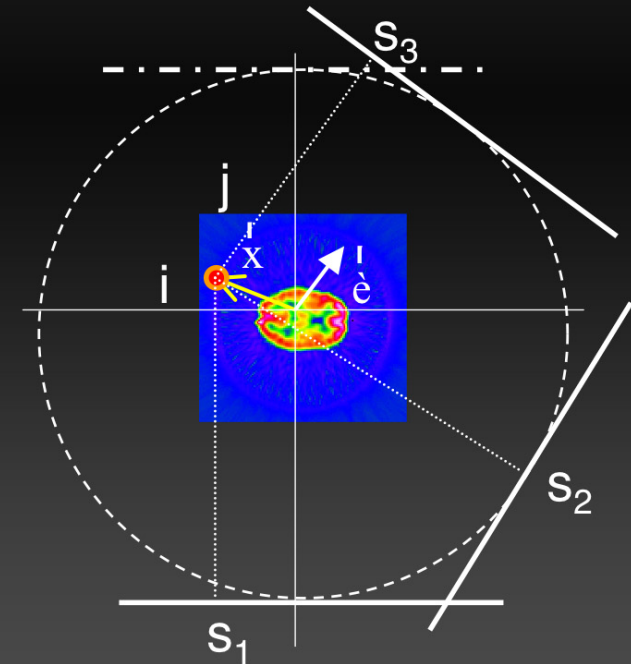
$$\vec{e}^r = \begin{pmatrix} \cos \theta \\ \sin \theta \end{pmatrix} \quad \vec{x} = \begin{pmatrix} i \\ j \end{pmatrix} \quad p_e^r(s) = p(\theta, s)$$

Modélisation analytique (II)

$$(R^* p) \left(\frac{r}{x} \right) = \int_{\vec{e}=0}^{\partial} p \left(\frac{r}{e}, \frac{r}{e} \cdot x \right) d\vec{e}$$

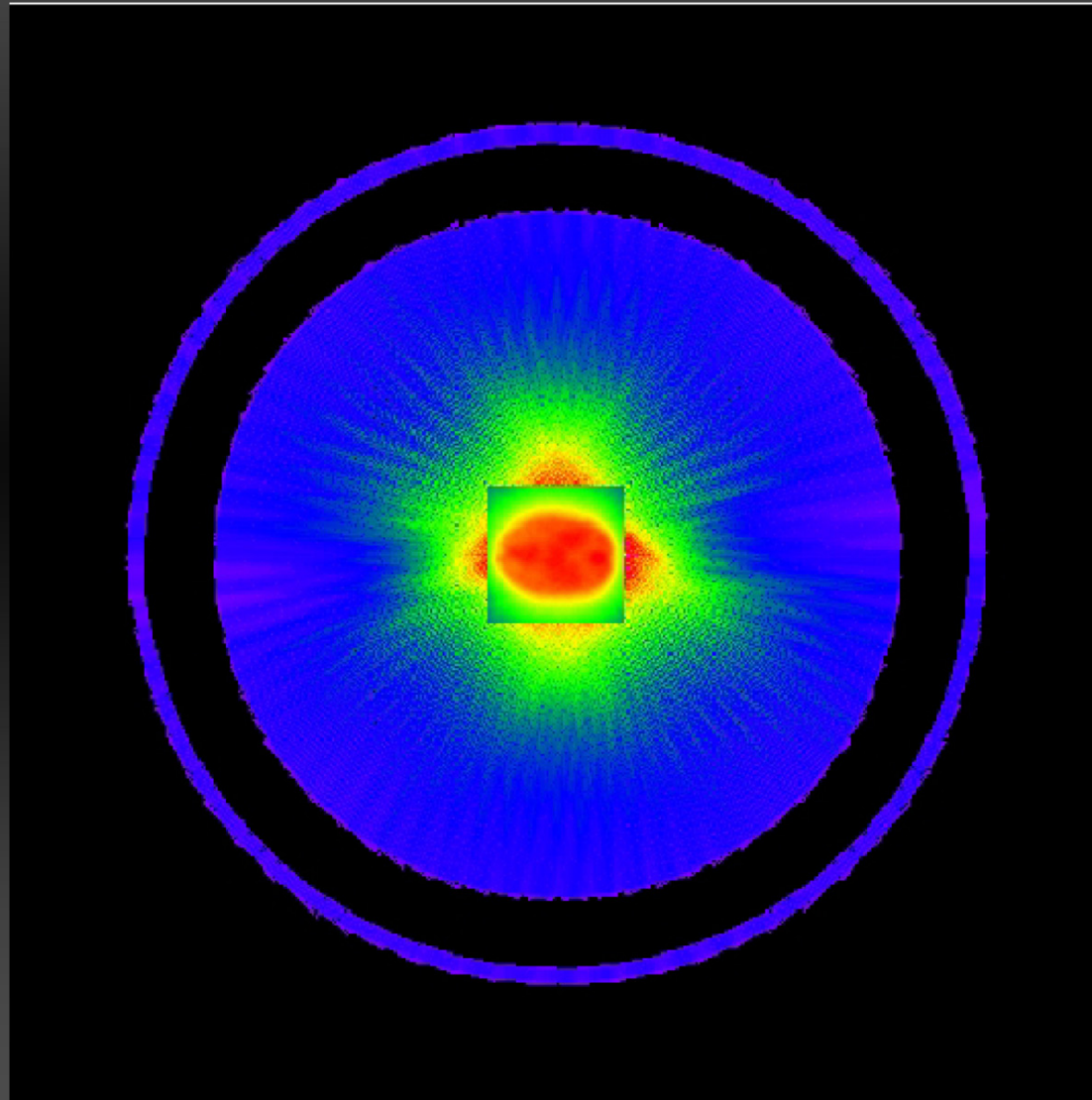
Rétroprojection ou épandage

$$Rm : \quad \langle R f_1, p_2 \rangle = \langle f_1 | R^* p_2 \rangle$$

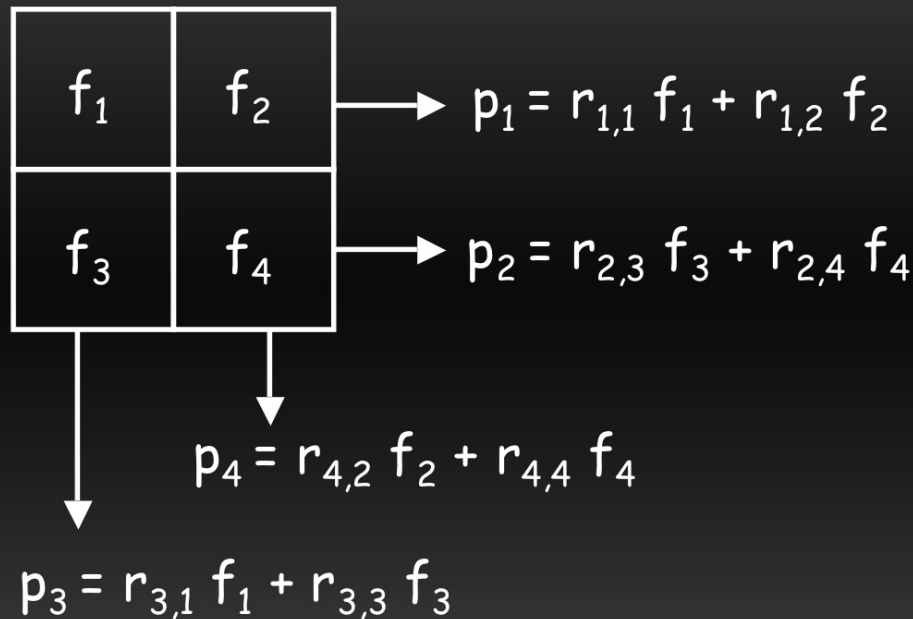


$$s = i.\cos\theta + j.\sin\theta$$

Illustration de l'épandage



Modélisation algébrique (I)



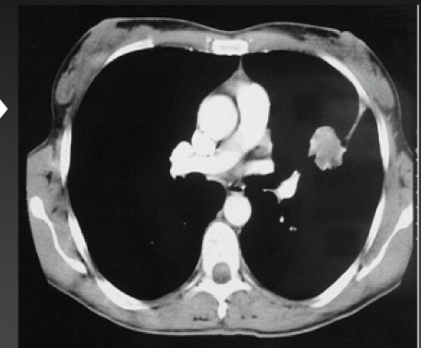
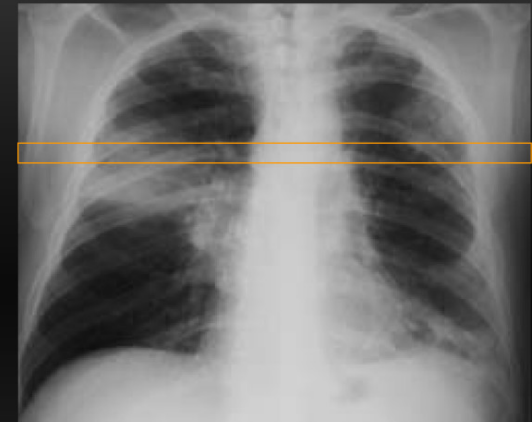
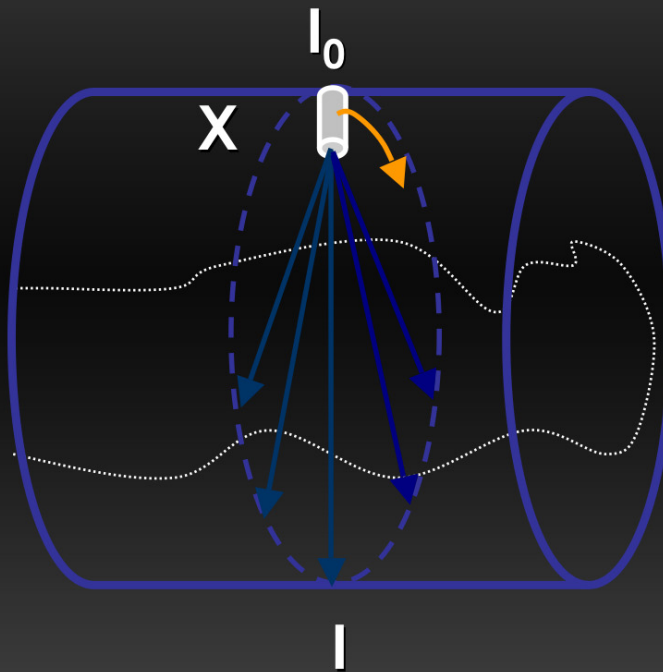
$$\begin{pmatrix} r_{1,1} & r_{1,2} & r_{1,3} & r_{1,4} \\ r_{2,1} & r_{2,2} & r_{2,3} & r_{2,4} \\ r_{3,1} & r_{3,2} & r_{3,3} & r_{3,4} \\ r_{4,1} & r_{4,2} & r_{4,3} & r_{4,4} \end{pmatrix} \cdot \begin{pmatrix} f_1 \\ f_2 \\ f_3 \\ f_4 \end{pmatrix} = \begin{pmatrix} p_1 \\ p_2 \\ p_3 \\ p_4 \end{pmatrix}$$

$r_{i,j}$ = % du pixel j intersecté par la projection i

$$\mathbf{R} \cdot \mathbf{f} = \mathbf{p}$$

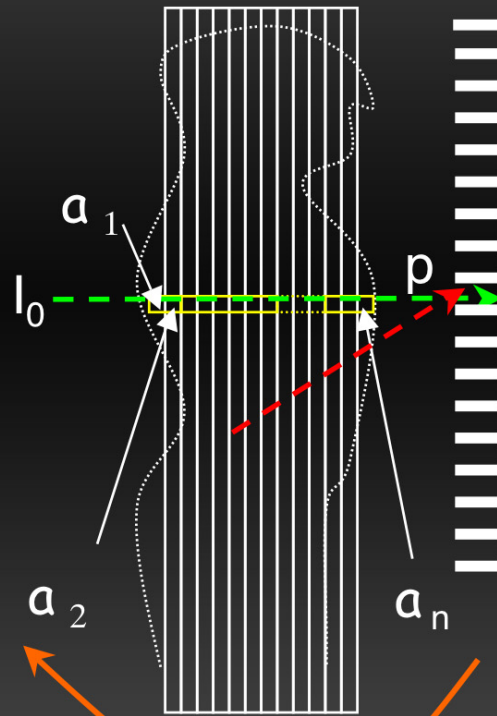
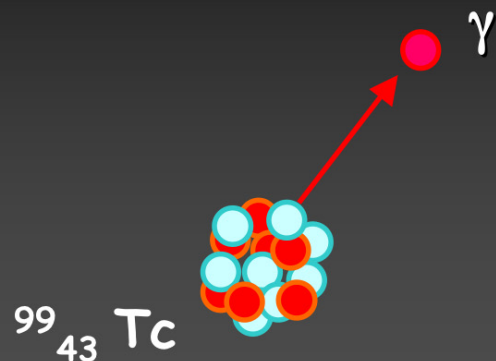
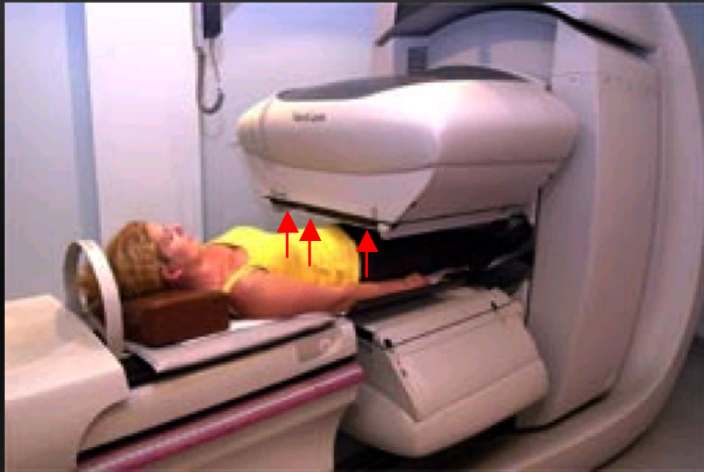
Système linéaire de $n_{\text{équations}}$ et $n_{\text{inconnues}}$

Tomodensitométrie (scanner X)

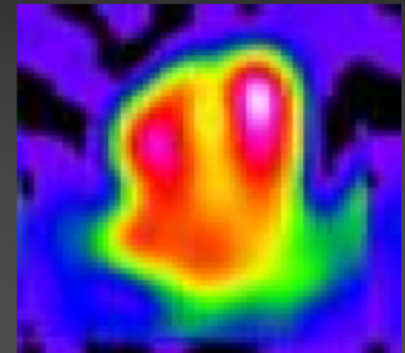
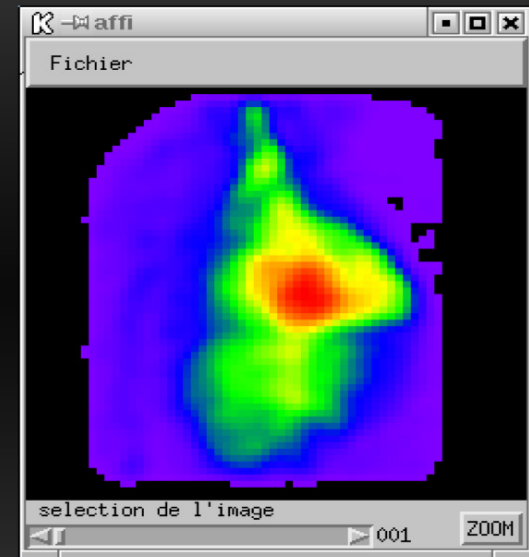


$$p = i_1 + i_2 + \dots + i_n$$

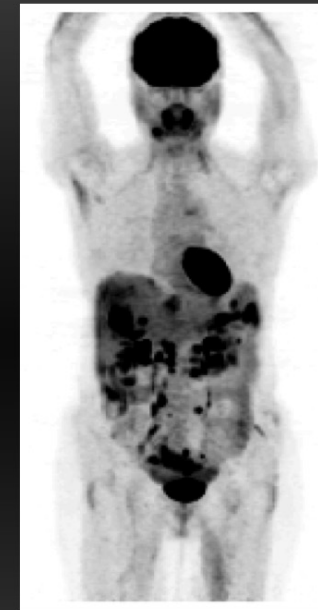
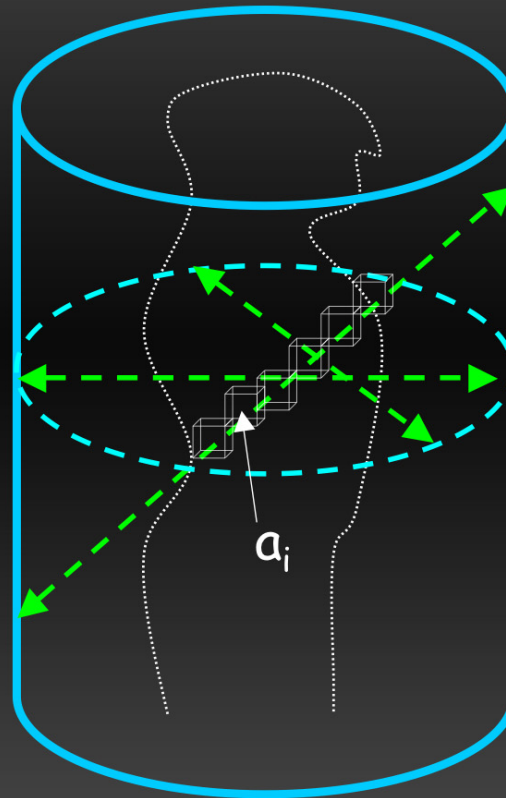
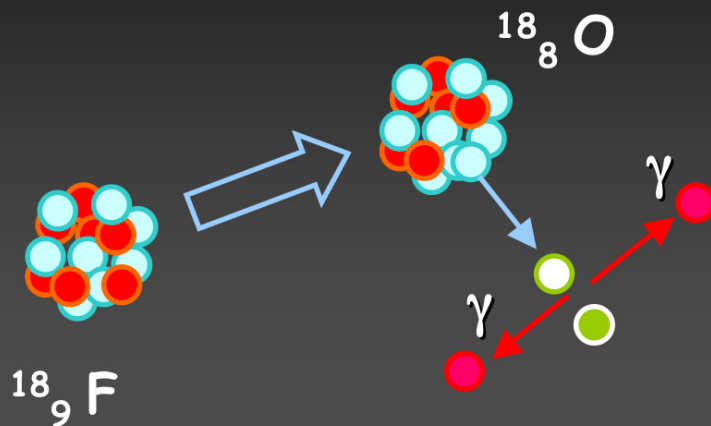
Scintigraphie d'émission mono- photonique γ



$$p = a_1 + a_2 + \dots + a_n$$

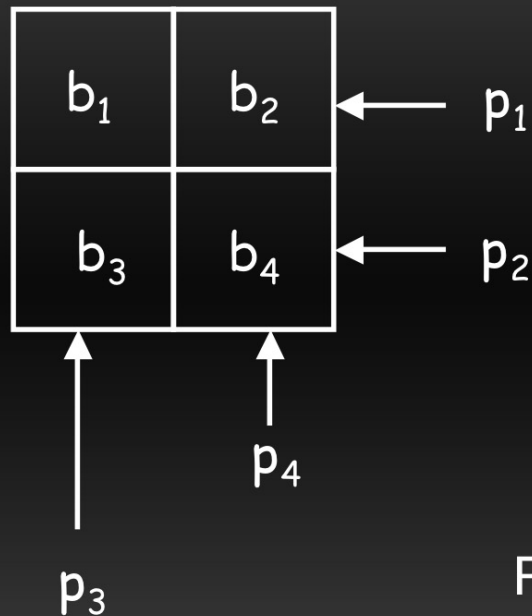


Tomographie en coïncidence 3D



$$p = a_1 + a_2 + \dots + a_n$$

Modélisation algébrique (II)



$$\begin{pmatrix} r_{1,1} & r_{2,1} & r_{3,1} & r_{4,1} \\ r_{1,2} & r_{2,2} & r_{3,2} & r_{4,2} \\ r_{1,3} & r_{2,3} & r_{3,3} & r_{4,3} \\ r_{1,4} & r_{2,4} & r_{3,4} & r_{4,4} \end{pmatrix} \cdot \begin{pmatrix} p_1 \\ p_2 \\ p_3 \\ p_4 \end{pmatrix} = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \\ b_4 \end{pmatrix}$$

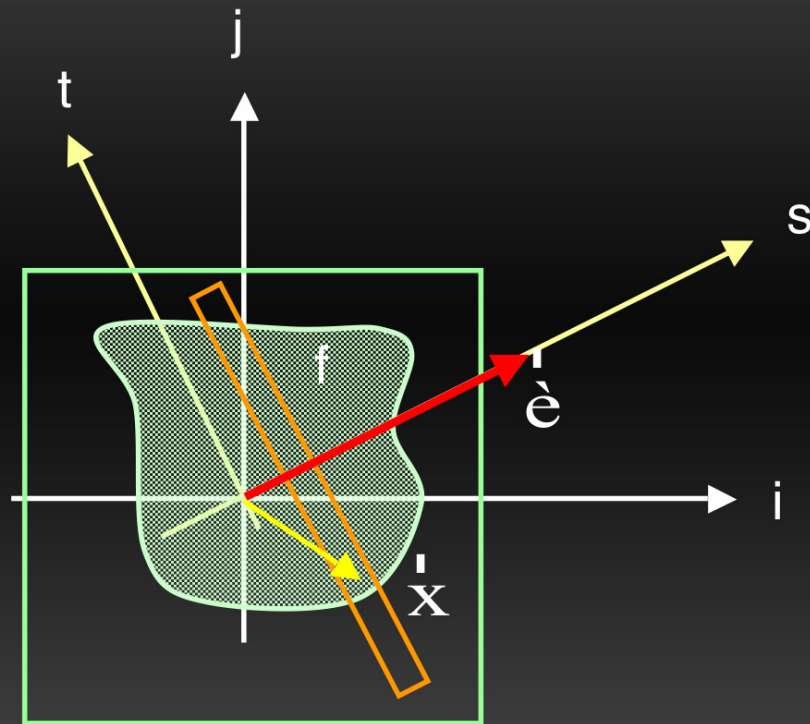
Rétroprojection ou épandage : $R^* \Leftrightarrow {}^tR$

$${}^tR \cdot p = b$$

Problème bien posé de Hadamard

- R surjectif: $({}^tRR)\phi = A\phi = {}^tR \pi = \theta$
 - $A = {}^tRR$ symétrique, définie positive
 - Revient à minimiser $\| R\phi - \pi \|_2$
- R injectif: initialisation du gradient
- R^{-1} continue
 - Si $R(\phi + d\phi) = \pi + d\pi$ alors $\| d\phi \| \leq \| R^{-1} \| \cdot \| d\pi \|$
 - $\| R^{-1} \|$ limitée

Théorème de Radon 2D



$$\begin{cases} p_{\vec{e}}^r(s) = \int_{\langle \vec{x}, \vec{e} \rangle = s} f(\vec{x}) \cdot d\vec{x} \\ U_s(p_{\vec{e}}^r)(\sigma) = \int_s p_{\vec{e}}^r(s) e^{-i.s.\sigma} ds \end{cases}$$

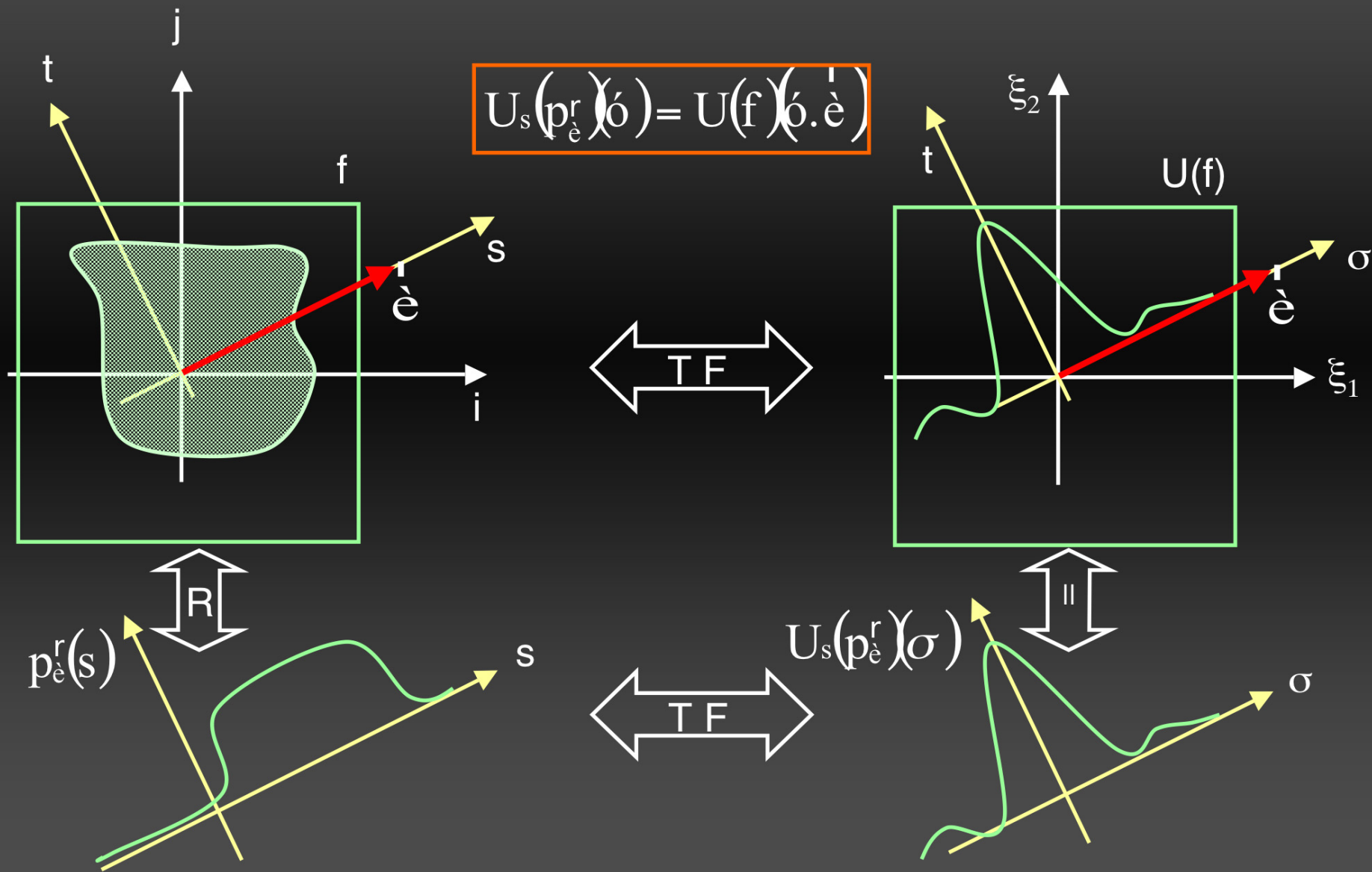
$$U_s(p_{\vec{e}}^r)(\sigma) = \int_s \int_{\langle \vec{x}, \vec{e} \rangle = s} f(\vec{x}) \cdot e^{-i.s.\sigma} ds \cdot dt$$

$$U_s(p_{\vec{e}}^r)(\sigma) = \iint_{\mathbb{R}^2} f(i,j) \cdot e^{-i.\sigma.(i.\cos\theta + j.\sin\theta)} di \cdot dj$$

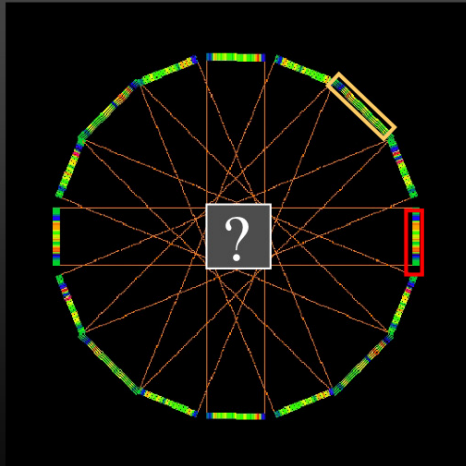
$$\vec{e} = \begin{pmatrix} \cos \theta \\ \sin \theta \end{pmatrix} \quad \vec{x} = \begin{pmatrix} i \\ j \end{pmatrix}$$

$$U_s(p_{\vec{e}}^r)(\sigma) = U(f)(\sigma \cdot \cos \theta, \sigma \cdot \sin \theta) = U(f)(\sigma \cdot \vec{e})$$

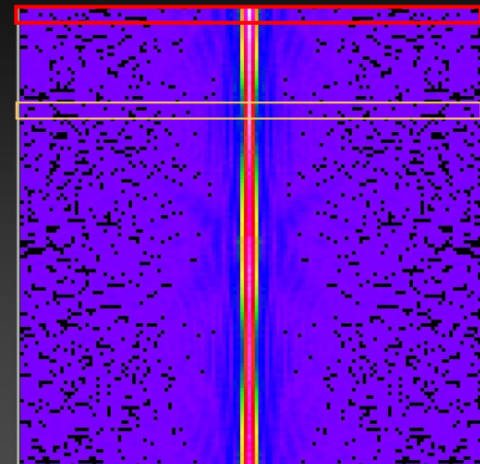
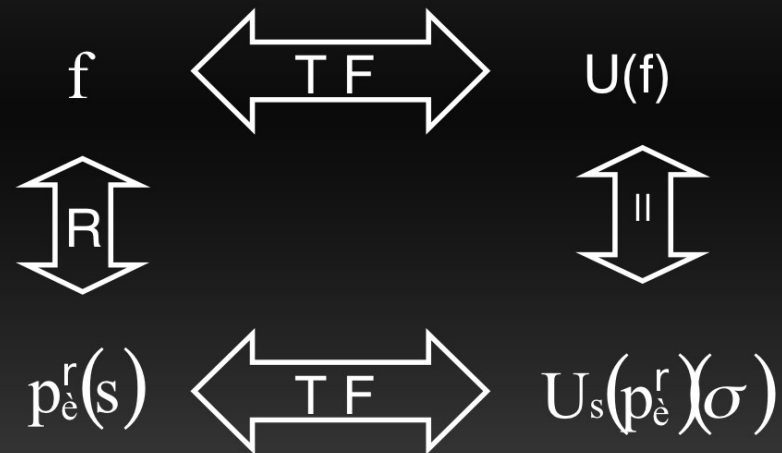
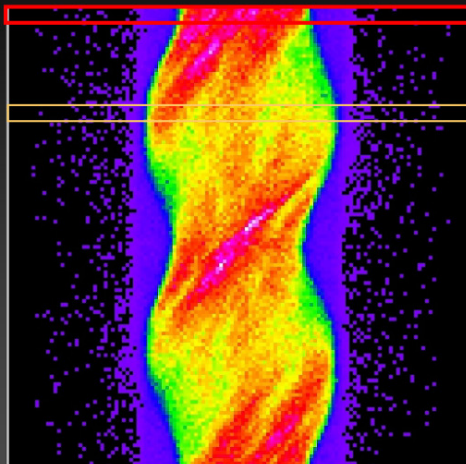
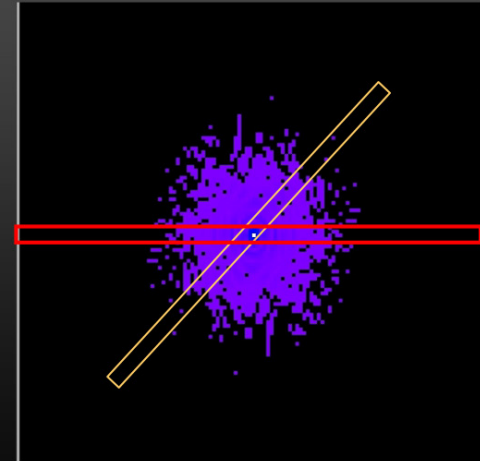
Interprétation (I)



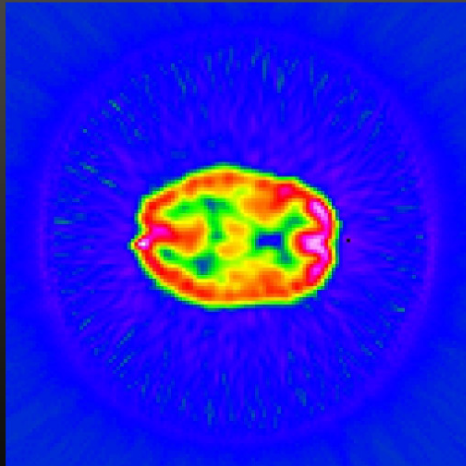
Interprétation (I)



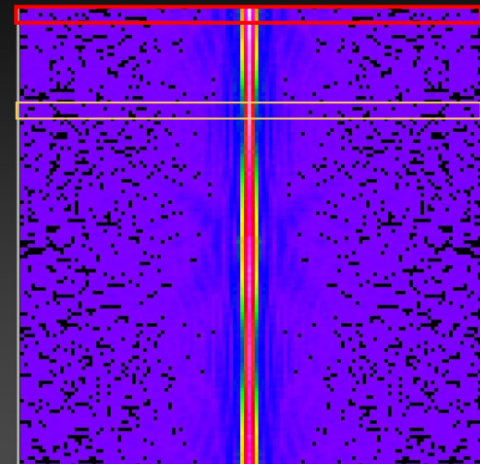
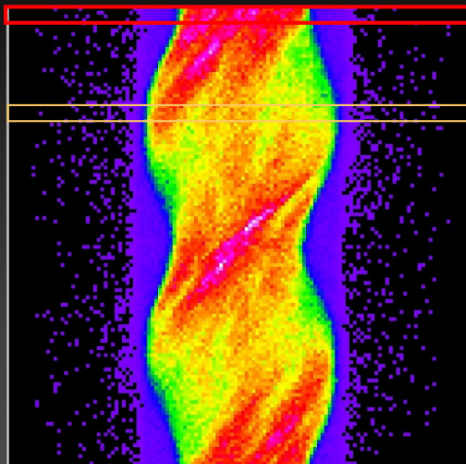
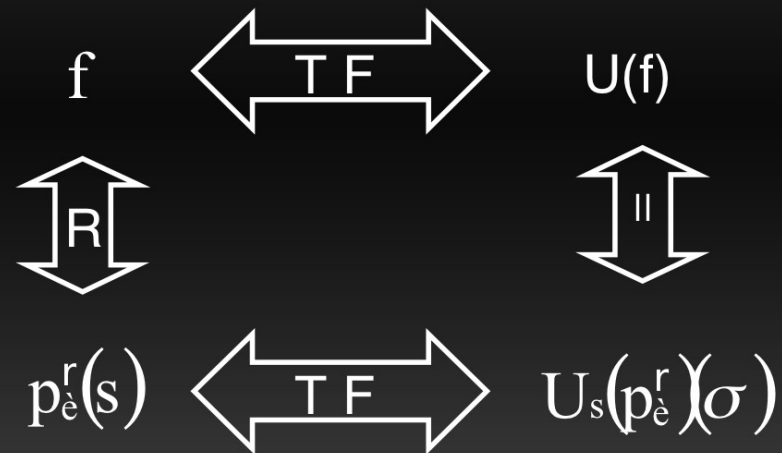
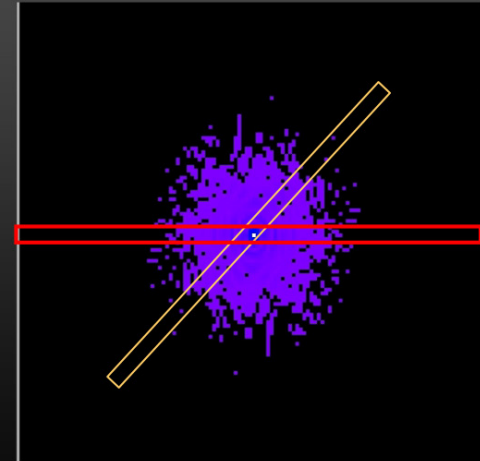
$$U_s(p_e^r)(\sigma) = U(f)(\sigma.e)$$



Interprétation (I)



$$U_s(p_e^r)(\sigma) = U(f)(\sigma, e)$$



Rétroprojection filtrée (I)

$$f(i,j) = \iint U(f)(\hat{i}_1, \hat{i}_2) e^{2i\varphi(i.\hat{i}_1 + j.\hat{i}_2)} d\hat{i}_1 d\hat{i}_2$$

$$f(i,j) = \int_{\varphi=0}^{\varphi=2\pi} \int_{\rho=-\infty}^{+\infty} U(f)(\rho, \varphi) e^{2i\varphi(i.\cos\varphi + j.\sin\varphi)} |\rho| d\rho d\varphi$$

$$f(i,j) = \int_{\varphi=0}^{\varphi=2\pi} \int_{\rho=-\infty}^{+\infty} U_s(p_\theta^r)(\rho) |\rho| e^{2i\varphi(i.\cos\varphi + j.\sin\varphi)} d\rho d\varphi$$

$$f(i,j) = \int_{\varphi=0}^{\varphi=2\pi} U_s^{-1}[U_s(p_\theta^r).abs](i.\cos\varphi + j.\sin\varphi) d\varphi$$

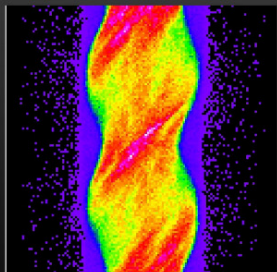
Rétroprojection filtrée (II)

$$f(i,j) = \int_{\vec{e}=0}^{\partial} U_s^{-1} [U_s(p_{\vec{e}}^r) \cdot \text{abs}] (i \cdot \cos \vec{e} + j \cdot \sin \vec{e}) d\vec{e}$$

$$\begin{cases} p_{\vec{e}}^r(s) = p'(\vec{e}, s) = U_s^{-1} [U_s(p_{\vec{e}}^r) \cdot \text{abs}](s) \\ (R^* p) \left(\begin{smallmatrix} r \\ x \end{smallmatrix} \right) = \int_{\vec{e}=0}^{\partial} p \left(\vec{e}, \begin{smallmatrix} r \\ \vec{e} \cdot x \end{smallmatrix} \right) d\vec{e} \end{cases}$$

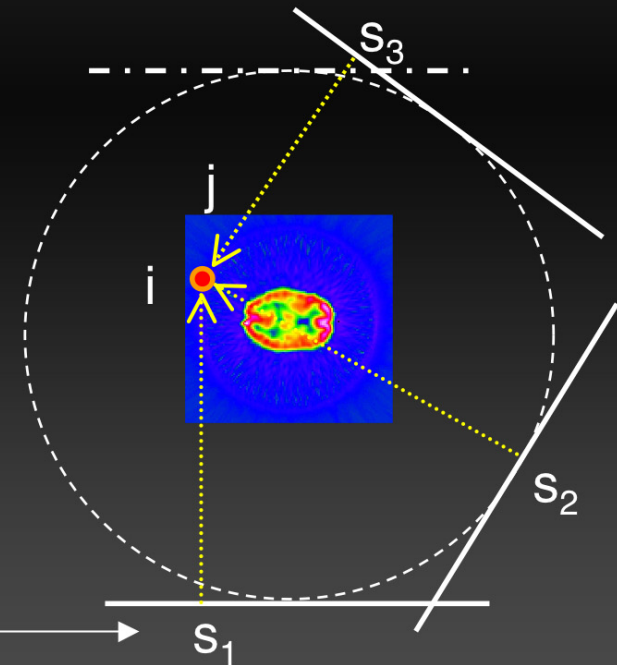
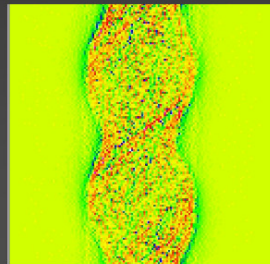
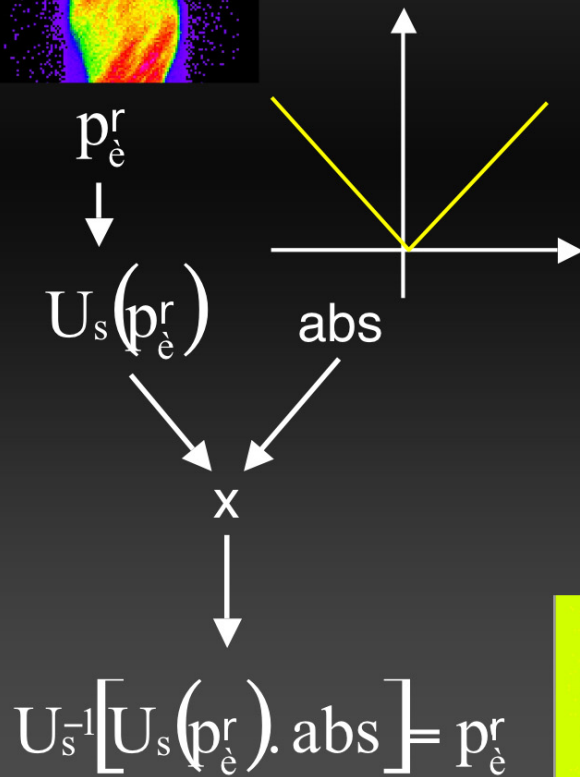
$$f \left(\begin{smallmatrix} l \\ x \end{smallmatrix} \right) = (R^* p') \left(\begin{smallmatrix} l \\ x \end{smallmatrix} \right)$$

Rétroprojection filtrée (III)



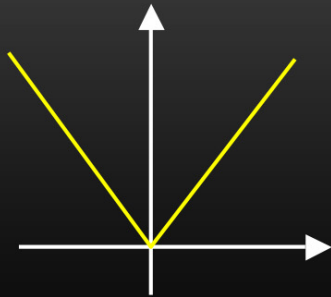
$$f(i,j) = (R^* p')(i,j)$$

Projections sur 180°

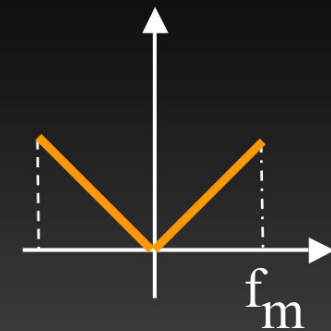


$$s = i.\cos\theta + j.\sin\theta$$

Rétroprojection filtrée (IV)



$$TF(|x|) = \frac{1}{2\partial^2} \frac{d}{dx} V_p\left(\frac{1}{x}\right)$$

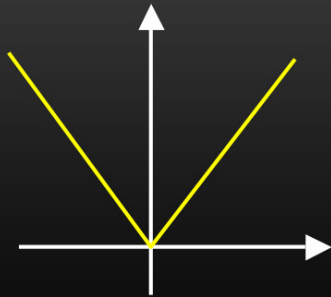


$$f_m = \frac{1}{2 \cdot \ddot{A}_r} = \frac{f_e}{2}$$

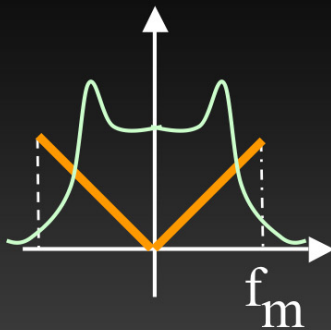
$$RL(x) = \frac{f_m \sin(2\partial f_m x)}{\partial x} - \frac{1 - \cos(2\partial f_m x)}{2\partial^2 x^2}$$

$$RL(k \cdot \Delta r) = \begin{cases} \frac{1}{4(\Delta r)^2} & k = 0 \\ 0 & k \neq 0 \text{ pair} \\ -\frac{1}{(k\partial \Delta r)^2} & k \text{ impair} \end{cases}$$

Rétroprojection filtrée (IV)



$$TF(|x|) = \frac{1}{2\partial^2} \frac{d}{dx} V_p\left(\frac{1}{x}\right)$$

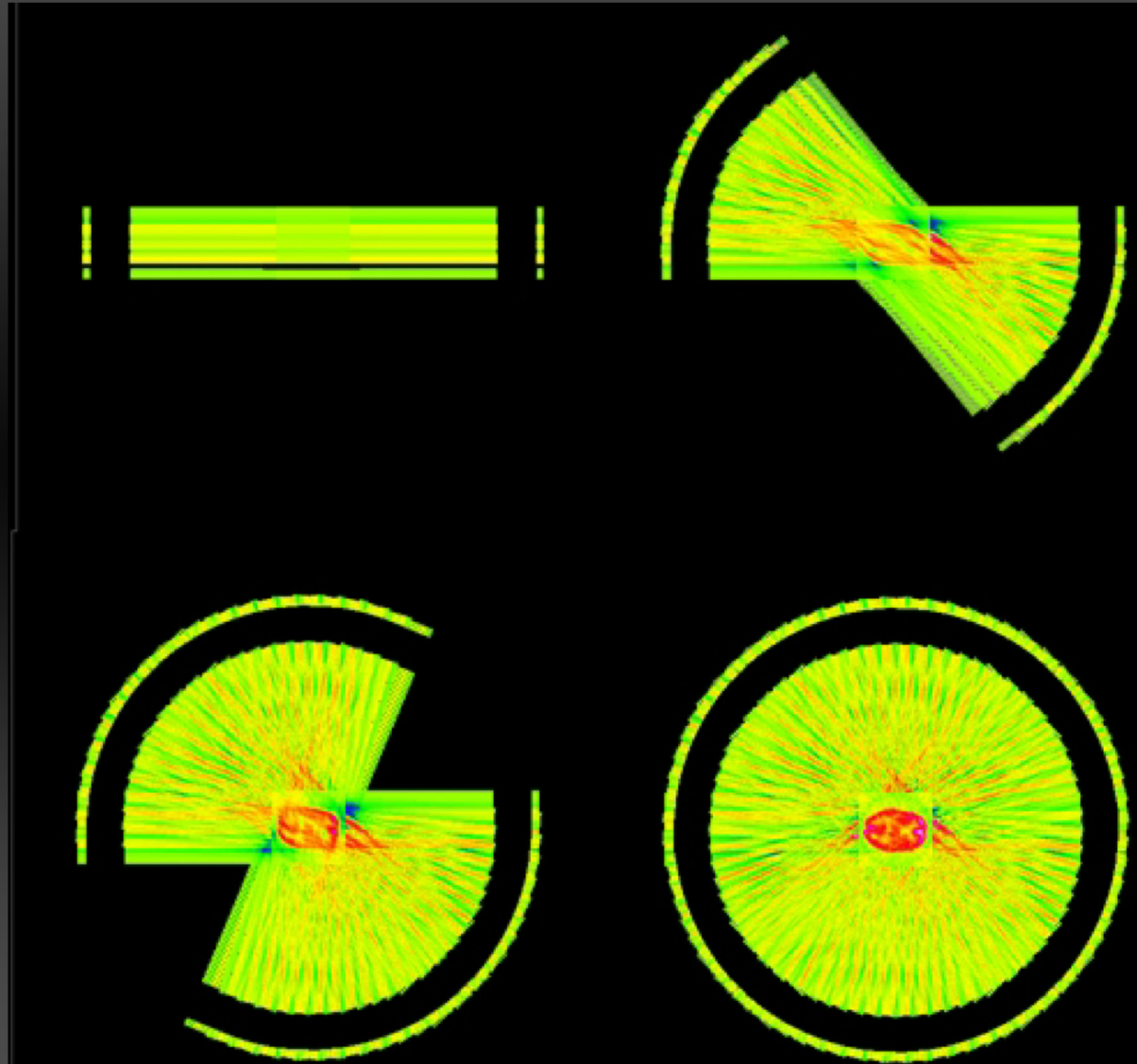


$$f_m = \frac{1}{2 \cdot \ddot{A}r} = \frac{f_e}{2}$$

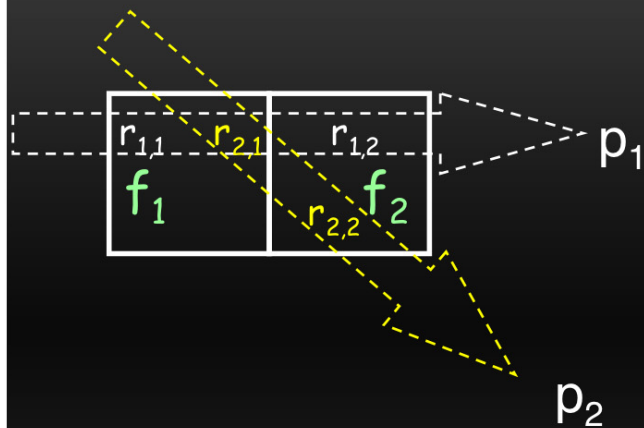
$$RL(x) = \frac{f_m \sin(2\partial f_m x)}{\partial x} - \frac{1 - \cos(2\partial f_m x)}{2\partial^2 x^2}$$

$$RL(k \cdot \Delta r) = \begin{cases} \frac{1}{4(\Delta r)^2} & k = 0 \\ 0 & k \neq 0 \text{ pair} \\ -\frac{1}{(k\partial \Delta r)^2} & k \text{ impair} \end{cases}$$

Rétro-Projection Filtrée

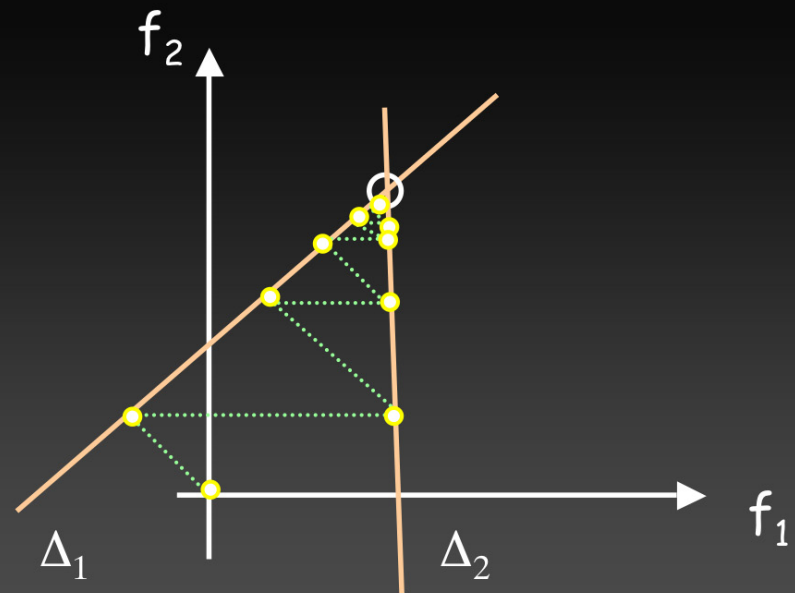


Algorithmes itératifs (ART)



$$\begin{aligned} \Delta_1 & \begin{cases} p_1 = r_{1,1} f_1 + r_{1,2} f_2 \\ p_2 = r_{2,1} f_1 + r_{2,2} f_2 \end{cases} \\ \Delta_2 & \end{aligned}$$

Kaczmarz



Algorithmes itératifs (ART)

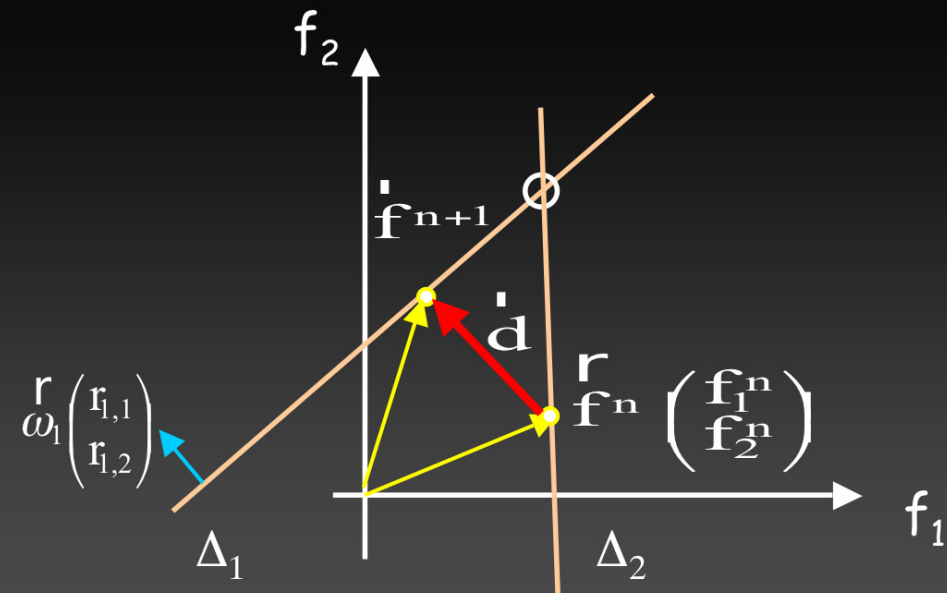
$$\begin{cases} \mathbf{f}^{n+1} = \mathbf{f}^n + d \frac{\mathbf{u}_1}{\|\mathbf{u}_1\|} \\ d = \frac{p_1 - \langle \mathbf{f}^n, \mathbf{u}_1 \rangle}{\|\mathbf{u}_1\|} \end{cases}$$

$$\mathbf{p}_1^n = r_{1,1} \mathbf{f}_1^n + r_{1,2} \mathbf{f}_2^n$$

$$\mathbf{f}^{n+1} = \mathbf{f}^n + \frac{p_1 - p_1^n}{\|\mathbf{u}_1\|^2} \mathbf{u}_1$$

$$\mathbf{f}^{n+1} = \mathbf{f}^n + R^* (p_1 - p_1^n)$$

$$\begin{cases} p_1 = r_{1,1} f_1 + r_{1,2} f_2 \\ p_2 = r_{2,1} f_1 + r_{2,2} f_2 \end{cases}$$



ART

0	0	0
0	0	0
0	0	0

$$\leftarrow 45 - 0 = 15 + 15 + 15$$

$$\leftarrow 90 - 0 = 30 + 30 + 30$$

$$\leftarrow 45 - 0$$

45 90 45

10	25	10
25	40	25
10	25	10

15	15	15
30	30	30
15	15	15

$\Rightarrow$



45 90 45

- 60 60 60

-15 30 -15



Méthodes Algébriques

- MLEM et OSEM :

$$\begin{bmatrix} \mathbf{r} \\ \mathbf{f}^{n+1} \end{bmatrix} = \begin{bmatrix} \mathbf{r} \\ \mathbf{f}^n \end{bmatrix} \cdot \mathbf{R}^* \left[\frac{\mathbf{p}_1}{\mathbf{p}^n} \right]$$

- Maximiser proba (i/p)
- Bruit de Poisson sur p

- Gradient conjugué :

$$\mathbf{f}^{n+1} = \mathbf{f}^n + a_n \cdot \mathbf{R}^* (\mathbf{p} - \mathbf{R} \mathbf{f}^n)$$

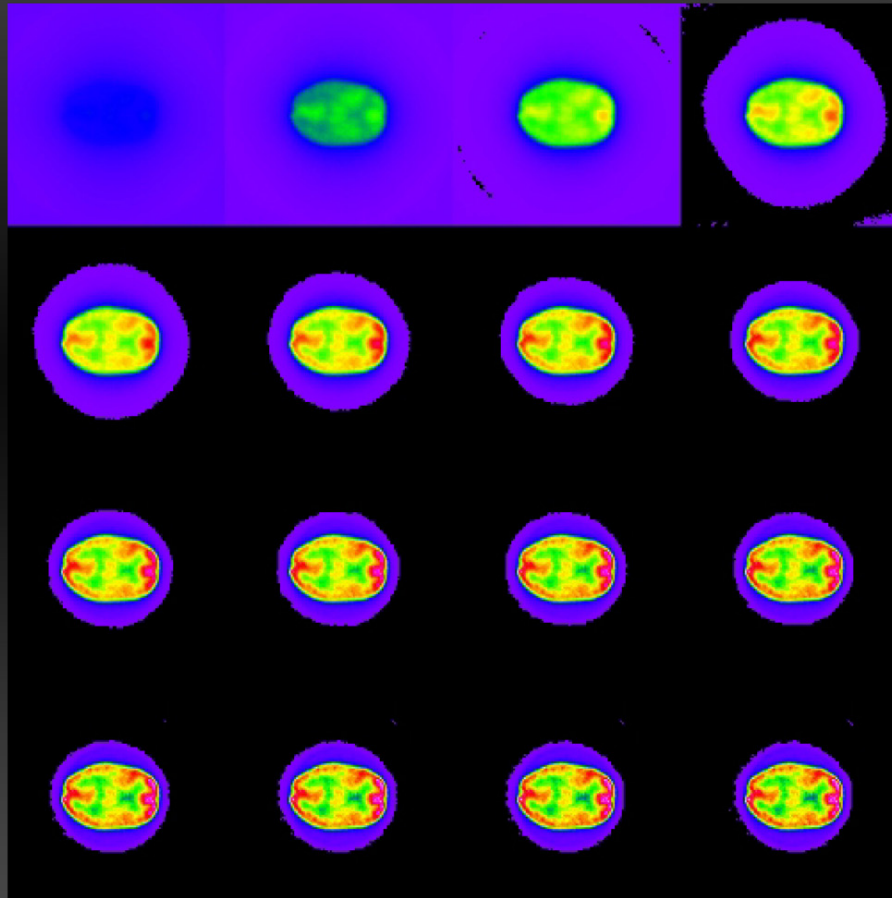
$$\chi^2(\mathbf{f}^n) = \|\mathbf{R} \mathbf{f}^n - \mathbf{p}\|^2$$

$$\nabla \chi^2(\mathbf{f}^n) = \mathbf{R}^* (\mathbf{p} - \mathbf{R} \mathbf{f}^n)$$

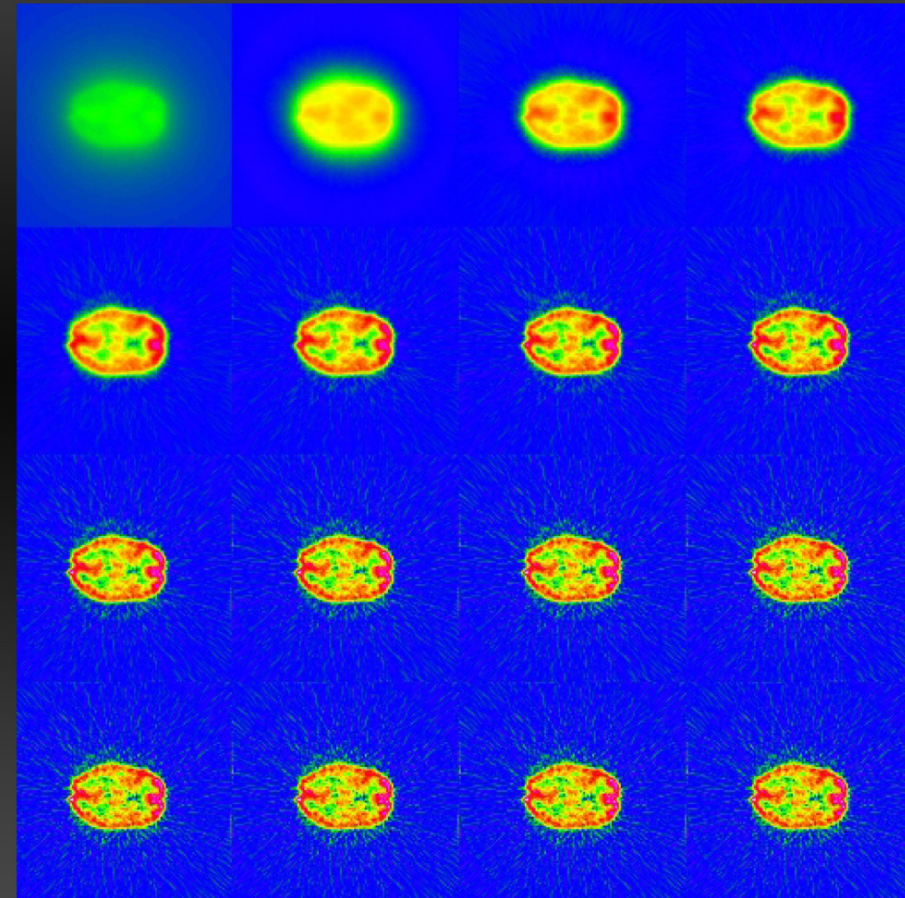
- Minimiser

$$a_n = \frac{\|\nabla \chi^2(\mathbf{f}^n)\|^2}{\langle \nabla \chi^2(\mathbf{f}^n), \mathbf{R} \cdot \mathbf{R}^* (\nabla \chi^2(\mathbf{f}^n)) \rangle}$$

Les itérations

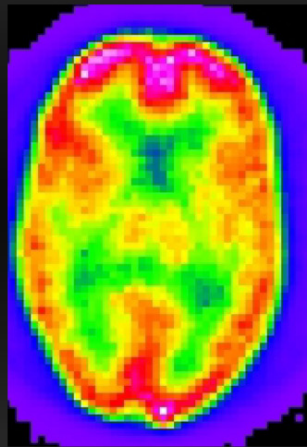


MLEM

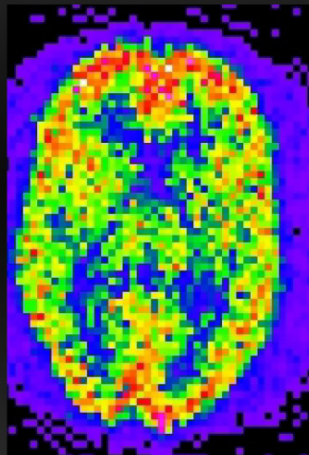


Gradient Conjugué

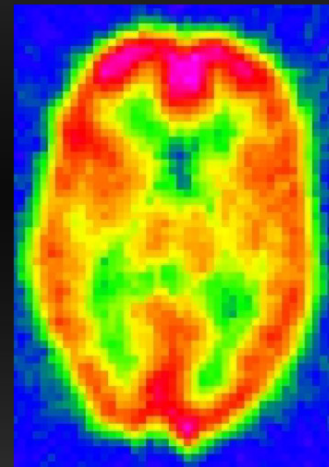
Comparaison des résultats



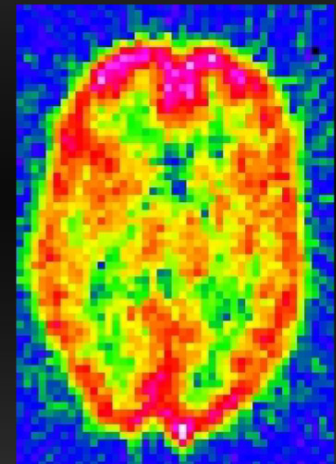
MLEM 6



MLEM 200



GC 6



GC 16

Stabilité

- Hadamard : R bijectif, R^{-1} continue

- Mais :

$$\hat{e}(R) = \|R\| \|R^{-1}\| = \frac{\ddot{e}_{\max}}{\ddot{e}_{\min}}$$

$$\frac{\|\ddot{a}f\|_r}{\|f\|} \leq \hat{e}(R) \frac{\|\ddot{a}p\|_r}{\|p\|}$$

$$\frac{\|\ddot{a}f\|_r}{\|f\|} \leq \frac{\hat{e}(R)}{1 - \hat{e}(R) \frac{\|\ddot{a}R\|}{\|R\|}} \left[\frac{\|\ddot{a}p\|_r}{\|p\|} + \frac{\|\ddot{a}R\|}{\|R\|} \right]$$

Exemple

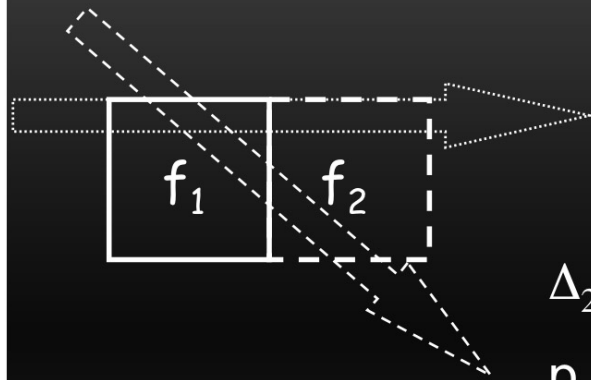
$$\begin{pmatrix} 10 & 7 & 8 & 7 \\ 7 & 5 & 6 & 5 \\ 8 & 6 & 10 & 9 \\ 7 & 5 & 9 & 10 \end{pmatrix} \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 32 \\ 23 \\ 33 \\ 31 \end{pmatrix}$$

$$\text{Sp}(\mathbf{R}) \approx \{0,010; 0,843; 3,858; 30,289\} \Rightarrow \kappa(\mathbf{R}) = \frac{30,289}{0,010} = 2984$$

$$\begin{pmatrix} 10 & 7 & 8 & 7 \\ 7 & 5 & 6 & 5 \\ 8 & 6 & 10 & 9 \\ 7 & 5 & 9 & 10 \end{pmatrix} \begin{pmatrix} 9,2 \\ -12,6 \\ 4,5 \\ -1,1 \end{pmatrix} = \begin{pmatrix} 32,1 \\ 22,9 \\ 33,1 \\ 30,9 \end{pmatrix}$$

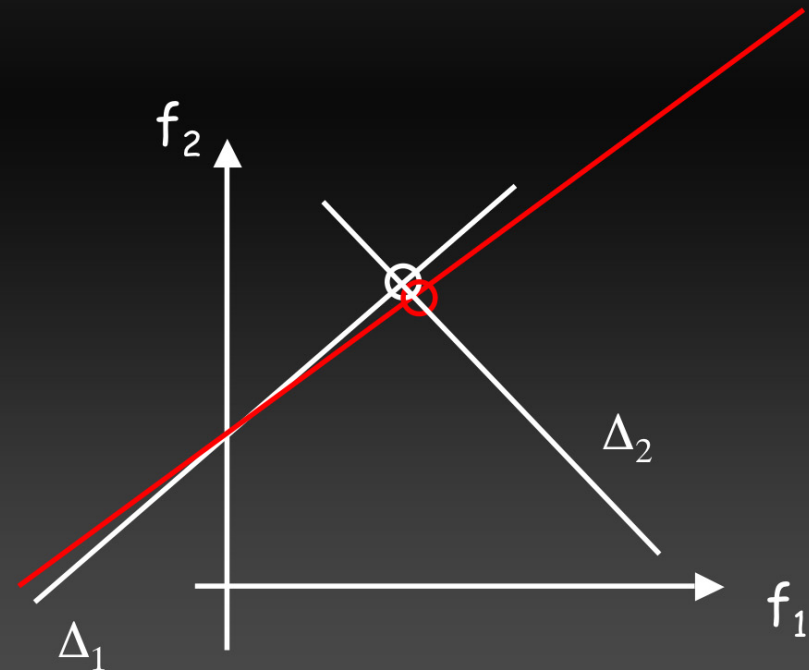
$$\begin{pmatrix} 10 & 7 & 8 & 7 \\ 7 & 5 & 6 & 5 \\ 8 & 6 & 10 & 9 \\ 7 & 5 & 9 & 10 \end{pmatrix} \begin{pmatrix} -7,2 \\ 14,6 \\ -2,5 \\ 3,1 \end{pmatrix} = \begin{pmatrix} 31,9 \\ 23,1 \\ 32,9 \\ 31,1 \end{pmatrix}$$

Conditionnement (I)

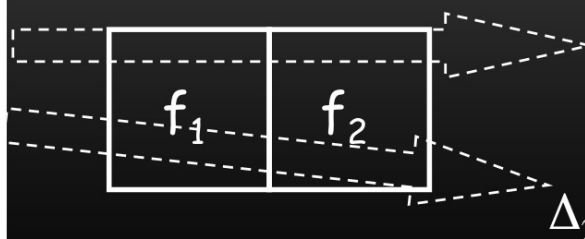


$$\Delta_1$$
$$p_1 = r_{1,1} f_1 + r_{1,2} f_2$$

$$\Delta_2$$
$$p_2 = r_{2,1} f_1 + r_{2,2} f_2$$



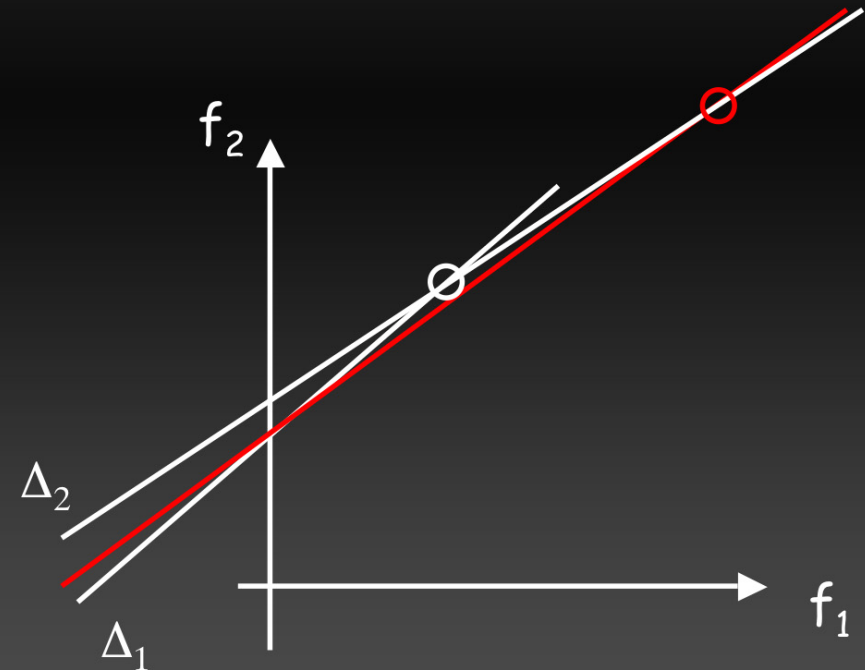
Conditionnement (II)



Δ_1
 $p_1 = r_{1,1} f_1 + r_{1,2} f_2$

Δ_2
 $p_2 = r_{2,1} f_1 + r_{2,2} f_2$

64_ = 4 096
128_ = 16 384
256_ = 65 536
512_ = 262 144



Objectifs

- Garantir la **convergence** de l'algorithme
 - Définir un critère d'arrêt
 - garantir la stabilité de l'algorithme au fil des itérations
- Optimiser la **régularisation** en terme de :
 - résolution
 - niveau de bruit (quantification)
- **Quantification** du bruit dans les coupes

Convergence: Critère d'arrêt

Erreur inverse (backward error) :

$$\hat{a}(\mathbf{r}) = \underset{\delta A, \delta \mathbf{q}}{\text{Min}} \left\{ \text{Max} \left(\frac{\|\delta A\|}{\hat{a}}, \frac{\|\delta \mathbf{q}\|}{\hat{a}} \right) / (A + \delta A) \mathbf{x} = \mathbf{q} + \delta \mathbf{q} \right\}$$

$$\hat{a}(\mathbf{r}) = \frac{\|A \mathbf{x} - \mathbf{q}\|}{\|A\| \|\mathbf{x}\| + \|\mathbf{q}\|}$$

$$\hat{a} = 0, \|\delta \mathbf{q}\| = 0 \quad \text{et} \quad \hat{a} = \|A\| \Rightarrow \hat{a}^j = \frac{1}{\ddot{e}_{\max}} \frac{\|\mathbf{r}^j\|}{\|\mathbf{f}^j\|}$$

Convergence: Stabilité

CONDITIONNEMENT :

$$\hat{e}(A) = \|A\| \|A^{-1}\| = \frac{\ddot{e}_{\max}}{\ddot{e}_{\min}}$$

$$\frac{\|\ddot{a}f\|_r}{\|f\|} \leq \frac{\hat{e}(A)}{1 - \hat{e}(A) \frac{\|\delta A\|}{\|A\|}} \left[\frac{\|\ddot{a}q\|_r}{\|q\|} + \frac{\|\ddot{a}A\|}{\|A\|} \right]$$

$$\frac{\|\ddot{a}f\|_r}{\|f\|} \leq \hat{e}(A) \frac{\|\ddot{a}q\|_r}{\|q\|}$$

Gradient conjugué

$$\bar{f} = \arg \min_{f \in \mathbb{C}} \left\| A^* f - q \right\|^2$$

$$d^0 = r^0 = A^* . q$$

$$\alpha^j = \frac{\|r^j\|^2}{\langle d^j | A^* . A . d^j \rangle}$$

$$r^{j+1} = r^j - \alpha^j . A^* . A . d^j$$

$$d^{j+1} = r^{j+1} + \frac{\|r^{j+1}\|^2}{\|r^j\|^2} . d^j$$

$$f^{j+1} = f^j + \alpha^j . d^j$$

Matrice de Galerkin

$$\left. \begin{aligned} \hat{u}^j &= \frac{\|r^j\|^2}{\langle d^j | R^* . R . d^j \rangle} \\ \hat{a}^j &= \frac{\|r^{j+1}\|^2}{\|r^j\|^2} \end{aligned} \right\} \rightarrow G^j = \begin{pmatrix} \frac{1}{\hat{u}^0} & -\frac{\sqrt{\hat{a}^0}}{\hat{u}^0} & 0 & 0 \\ -\frac{\sqrt{\hat{a}^0}}{\hat{u}^0} & \frac{1}{\hat{u}^1} + \frac{\hat{a}^0}{\hat{u}^0} & 0 & 0 \\ 0 & 0 & 0 & -\frac{\sqrt{\hat{a}^{j-1}}}{\hat{u}^{j-1}} \\ 0 & 0 & -\frac{\sqrt{\hat{a}^{j-1}}}{\hat{u}^{j-1}} & \frac{1}{\hat{u}^j} + \frac{\hat{a}^{j-1}}{\hat{u}^{j-1}} \end{pmatrix}$$

Convergence: Spectre

- Erreur inverse sur les coupes :

$$\hat{\alpha}^j = \frac{1}{\ddot{e}_{\max}} \frac{\left\| \begin{matrix} r^j \\ r \\ f^j \end{matrix} \right\|}{\left\| \begin{matrix} r^j \\ r \\ f^j \end{matrix} \right\|}$$

- Erreur inverse sur les valeurs propres :

$$\left| \ddot{e} - \tilde{e} \right| \leq \zeta(\tilde{e}) = \frac{\sqrt{\hat{a}_{i+1}} \ddot{v}_{\text{der}}}{\ddot{e}_{\max}^i \left\| \begin{matrix} r \\ v^{\tilde{e}} \end{matrix} \right\|}$$

- Contrôle du nombre de conditionnement:

$$N = \frac{\ddot{e}_{\max}}{\ddot{e}_{\min}}$$

Régularisation

Remplacer : $\bar{f} = \arg \min_{f \in \mathcal{C}} \left\| \overset{r}{p} - R \overset{r}{f} \right\|^2$ par

$$\bar{f} = \arg \min_{f \in \mathcal{C}} \left\{ \left\| \overset{r}{p} - R \overset{r}{f} \right\|^2 + \alpha \cdot \tilde{n}(\overset{r}{f}) \right\}$$

Exemples :

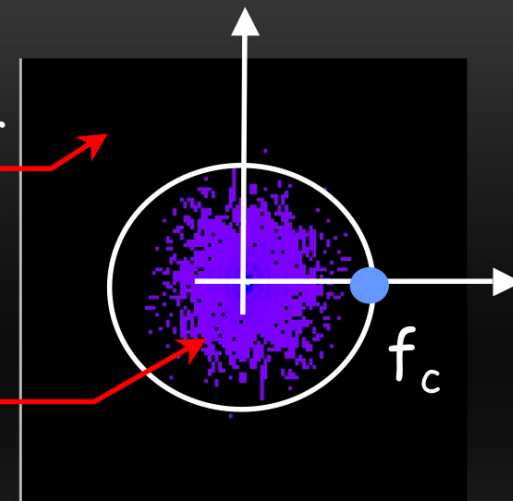
$$\tilde{n}(\overset{r}{f}) \in \left\{ \frac{\left\| \overset{r}{f} \right\|^2}{2} ; \frac{\left\langle \overset{r}{f} | Q \overset{r}{f} \right\rangle}{2} ; \sum_i f_i \ln(f_i) ; \dots \right\}$$

Exemple (I)

$$Uf' = B.Uf' + (1-B)Uf'$$

Adéquation à $p' = R.f'$

à régulariser



$$U_s \overset{r}{p}' = Uf' = B.Uf' = B.U_s \overset{r}{p} \Rightarrow \overset{r}{p}' = U_s^{-1} B . U_s \overset{r}{p}$$

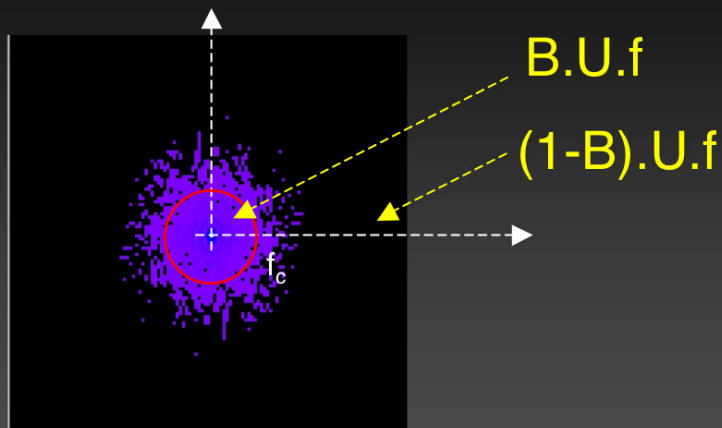
Example (II)

$$E(f^r) = \| U_s^{-1}(B \cdot U_s p^r) - R f^r \|^2 + \| (1-B)U f^r \|^2$$

$$E(f^r) = \left\| \begin{pmatrix} R \\ (1-B)U \end{pmatrix} f^r - \begin{pmatrix} b^*p^r \\ 0 \end{pmatrix} \right\|^2 = \|A.f^r - q\|^2$$

$$\left[{}^t\mathbf{R}\mathbf{R} + \mathbf{U}^{-1} (1-\mathbf{B})^2 \mathbf{U} \right]_{\mathbf{f}}^{\mathbf{r}} = \begin{pmatrix} \mathbf{b}^* & \mathbf{p} \\ 0 & \end{pmatrix}$$

Résolution par BFGS ou GC

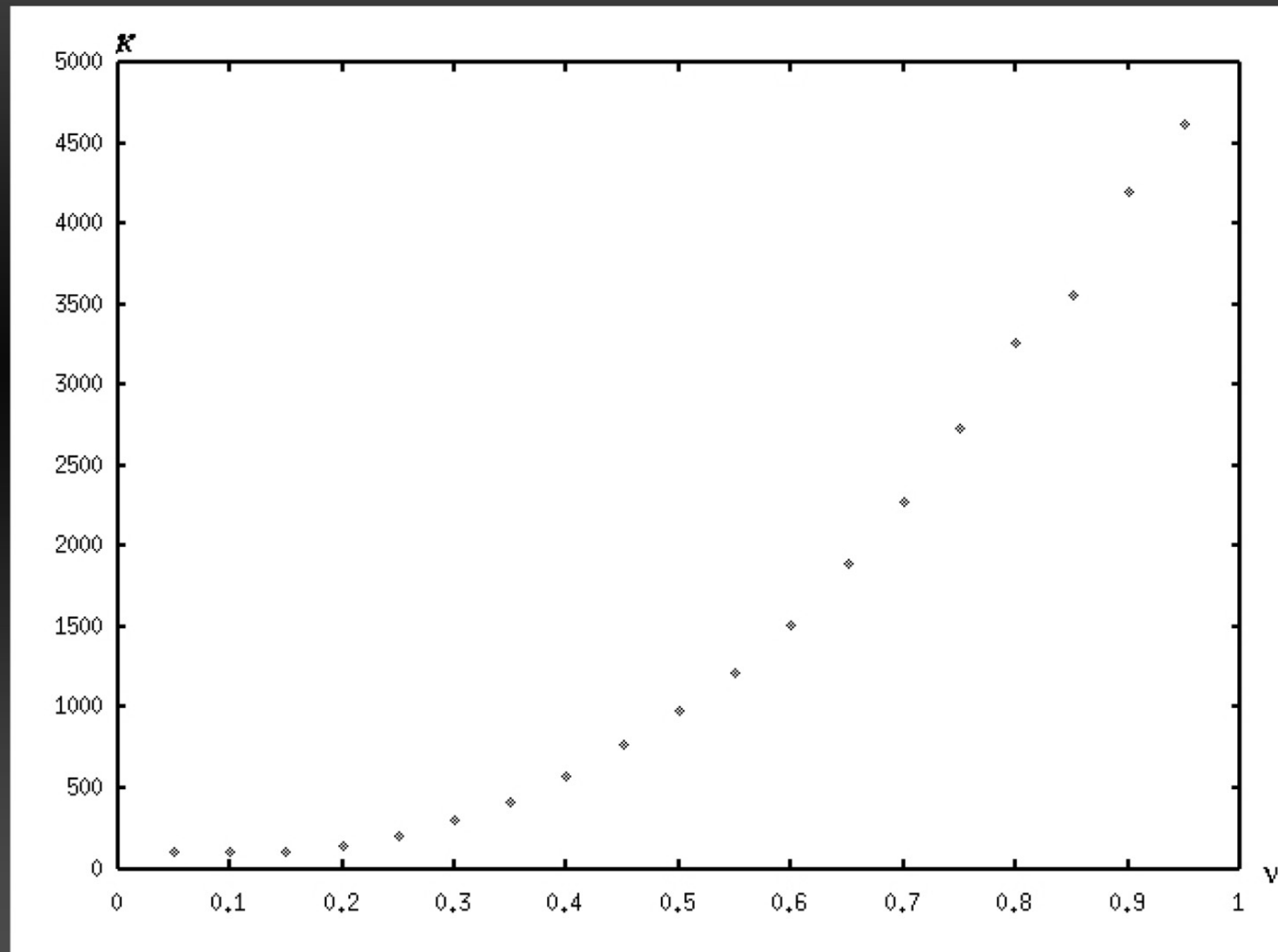


ALGORITHMIQUE (I)

- Résolution par GC

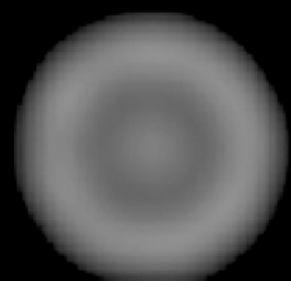
- $\delta^0 = \rho^0 = \text{RETRO} [U^{-1} B U (\pi)]$
- ${}^t R R \delta^j = \text{RETRO} [\text{PRO} (\delta^j)] + U^{-1} (1-B) U (\delta^j)$
- Calcul et diagonalisation des matrices de Galerkin
- Arrêt si erreur inverse $< \varepsilon$
- Nombre de conditionnement en fonction de f_c

Example (III)

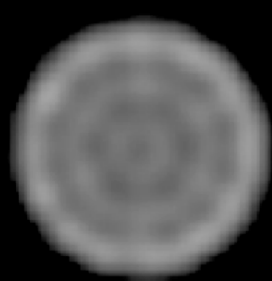


ALGORITHMIQUE (II)

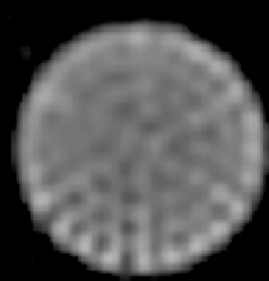
- Résolution par BFGS (contrainte de positivité)
 - $BF = U^{-1} B U (\pi)$
 - Boucle {
 $HF = U^{-1} (1-B) U (\phi)$
 $r = PRO(\phi) - BF$
 $E = (||\rho|| + ||HF||)/2$
 $Grad(E) = RETROPRO(\rho) + HF$
 BFGS [E, Grad(E)]
}
 - Arrêt par seuil sur résidu (car convergence)



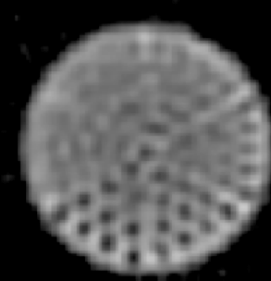
0.1



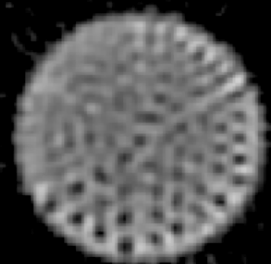
0.2



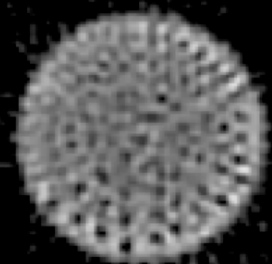
0.3



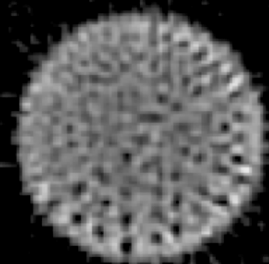
0.4



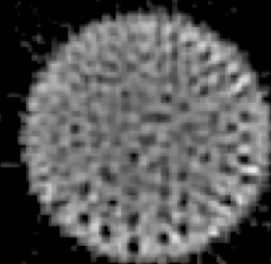
0.5



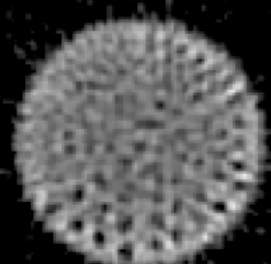
0.6



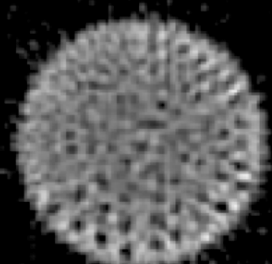
0.7



0.8

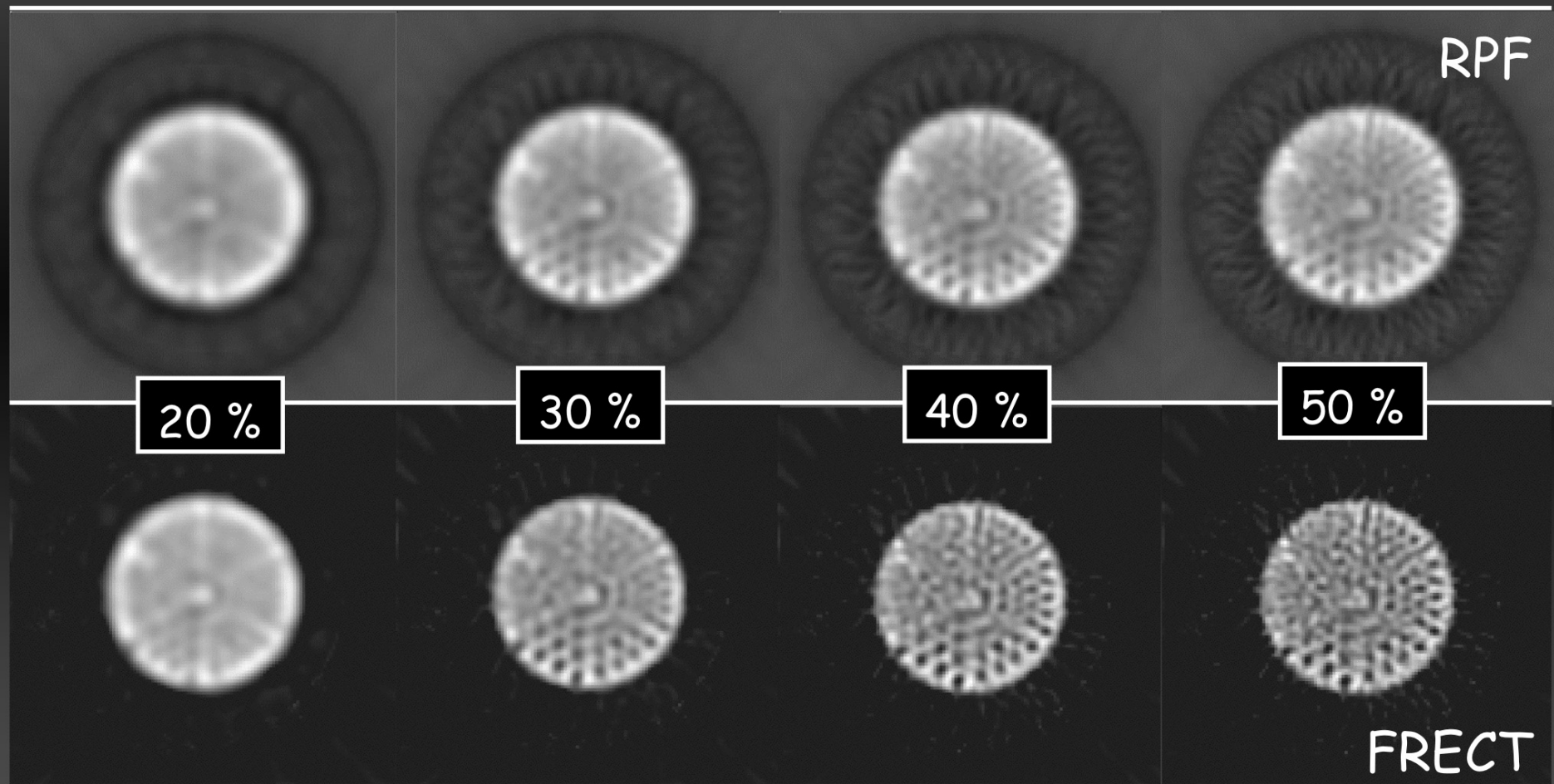


0.9

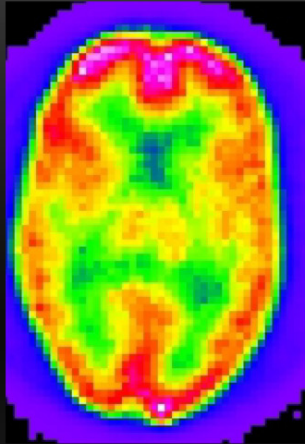


1.0

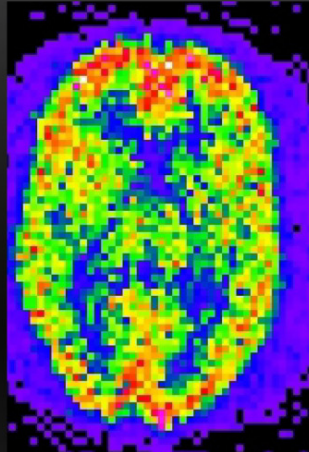
Example (IV)



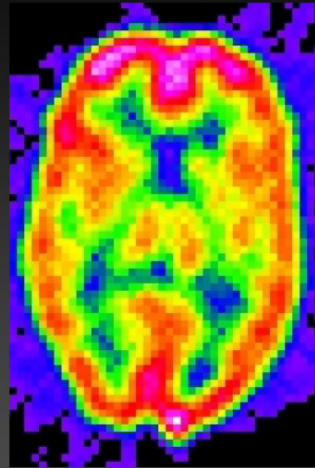
Example (V)



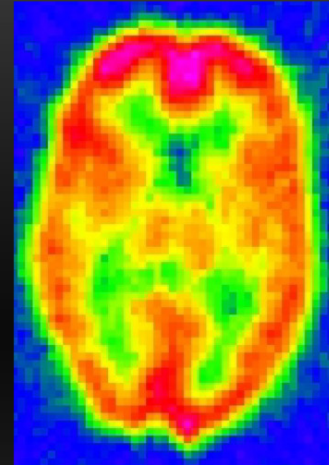
MLEM 6



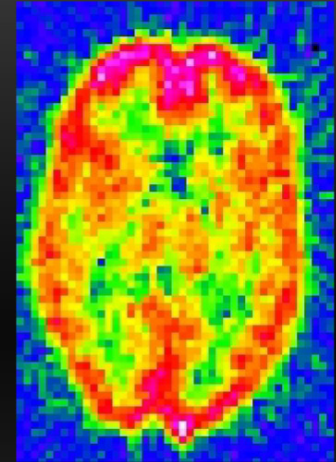
MLEM 200



FRECT 34 (CV)



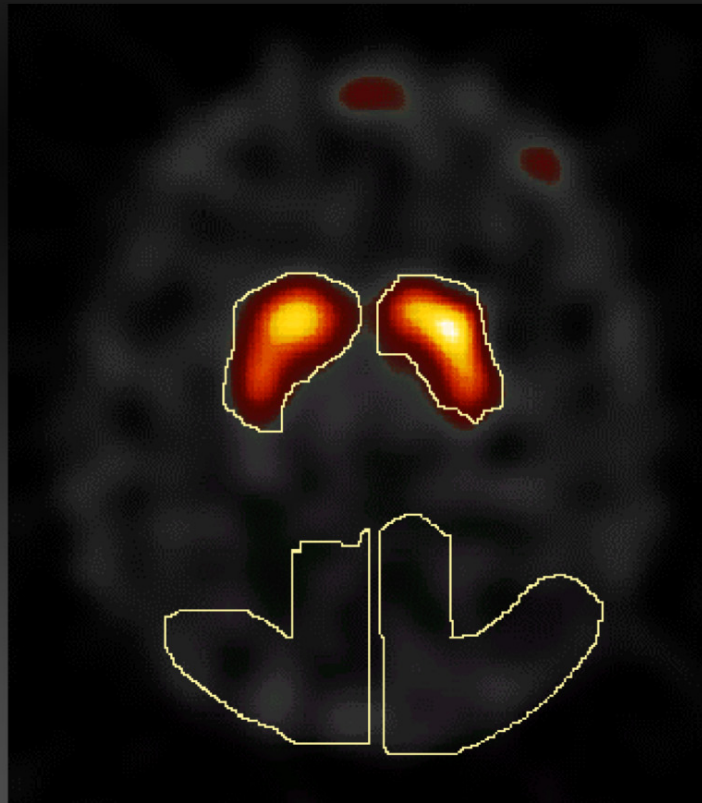
GC 6



GC 16

Exemple (VI)

$$S = (R^*R + H^*H)^{-1} R^* (U^{-1} BU)$$



$$d\mathbf{f}^i = S d\mathbf{q}^r \Rightarrow \|d\mathbf{f}^i\| \leq \|S\| \cdot \|d\mathbf{q}^r\|$$

$$\text{Cov}(d\mathbf{f}^i) = S \cdot \text{Cov}(d\mathbf{q}^r) \cdot S^*$$