# Modèles déformables pour l'analyse d'images

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# Context

Acquisition: Measurement of physical properties

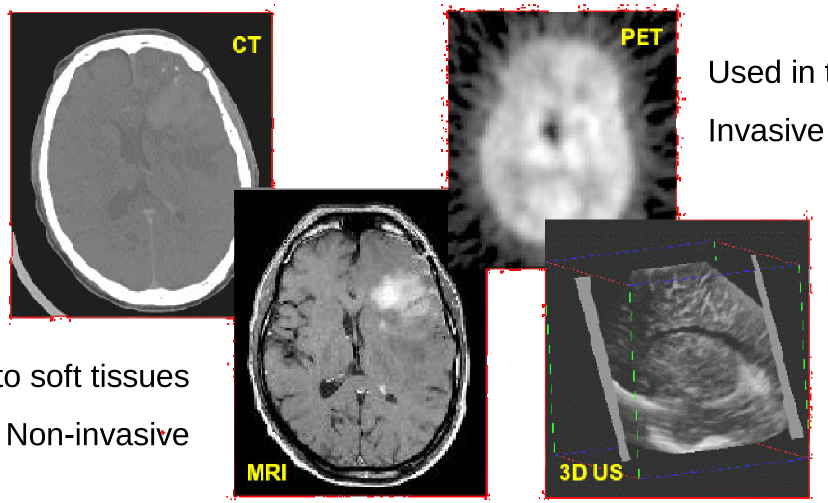
Several modalities:

Bone specific

harmful (X-rays )

Used in tumor detection

Invasive




Well adapted to soft tissues

Non-invasive

Used for fetus imaging

Cheap

Non-invasive

Real-time

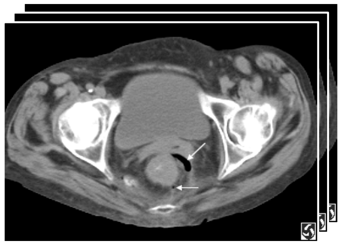
For computation, images are discretized (digitalized) :

In space : $(x,y,z) \rightarrow (n_x,n_y,n_z)$ samples

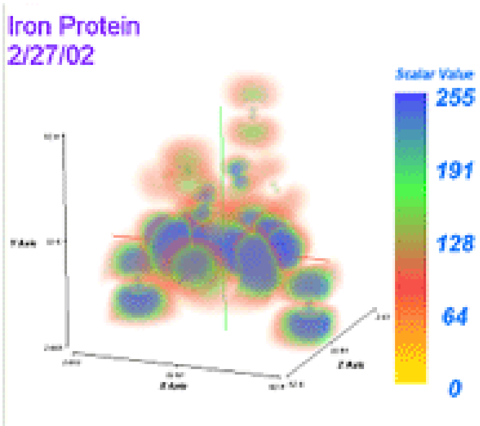
In time : $t \rightarrow n_t$ samples

In Intensity : Generally 256 levels (8 bits) or 2048 levels (11bits) = Grey levels

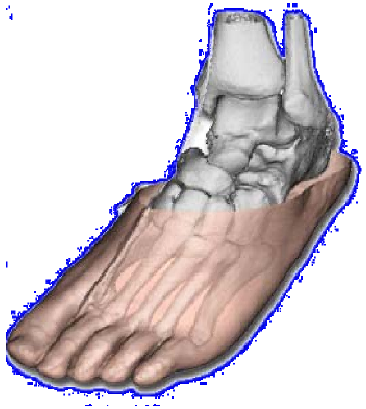
# Raw data visualization



2D-slices




3D ray-casting



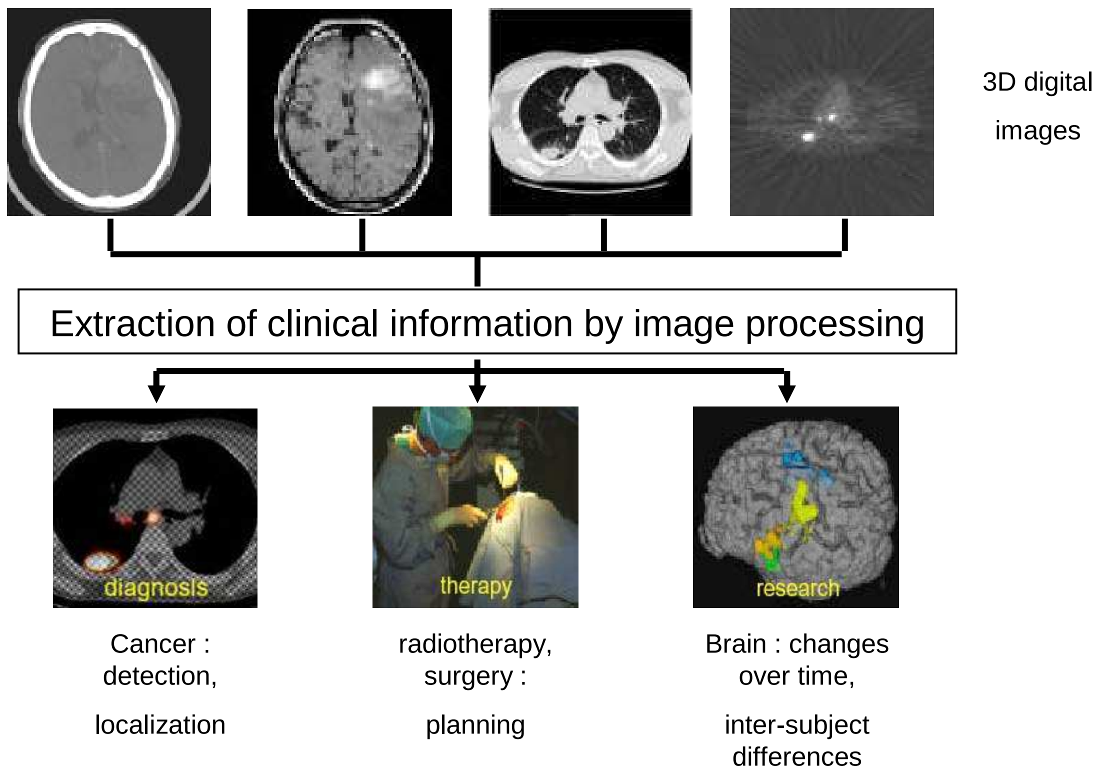
Iso-surfaces

# Context

Today, imaging is a routine clinical tool

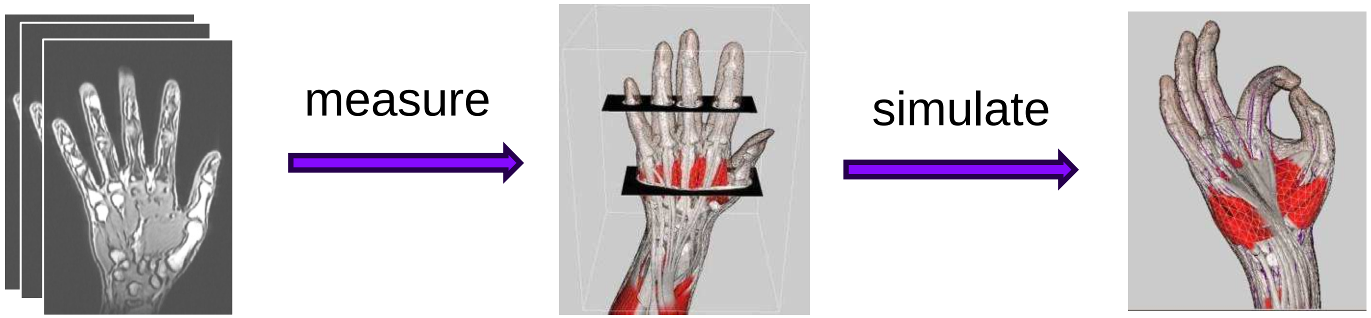
But we measure much more than we can understand...

→ Image analysis is required




Images: D. Vandermeulen, KUL

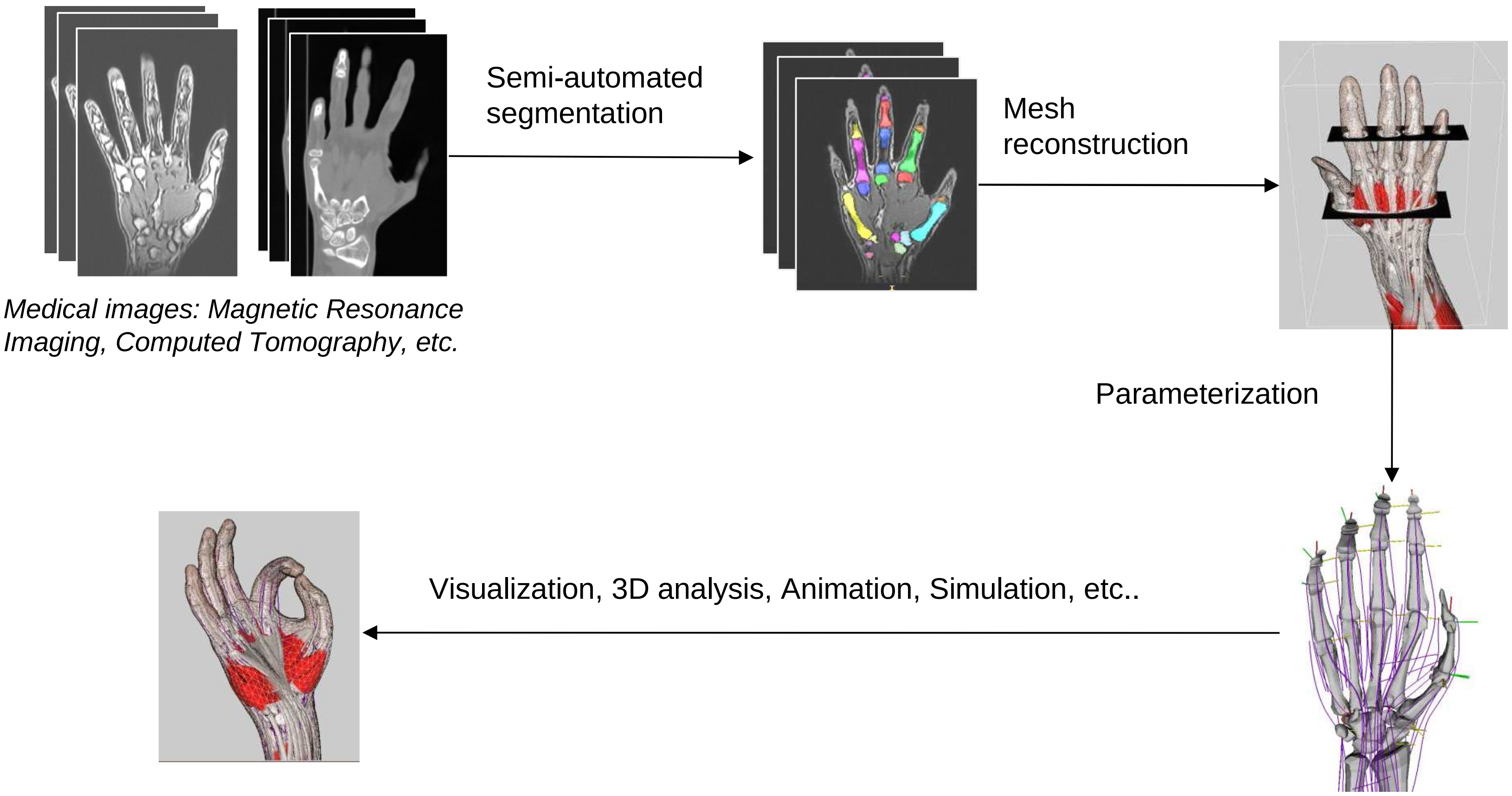
# Images to Models to Simulations



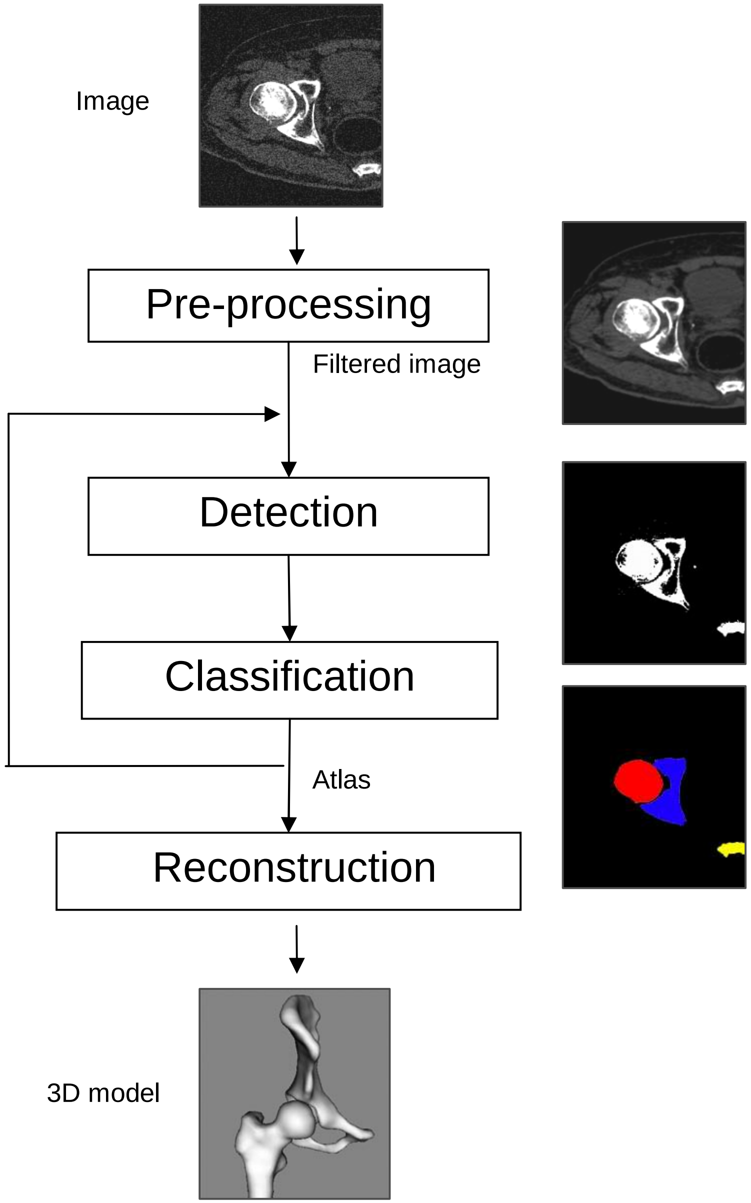

- Visualization
- Diagnosis
- Comparative anatomy
- Data fusion

- Computer animation
- Simulation
- Surgical planning

# Standard modeling pipeline



# Direct segmentation




## Pre-Processing:

- noise removal [perona90]
- contour enhancement
- bias filtering

## Detection:

- contour detection/closing
- histogram analysis
- texture analysis
- statistical approaches [staib92]

## Classification:

- region growing
- region splitting

## Reconstruction:

- Marching cubes [lorensen87]
- Constrained deformable models

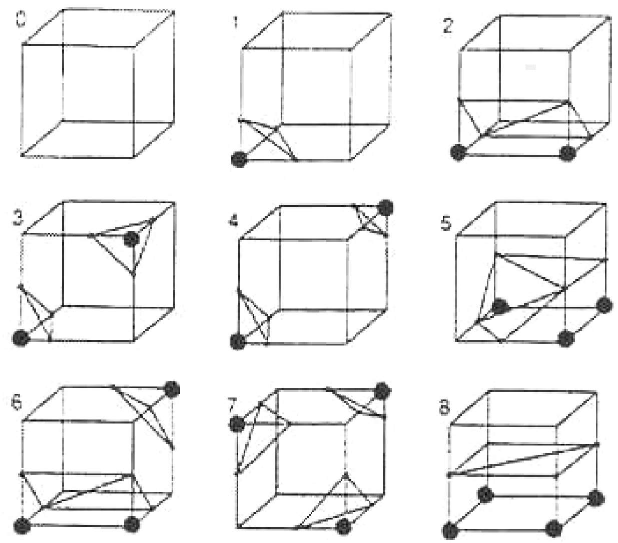
# Reconstruction

Transformation from binary volumes to surfaces :

Marching-cubes algorithm

[lorensen87]

Spatial voxels configurations
and associated surfaces :




Marching
cubes

Constrained
reconstruction

# Main issues

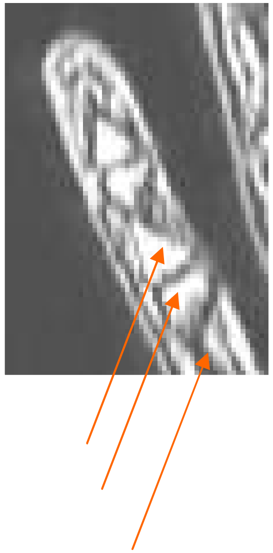
Segmentation step:

One organ = several intensities

→ Thresholding + morphological operations + manual corrections

One type of organ = same intensities

→ Manual separation + labeling

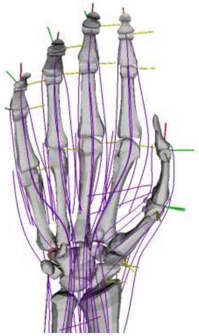


Parameterization step

Interactive placement of the joint coordinate systems

Definition of soft tissues / bones attachments

Definition of material parameters

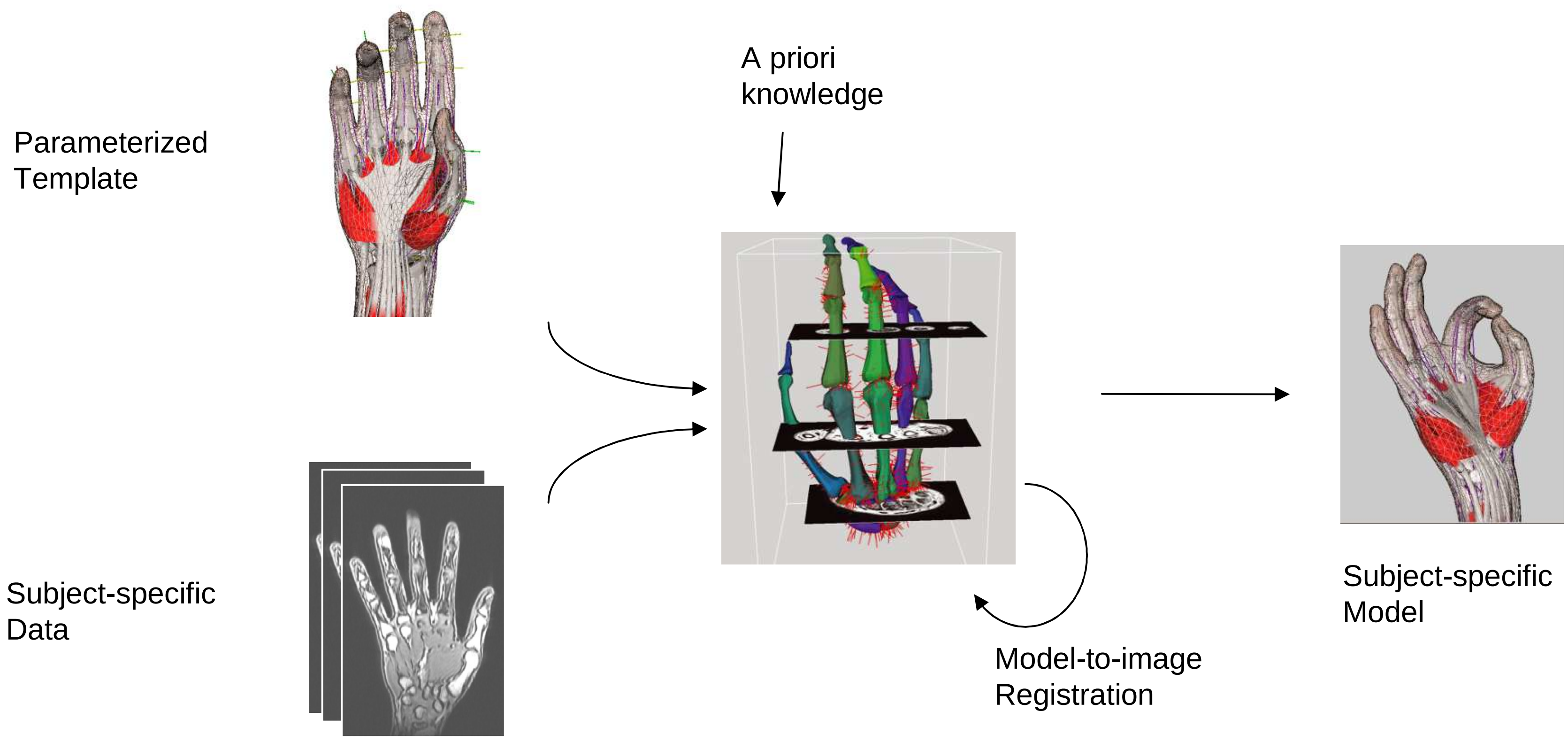


→ Time consuming

→ Requires a lot of anatomical knowledge

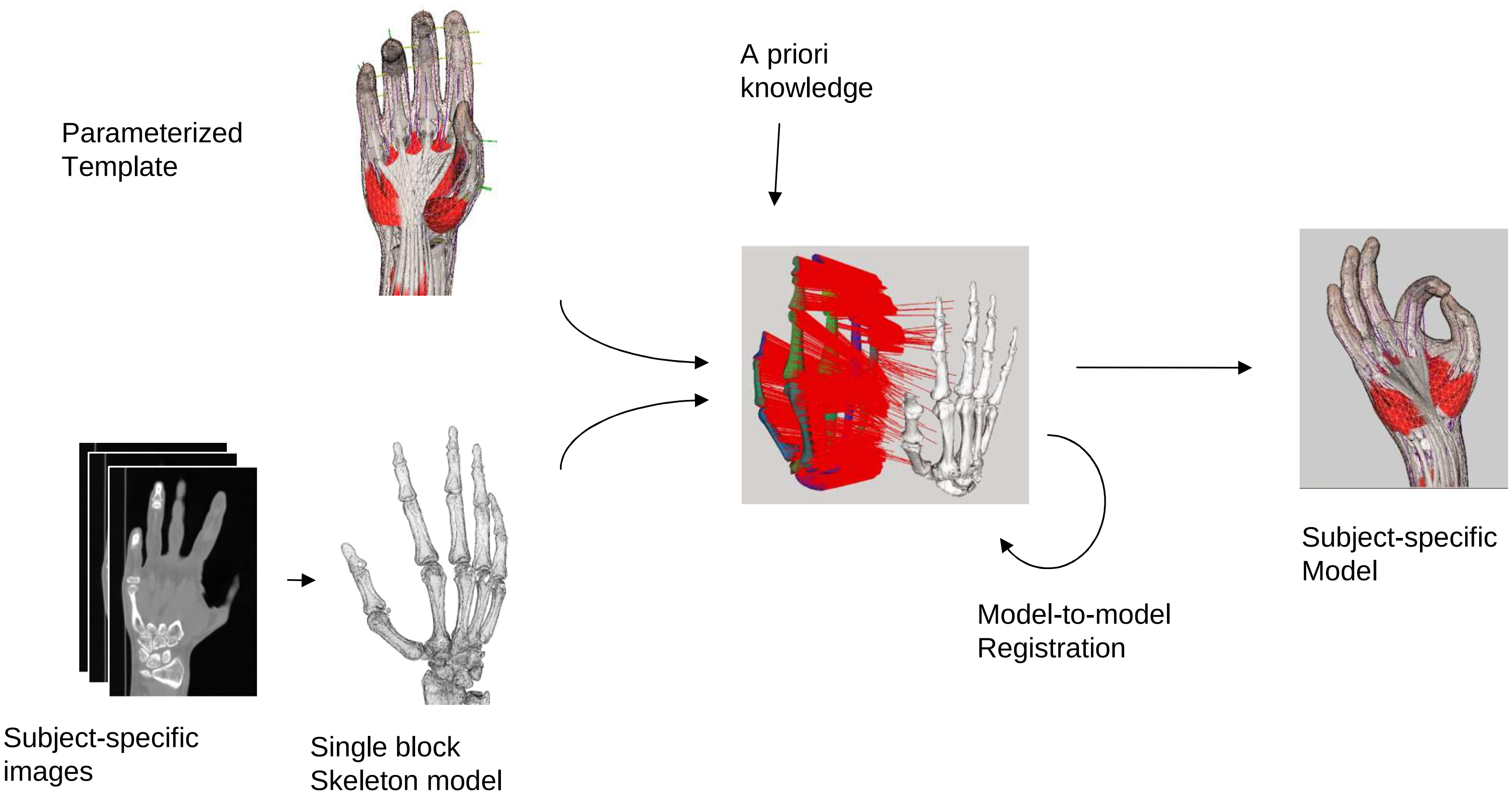
# Template registration approach

Registration to images:



# Segmentation + Registration

Registration to surfaces:



# Models for registration

Two approaches:

Model extraction in the two datasets

+ geometric registration [audette00]

→ direct segmentation

}

Ad-hoc parameters for region/ contour detection

→ sensitive to noise and global intensity variations

Model extraction in the source dataset

+ image registration [Zitova01], [maintz98], [cachier02]

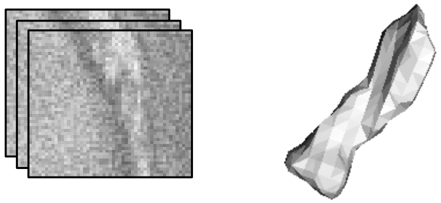
→ indirect segmentation

}

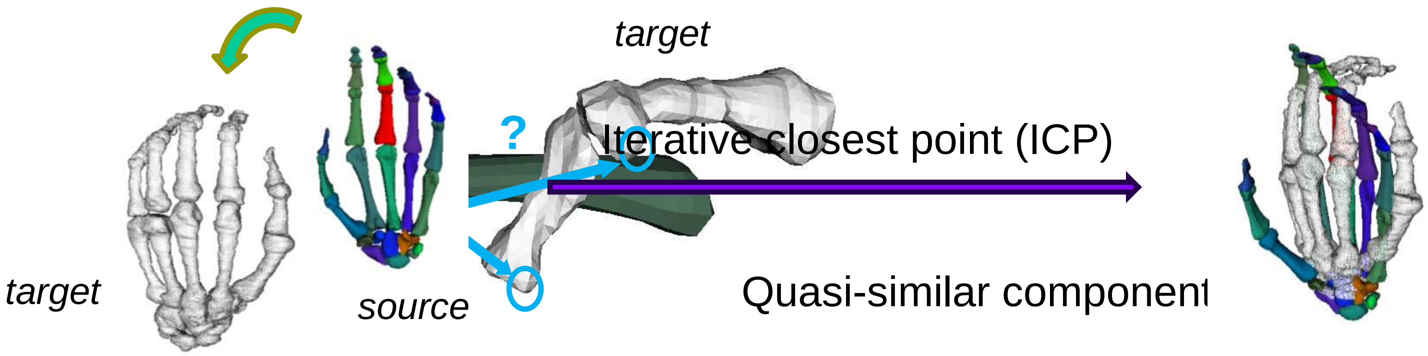
Use of prior knowledge

# Main issues

Input noise



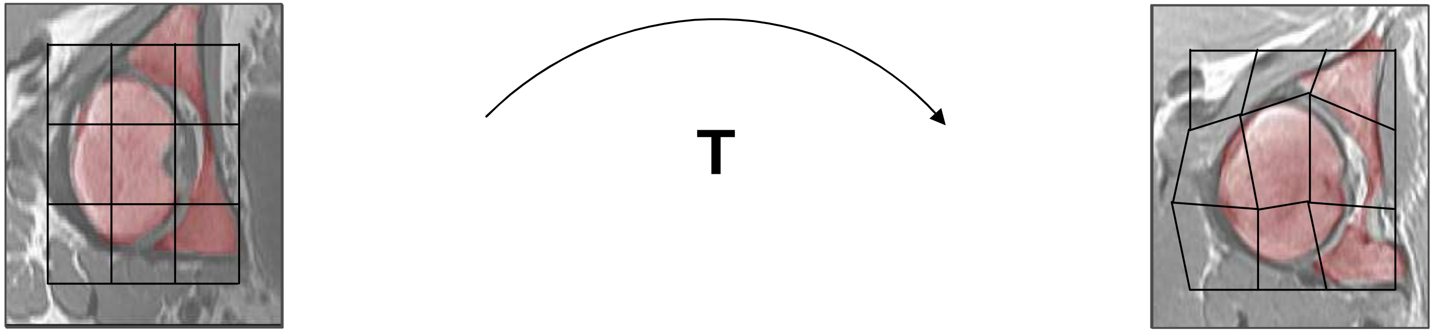
Local minima when searching for correspondences




Complex deformation field :

smooth elastic deformations + displacements discontinuities

# Registration




**Source J** **Target I**

Problem: find a transformation T that

- maximises the similarity between T(J) and I
- is admissible in the application context

Indirect segmentation

**Model, dofs**

**Criterion**

**Regularization**

**Optimisation strategy**

# Outline

What is registered: **Registration features**

Registration criterion: **Similarity measure**

How to constrain the problem: **Regularisation**

How the registration is performed: **Evolution**

**Examples**

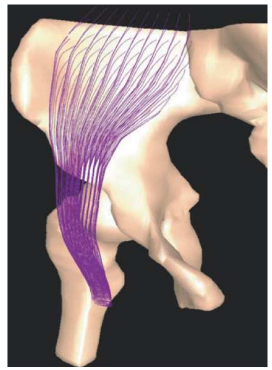
# Registration features

## Iconic features

photometric information: image intensities, gradient

Regions of interest: voxel, template, intensity profile

Feature vectors

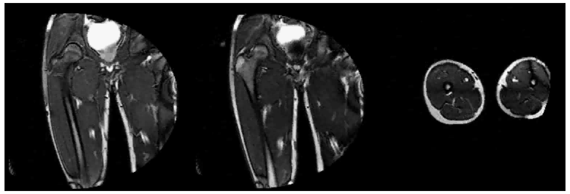


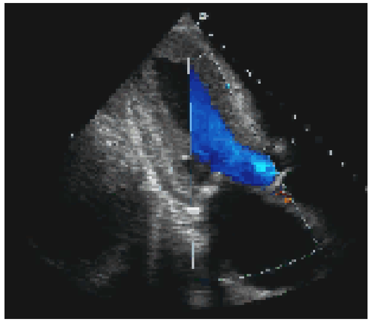
## Geometric features

Points, curves, surfaces, volumes

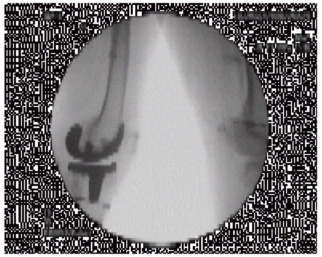
# Acquisition modalities

| Data | MRI | US | X-Rays/ CT | Other |
|---|---|---|---|---|
| Static | ++ | + | + | |
| Kinematics | + | ++ | + | MoCap |
| Dynamics | | | | Force plates<br>Strain gauges |
| Mechanics | + | + | | Mech. devices |
| Physiology | | | | EMG |

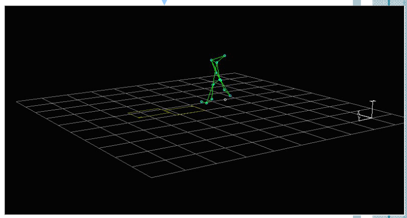




[anaesthesiaUK]

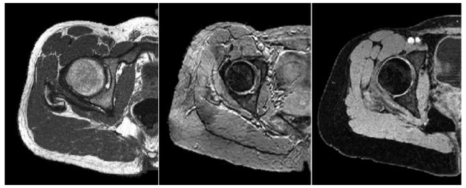


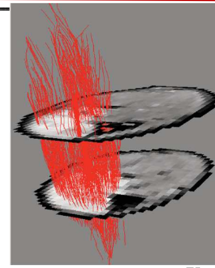
[ETHZ]



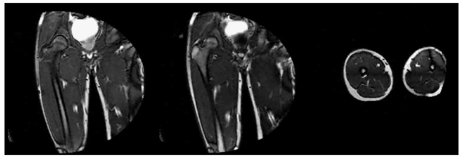
# Acquisition modalities

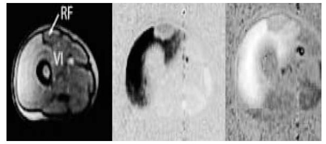
| Data | MRI |
|---|---|
| Static | ++ |
| Kinematics | + |
| Dynamics | |
| Mechanics | + |
| Physiology | |



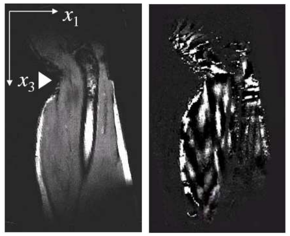


*[heemskerk05]*






*[delp02]*

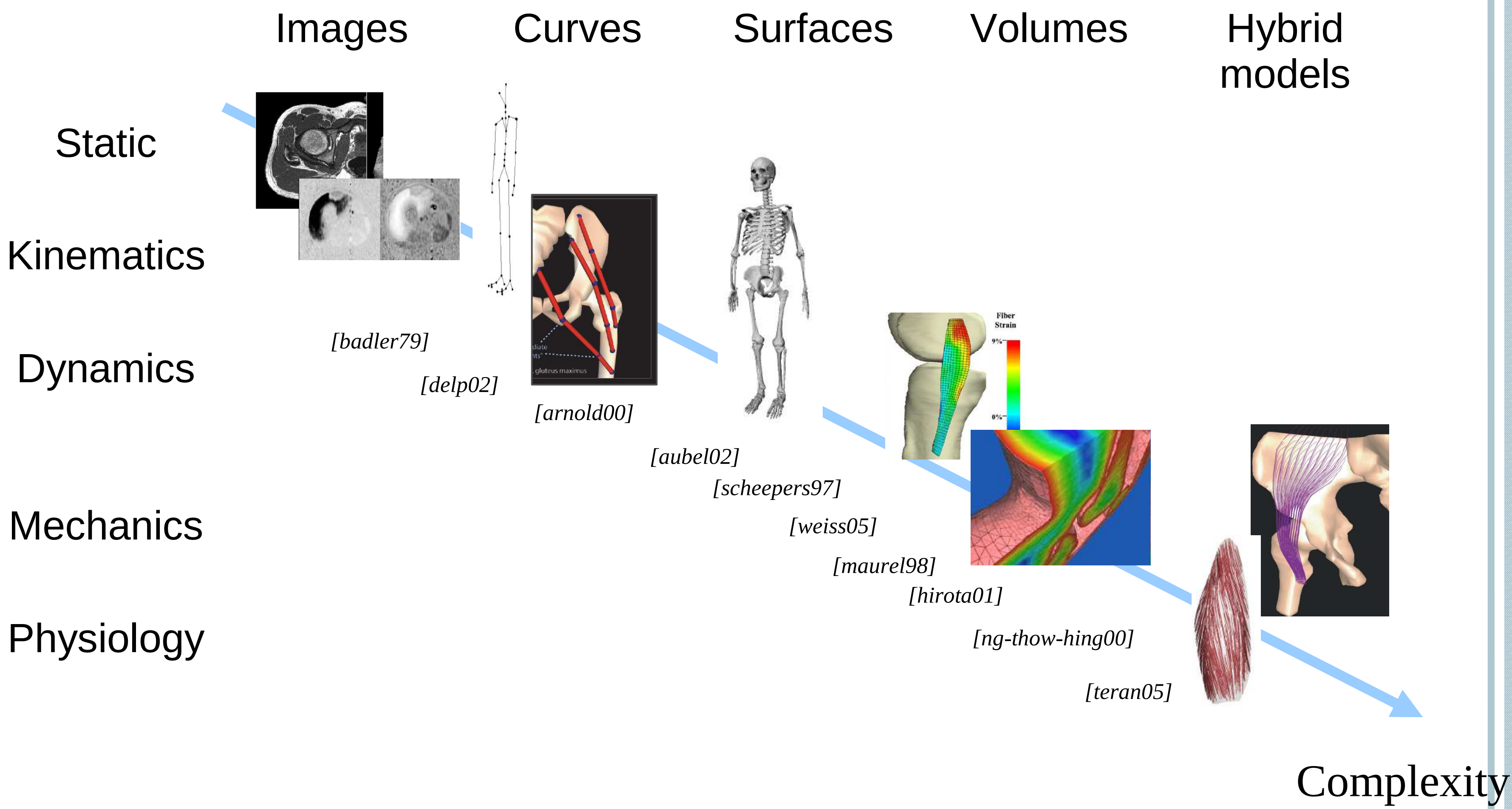



*[papazoglou05]*

Magnetic Resonance Imaging (MRI):

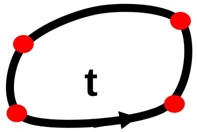
- Non-invasive
- most flexible imaging modality

# Model vs. data



# Deformable models

## Continuous models [kass88], [terzopoulos88], [cootes01]




- Mapping between material parameters and spatial coordinates
  - For example, in 3D: $\mathbf{u} \in [0,1]^p \rightarrow [x(\mathbf{u}),y(\mathbf{u}),z(\mathbf{u})]^T \in \Re^3$
  - Explicit mapping (snakes) or use of parametric functions (splines)

☺ Simple shape description through parametric function derivation → analytic

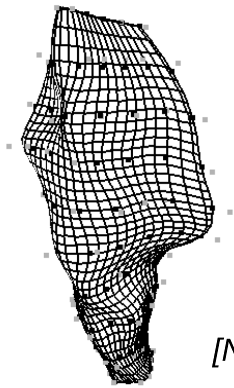
☺ Interpolation

☺ Few degrees of freedom (e.g. control points) → intrinsic regularisation

☹ Shapes are limited by the parametric function

☹ Parameters ≠ geodesic coordinates

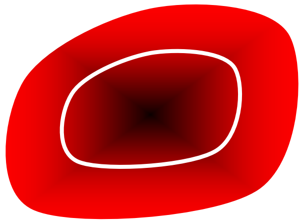
☹ Spatial interactions



*[Ng-thow-hing01]*

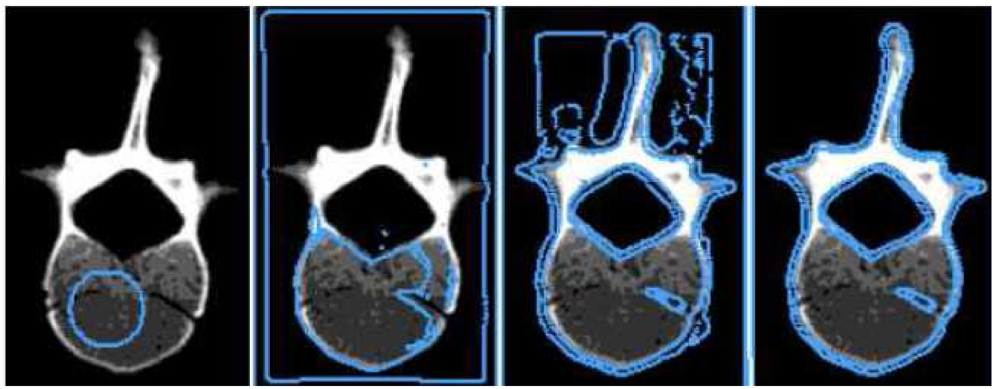
# Deformable models

## Implicit models [osher88], [vemuri03]



- Iso-value of a potential field
  - For example, in 3D: $\{ \mathbf{p} \in \Re^3 \mid F(\mathbf{p})=0 \}$
- Level sets, blobs, convolution surfaces, etc.

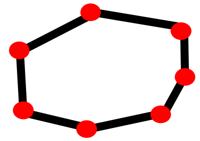
☺ Topological changes
☹ Spatial interactions
☹ Computational cost
☹ Rendering



*[Montagnat01]*

# Deformable models

Discrete models [delingette94], [montagnat05], [lotjonen99], [szeliski96]



- Explicit positions in space (vertices)

  + connectivity relationships

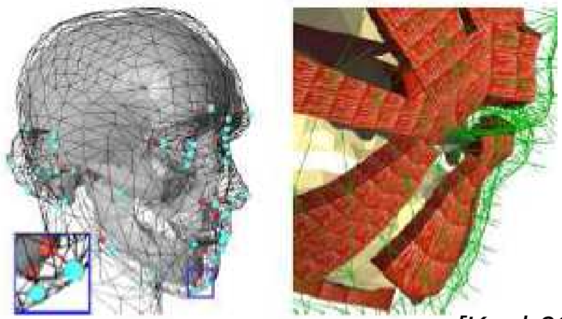
☺ Flexibility

☺ Spatial interactions

☺ Computational cost

☺ Rendering

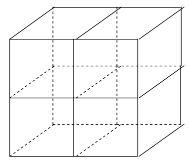
☹ Approximating



*[Koch02]*

# Discrete Models

- Abstract lattices:

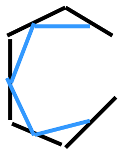


Do not match object contours

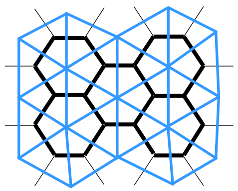
→ problem to handle transformation discontinuities at boundaries

- Polygonal meshes:

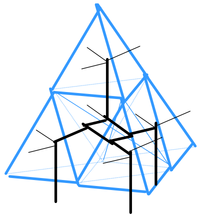
- Constant cell connectivity vs. Constant vertex connectivity



Polyline / 1-simplex meshes



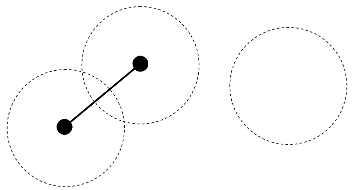
Triangle / 2-simplex meshes



Tetrahedral / 3-simplex meshes

- Particle systems:

- Non-constant connectivity



# Mixed implicit/discrete: medial axis

**Medial axis = medial vertices + thickness**

Reversible

Simpler representation for smooth model

Extension of action lines

The thickness is a relevant parameter

**Two approaches:**

Pruning

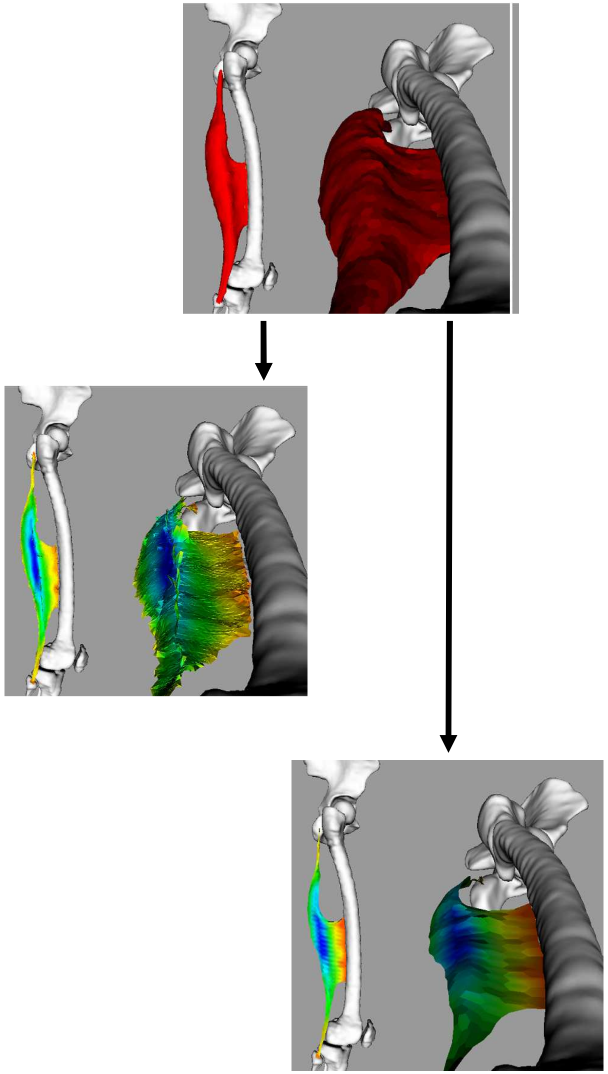
- Exact computation + Simplification

  + Direct computation
  - No homotopy equivalence

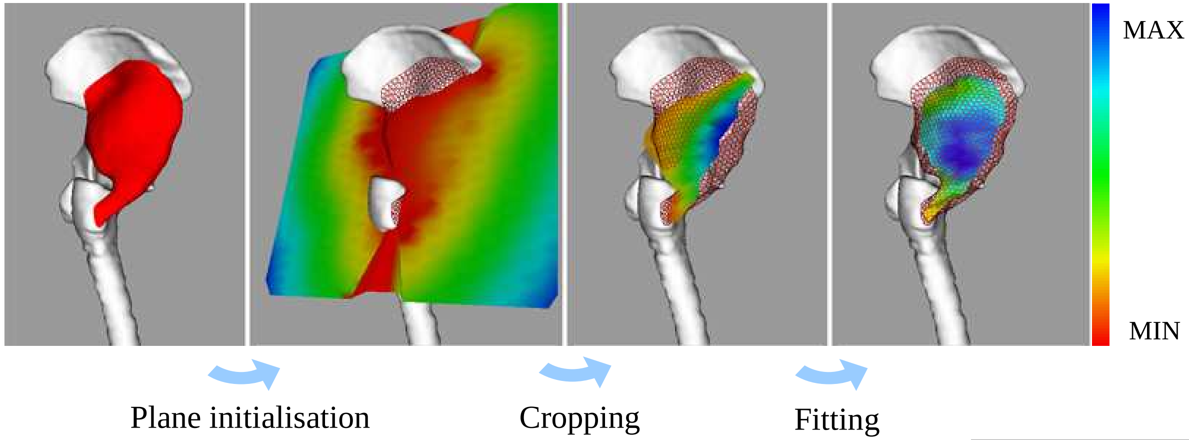
Shape constraints

- Fitting of a simplified model

  - Iterative computation
  + Homotopy equivalence



# Mixed implicit/discrete: medial axis



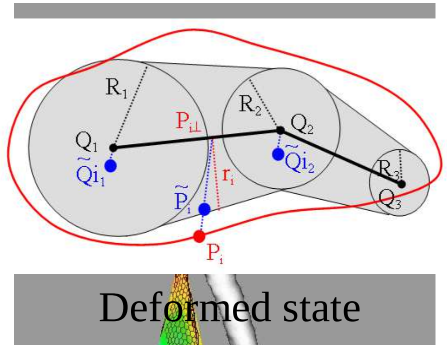

–Radii estimation

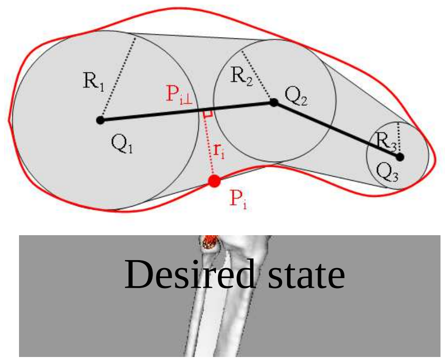
–Model/ axis interactions

- Model → Axis : convergence towards the true MA
- Axis → Model : shape constraints

–Results:

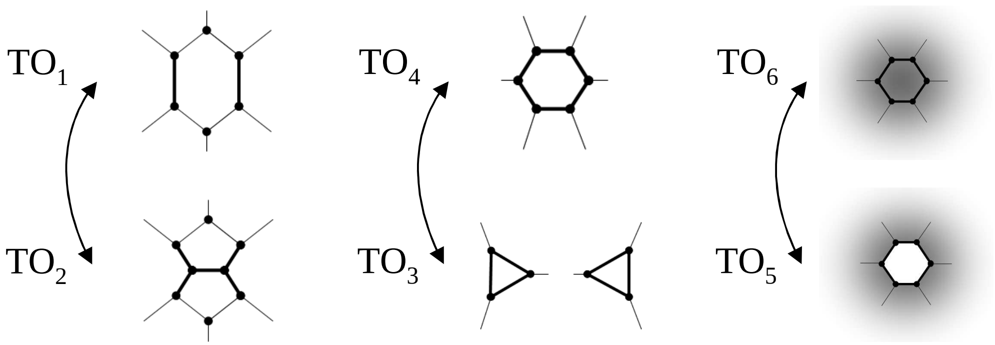
- Error = 0.6 ± 0.6 mm
- Compression factor = 14



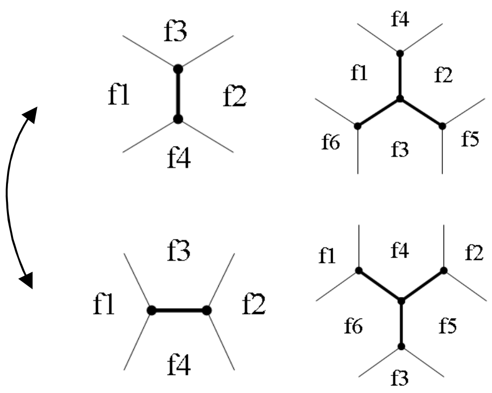



# Topology of simplex meshes (1/3)

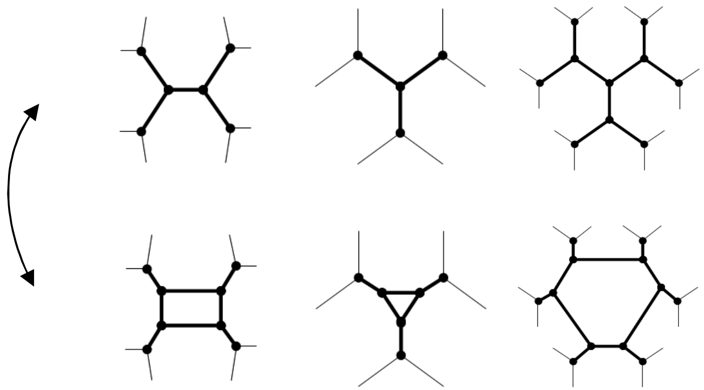
- Simplex meshes -> simple topology description : each vertex → (k+1) neighbors
- 6 Basic operators *[delingette94] [montagnat00]*



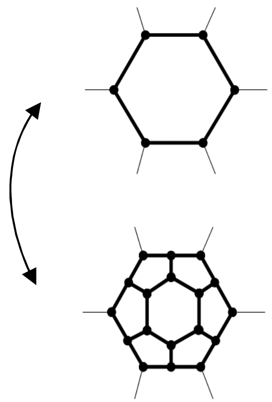

Macro-operators




Exchange operation



Vertex-based resolution change



Cell-based res. change

# Topology of simplex meshes (2/3)

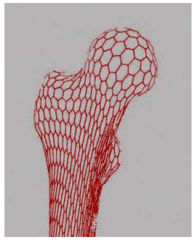
Regular mesh generation:

→ Optimize

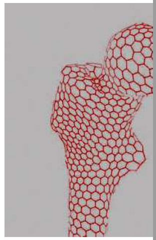
- topological quality (number of vertex per face)
- geometric quality (vertex repartition) according to a target edge length

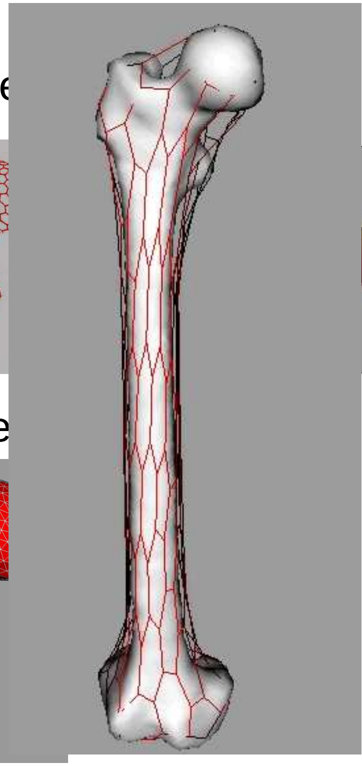
– Results

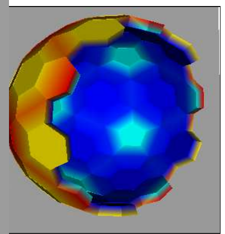
- Fast mesh adaptation to prede



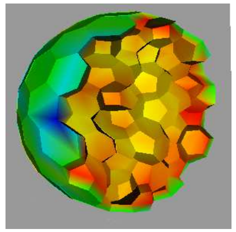
~2s →





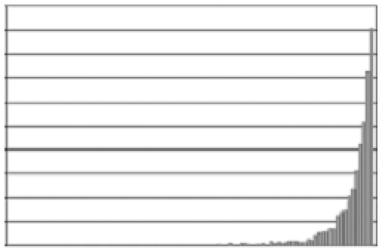


~4s →

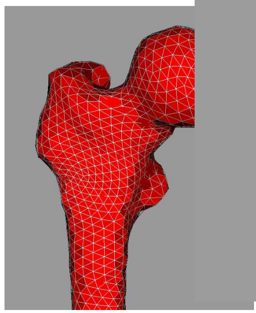


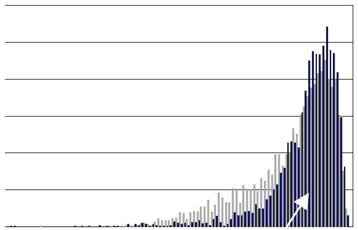
→ Quasi-regular triangulation/ te

Radius ratio histogram



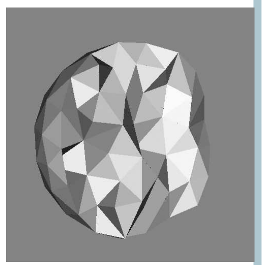
0 1





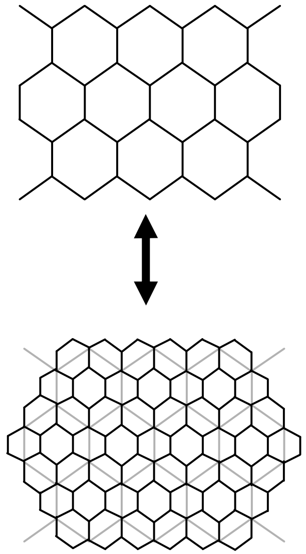
0 1

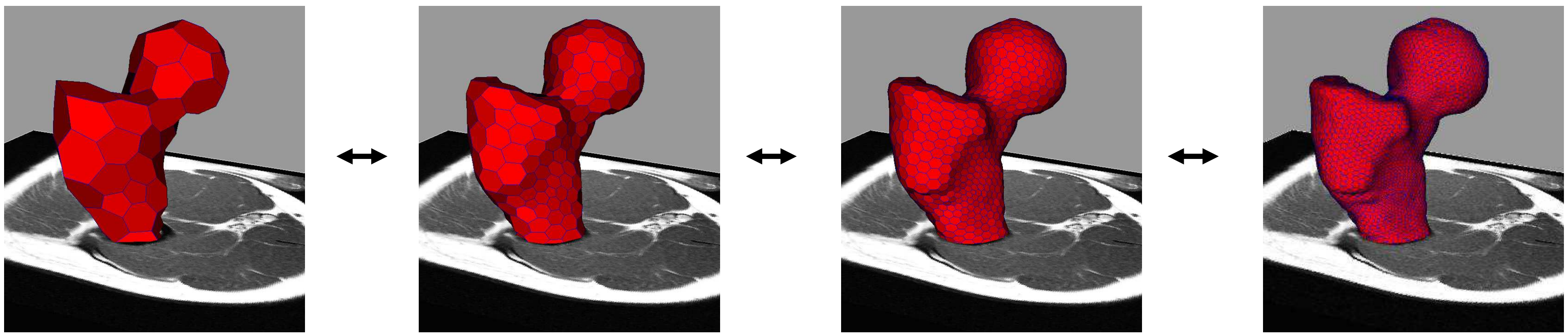
*[alliez05]*



# Topology of simplex meshes (3/3)

- Multi-resolution scheme
  - Global topology adaptation -> semi-regular mesh
  - Level of details (LOD) generation
    - Simple and systematic method: points linear combination
    - Shape features preservation





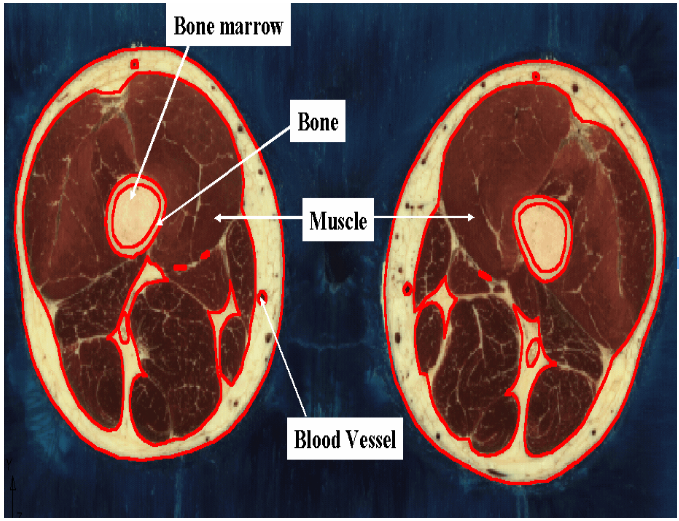
Global constraints
Collision handling

Local constraints
Image forces

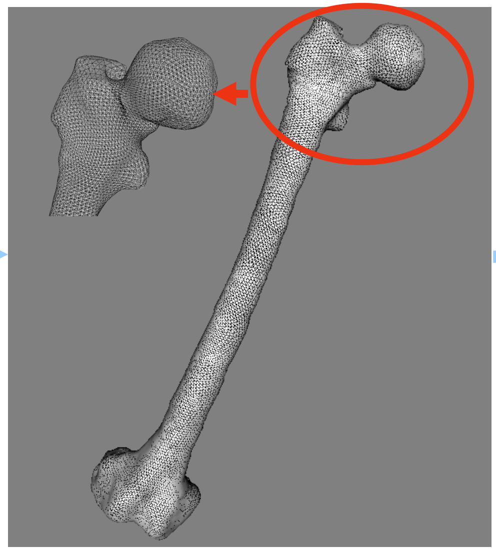
# Volumetric mesh generation (1/4)

Construct volumetric mesh from surface Mesh

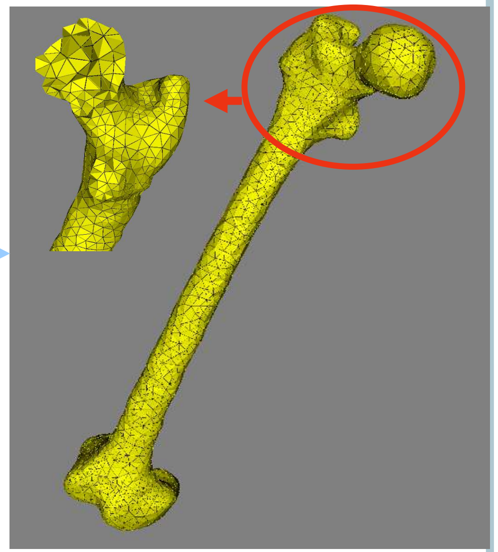
Problem: regular tetrahedra do not tile space




*Segmented MRI*



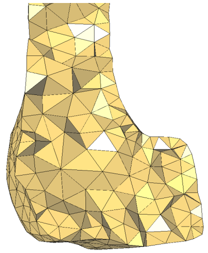
*Surface mesh*



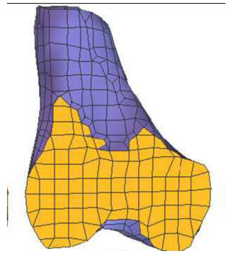
*Volumetric mesh*

# Volumetric mesh generation (2/4)

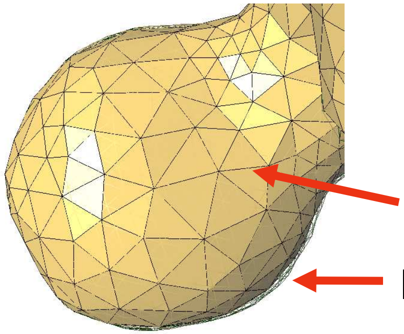
- Requirements
  - –Element type: **Tetrahedron**, Hexahedron, etc.
  - –Element density
- Quality measure
  - –Boundary / input surface matching
  - –Element quality: solid angle, **radius ratio**, etc.

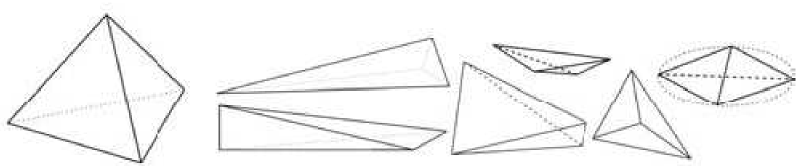


***Tetrahedral mesh***



***Hexahedral mesh***



**Different types of degeneracy (slivers, caps, needles and wedges)**

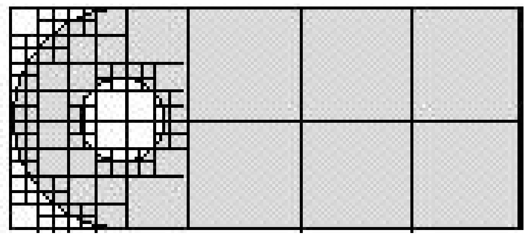
# Volumetric mesh generation (3/4)

Meshing techniques

Octree recursively Subdivision [molino03]

$\rightarrow$ Poor quality elements generated near the boundary

$\rightarrow$ Require a large number of surface intersection calculations

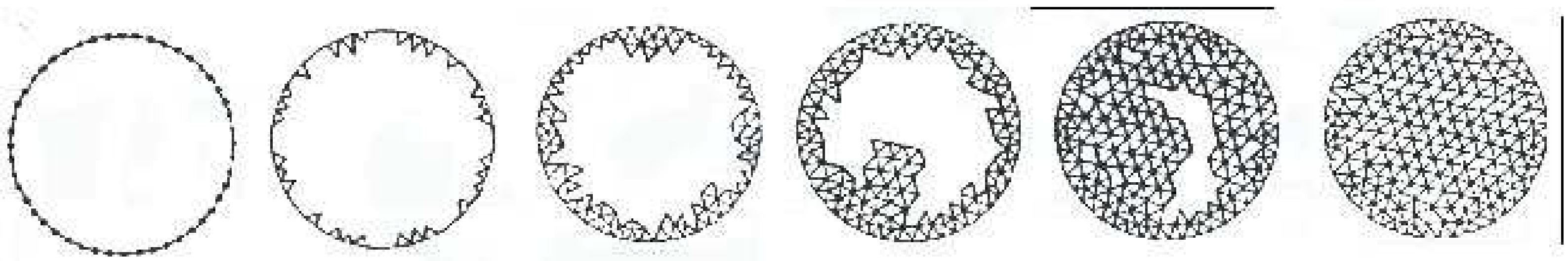


Owen (1998)

Advancing front: cells propagation from boundaries [li00]

$\rightarrow$ Difficult to compute ideal cell locations (local)

$\rightarrow$ Difficult to merge elements when they collide



Owen (1998)

# Volumetric mesh generation (4/4)

Delaunay → optimal connectivity

- The Delaunay criterion 'empty sphere' : no node is contained within the circumsphere of any tetrahedron of the mesh.

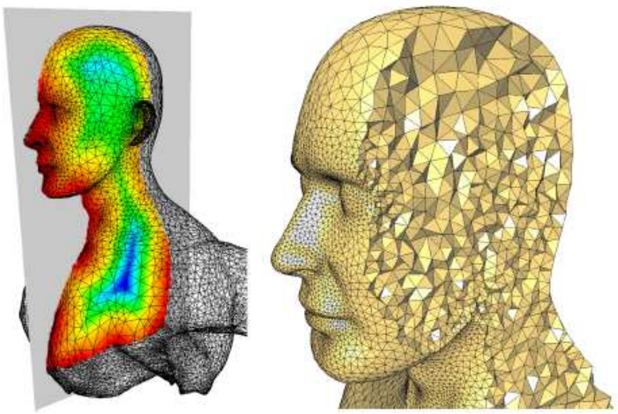
- Refine the tetrahedra locally by inserting new nodes to maintain the Delaunay criterion

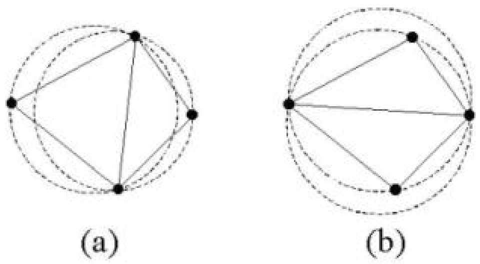
→ Degenerate tetrahedra 'slivers' appear

Variational approach [alliez05]:

Global energy minimization

Vertex repositioning






***2D Delaunay criterion***

***(a) Maintained***

***(b) Not maintained***

**Owen (1998)**

# Conclusion

Choice of model and discretization driven by:

- Geometry: large/small variability ?
- Topology: constant or not ?
- Deformations: large/small ? discontinuities ?

# Outline

What is registered: **Registration features**

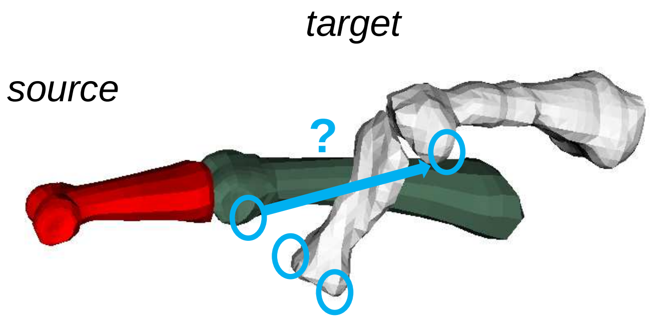
Registration criterion: **Similarity measure**

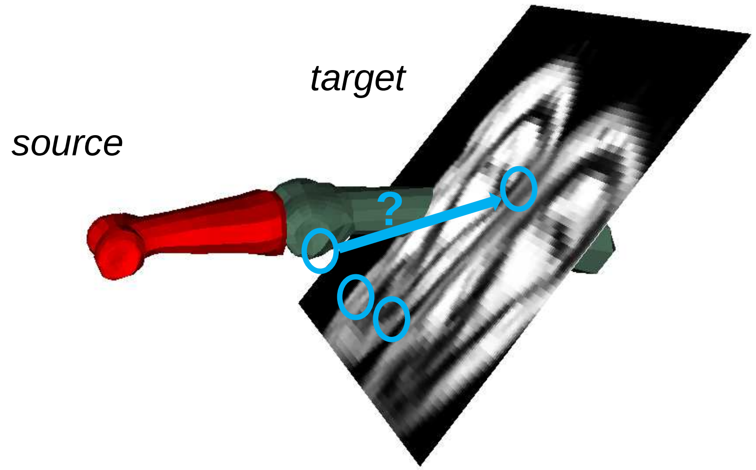
How to constrain the problem: **Regularisation**

How the registration is performed: **Evolution**

**Examples**

# Correspondences

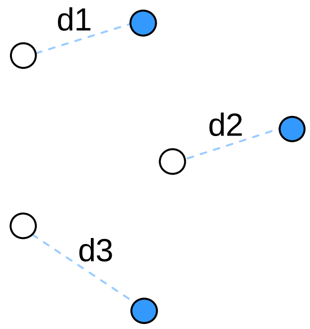



- Requirements:
  - Distinctiveness
  - Accuracy
  - Large capture range
  - Small number of local minima
  - Invariance :
    - Spatial transformations: rotations, translations, scale, shear, angles, isomorphism
    - Intensity changes, noise, topology

[Skerl06]

# Closest point correspondences

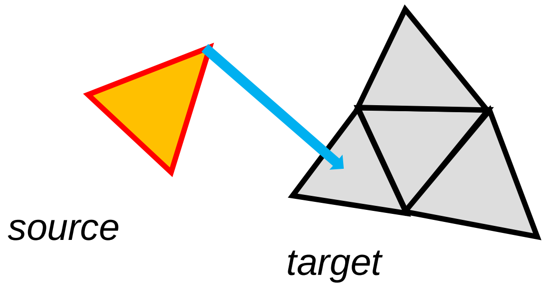



point-to-point:

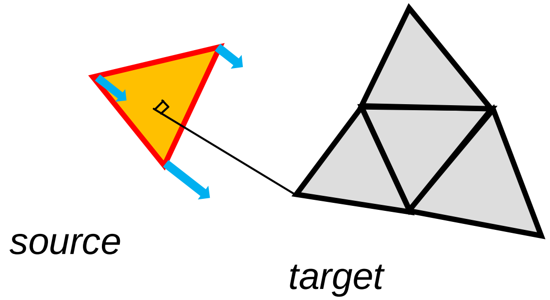
- Euclidian distance: $d = [\Sigma_3(x_j\text{-}y_j)^2]^{1/2}$
- p-order Minkowski distance: $d = [\Sigma_3(x_j\text{-}y_j)^p]^{1/p}$

Point-to-mesh:

Projection




Attraction



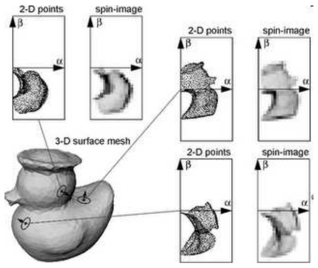

Mesh-to-mesh:

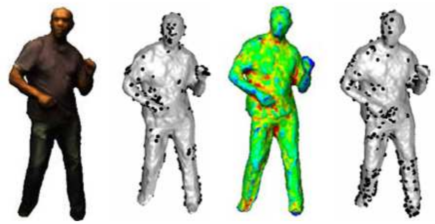
- Hausdorff distance: $d = max_{x \in X} \{ min_{y \in Y} \{d(\mathbf{x},\mathbf{y}\} \}$
- Probabilistic measures (e.g. Mahalanobis)

# Global correspondences using descriptors

– Euclidean:

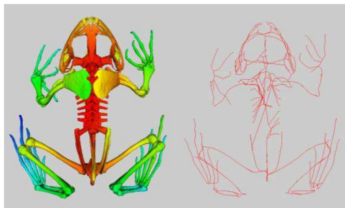
- Spin images *[Chang08],*
- *Shape context [Belongie00],*
- SIFT *[Lowe04], [Zaharescu09]*

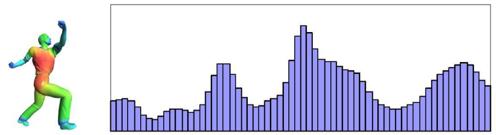


– G*eodesic:*

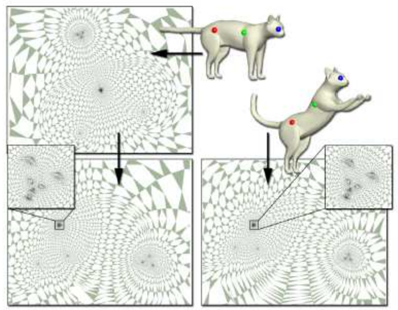
- Multidimensional scaling *[Elad01][Gal07][Bronstein08]*
- Reeb Graphs

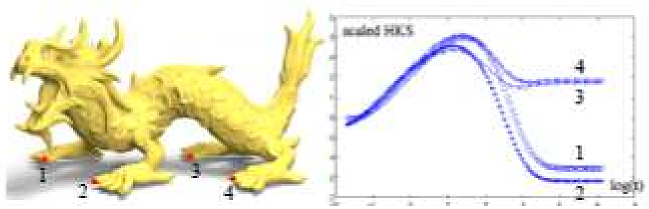




– Spectral methods:

- Laplacian Embedding *[Belkin03] [Mateus07]*
- Mobius maps *[Lipman09],*
- Diffusion distance [Lafon04]
- Global Point Signature [Rustamov07]
- Heat Kernel Signature *[Sun09]*





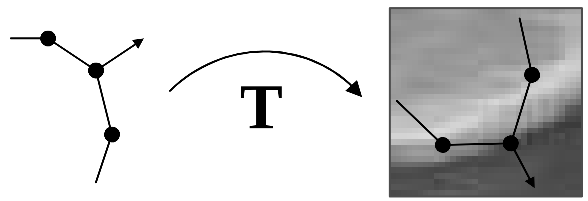
*-> coupled with feature detection*

# Model/Image correspondences

→ Align the source model to contours in the target image

Maximise gradient magnitude : $d = -\ ||\nabla I||$

Align model and image gradient : $d = \pm\ \nabla \mathbf{I.n}$




→ Maximise the similarity btw icons

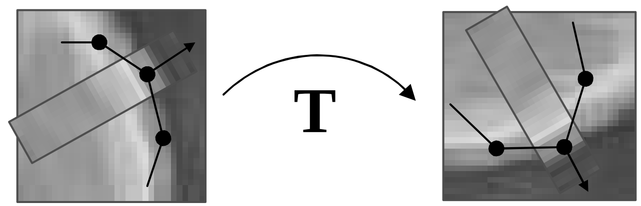
Region of Interest (vertex neighbourhood) :

- Blocks → template matching [ding01]

  Pre-processing: 3D convolution

- Direction of expected changes → Intensity profile matching [montagnat00]

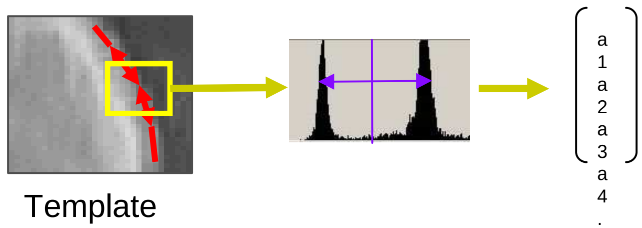
  Pre-processing: 1D convolution ( e.g. [-1 0 1] or [1 2 1] )




Similarity between:

- scalars (e.g. intensities, gradient magnitudes, gradient cosines, etc.)
- Gradients
- Feature vector :

  e.g. : SIFT [Lowe04], Histogram moments [Shen07]



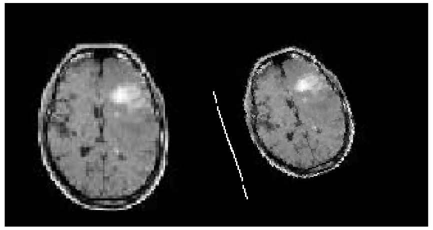
# Similarity measures

## Intensity differences

→ Assume intensity conservation: $I \approx T(J)$

Sum of absolute differences: $d_{SAD} = \Sigma_i |I_i - T(J)_i| / N$

Sum of squared differences: $d_{SAD} = \Sigma_i (I_i - T(J)_i)^2 / N$

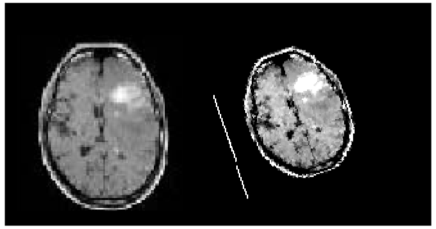


*Optical Flow [Horn81], Demon algorithm [Thirion95]: combined with pairing* $\mathbf{U}_i = (I_i - T(J)_i) . \nabla(\mathbf{T(J)}_i)$

## Intensity correlation [holden00]

→ Assume affine correlation btw intensities: $I \approx \alpha T(J) + \beta$

Normalised cross-correlation: $d_{NCC} = Cov(I, T(J)) / (\sigma_I \sigma_{T(J)})$



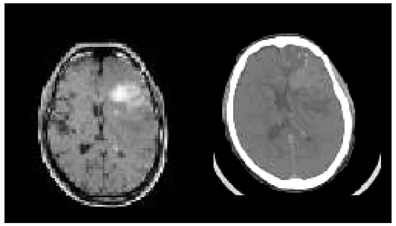
## Histogram correlation [viola95], [wells96], [maes97], [roche00], [woods92]

→ Assume functional relation btw intensities: $I \approx \Phi(T(J))$

Normalised mutual information: $d_{NMI} = [H(I) + H(T(J))] / H(I, T(J))$

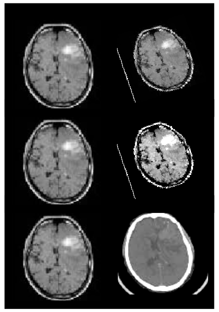
Correlation ratio: $d_{CR} = Var(I - \Phi^*(T(J))) / Var(I) = \Sigma_i N_i \sigma_i^2 / (N\sigma^2)$

Woods criterion: $d_W = \Sigma_i N_i \sigma_i / (m_i N)$



# Similarity measures

| | Different modalities | Different protocols | Large displacements |
|---|---|---|---|
| Gradient *[kass88] [xu98]* | + | + | |
| Intensity differences *[horn81], [thirion95]* | | | + |
| Intensity correlation *[holden00]* | | + | + |
| Histogram correlation *[viola95], [woods92]* | + | + | + |



# Conclusion

Choice of similarity measure and discretization :

- Input data: surface/image?

- Appearance: Large/small variability ?
  Spatial properties ?
  Invariance ?

- Initialization ?

# Outline

What is registered: **Registration features**

Registration criterion: **Similarity measure**

How to constrain the problem: **Regularisation**

How the registration is performed: **Evolution**

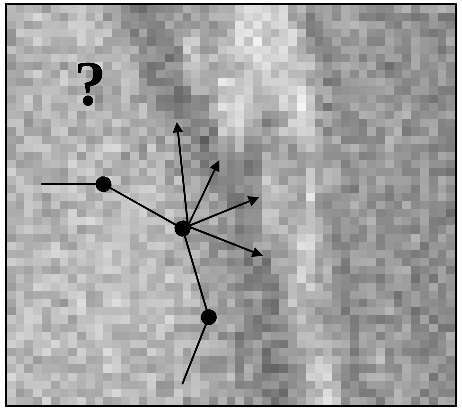
**Examples**

# Regularisation

Noise

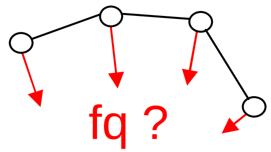
+ Local solutions

+ Aperture problem




→ The problem need to be constrained through parameterisation and internal forces

# Mapping



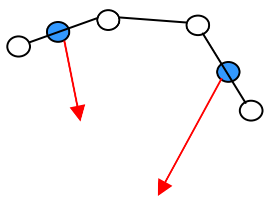

Kinematic DOFs **q**

Mapping:
$p = \Phi(q)$

Jacobian:
$dp = \underbrace{(\partial \Phi / \partial q)}_{J} \, dq$

Power conservation:
$dp^T f_p = dq^T f_q$

-> Jacobian tanspose:
$f_q = J^T f_p$

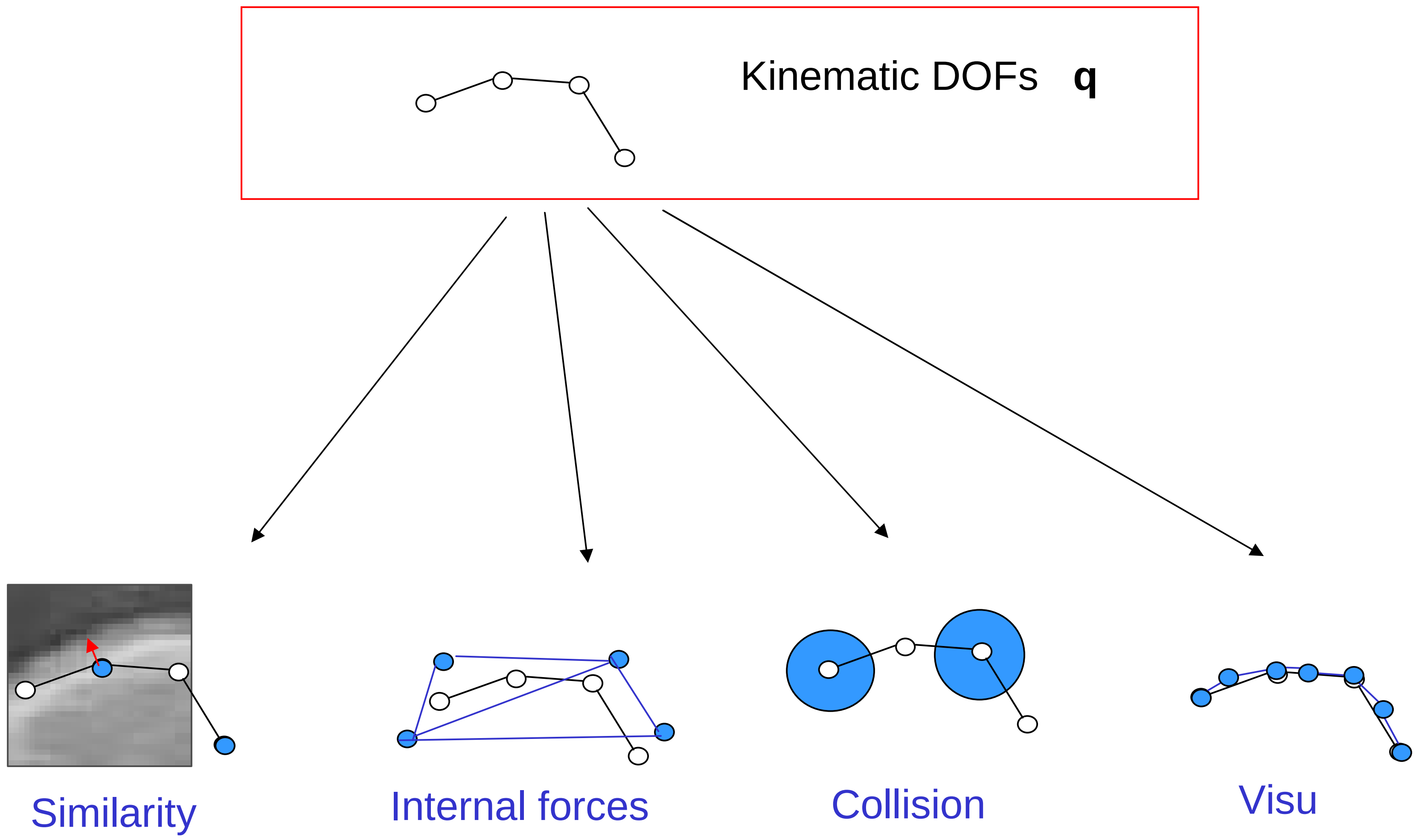


Mapped DOFs **p**

Energy: E(p)

Forces fp = - dE / dp

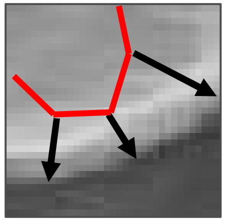
# Mapping -> separate problems



# Regularisation using parameterisation

Hypothesis about the form of the solution **T**

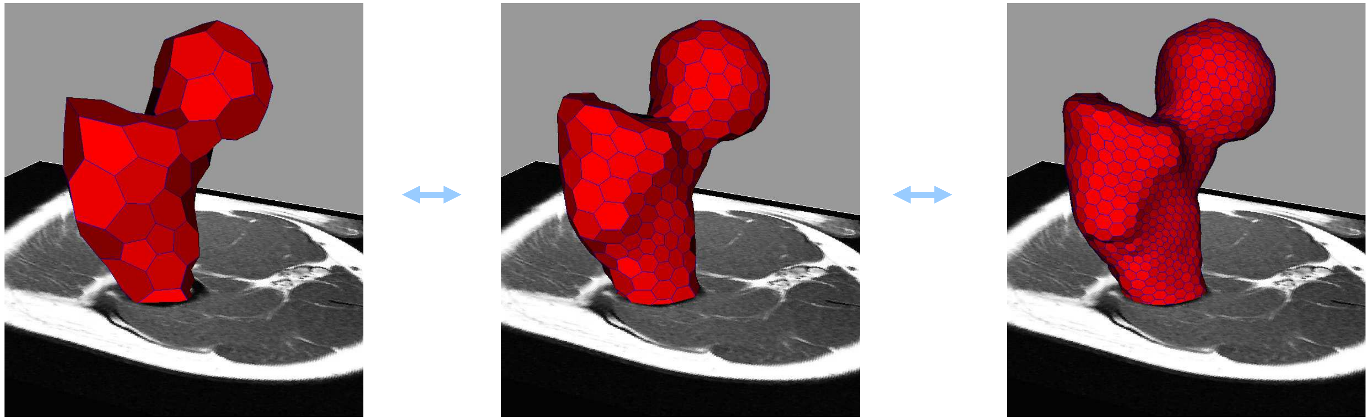
→ Reduce the search space (DOF)



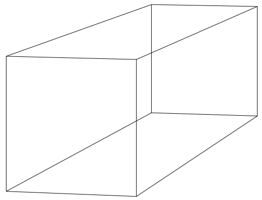
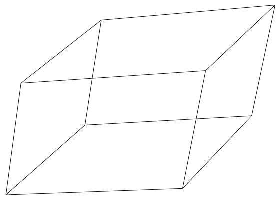
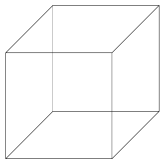
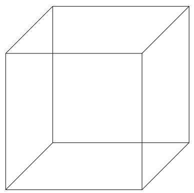
Subject

Coarse-to-fine approaches

→ Improve robustness and computational speed

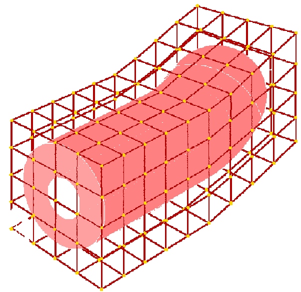


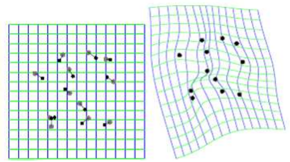
# Linear Transformations

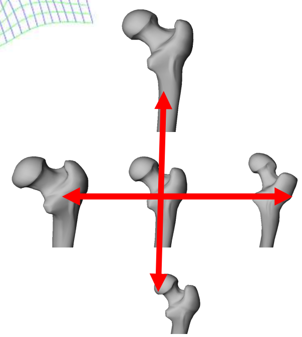
| Group | Matrix | Distortion | Invariant Properties |
|---|---|---|---|
| Projective (15DoF) | $\begin{bmatrix} A & t \\ w^T & v \end{bmatrix}$ |  | Intersection of surfaces<br>Tangency of surfaces<br>Sign of Gaussian curvature |
| Affine (12DoF) | $\begin{bmatrix} A & t \\ 0^T & 1 \end{bmatrix}$ |  | Parallelism of planes<br>Volume ratios<br>Plane at infinity |
| Similarity (7DoF) | $\begin{bmatrix} sR & t \\ 0^T & 1 \end{bmatrix}$ |  | Absolute conics |
| Euclidean (6DoF) | $\begin{bmatrix} R & t \\ 0^T & 1 \end{bmatrix}$ |  | Volume |

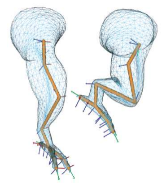
# Non-linear methods

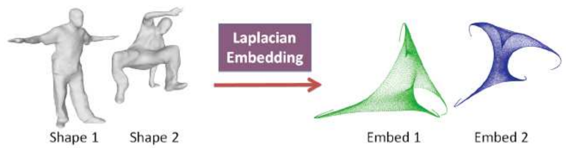
| Transform | General form |
|---|---|
| Free form deformation<br>*[sederberg86], [rueckert99]* | $\delta \mathbf{p} = \sum_{i,j,k}^{Nx,Ny,Nz} f_{i,j,k}(\mathbf{p})\delta \mathbf{p_{i,j,k}}$ |
| Radial Basis functions<br>*[rohr96 ], [rohde03 ], [Lewis01]* | $\delta p = \sum_i^N w_i(\delta p_i)\phi(\|p - p_i\|) + f(p)$<br>$\mathbf{W} = (\Phi^T \Phi)^{-}1\mathbf{\Phi^T}\delta \mathbf{P}$ |
| Example-based<br>*[szekely95 ], [cootes01 ]* | $\delta \mathbf{p} = \sum_i^N \delta w_i \mathbf{p}_i$ |
| Skinning deformation<br>*[Vlasic08][Chang09][Huang08]* | *dp=Σwi **Ai** p0* |
| Poly-Rigid, Affine<br>*[arsigny06]* | |
| Spectral embedding<br>*[Umeyama88][Mateus07]* | |
| Moving Least Squares<br>.... | |





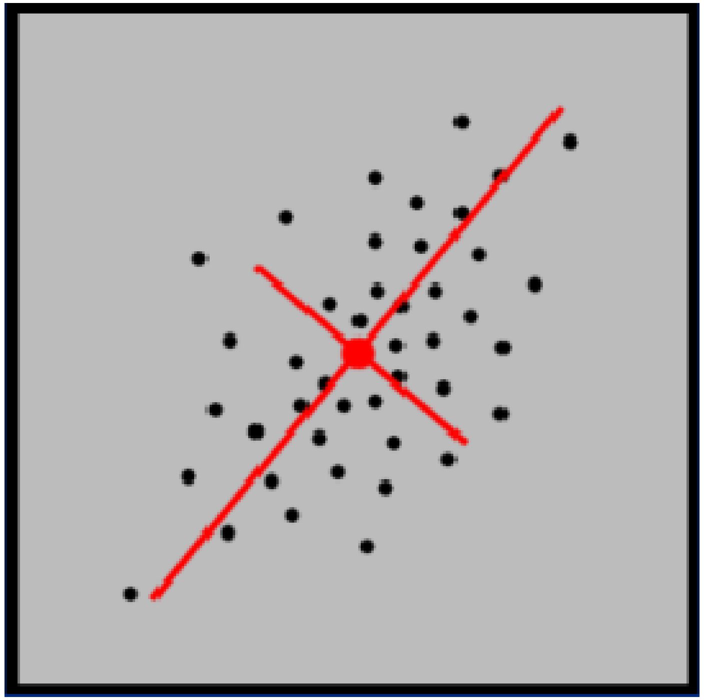




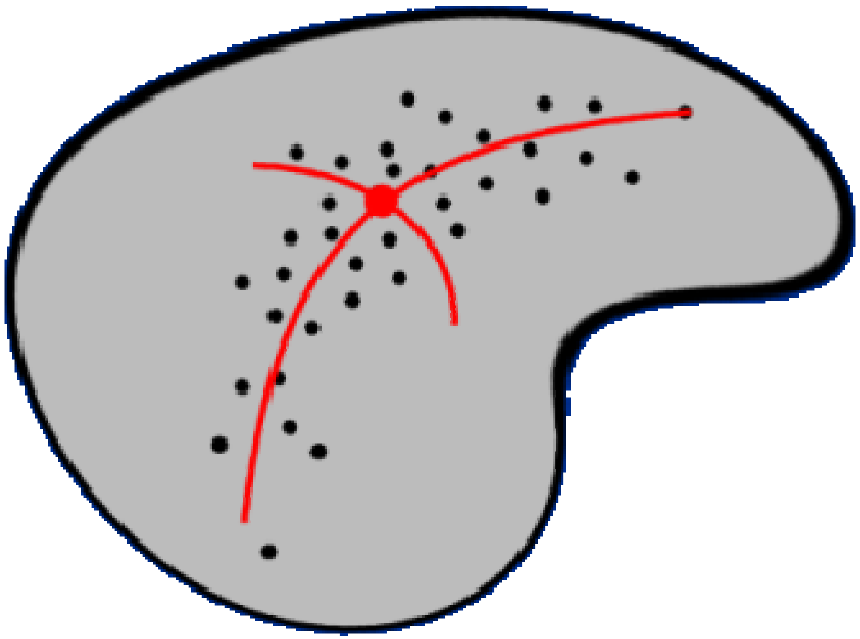


# Example-based DOFs

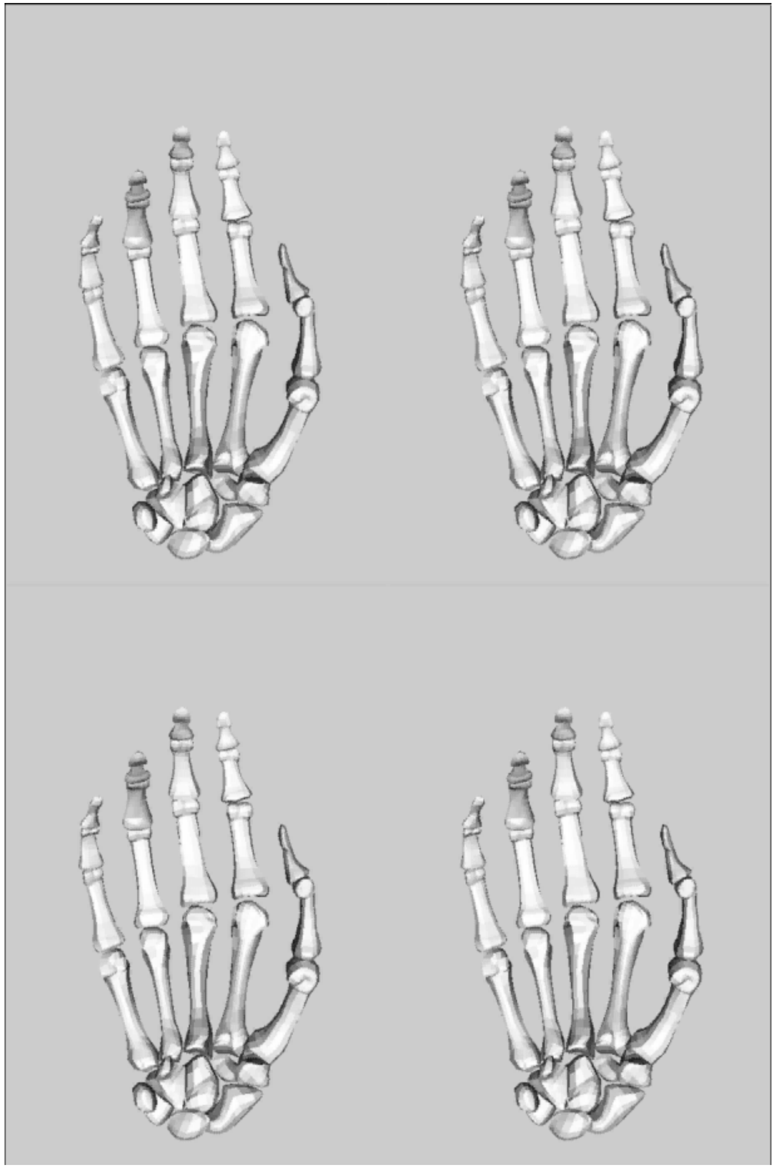
Linear Statistics : PCA



Curved Statistics : PGA

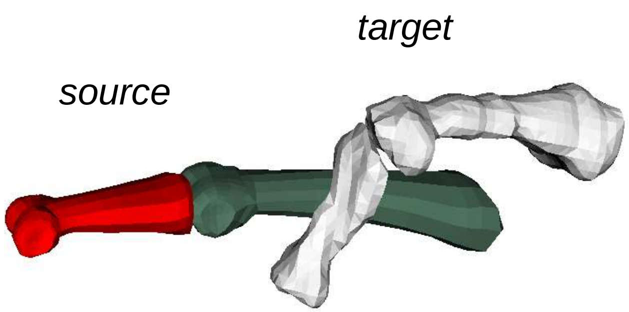


# Example-based DOFs

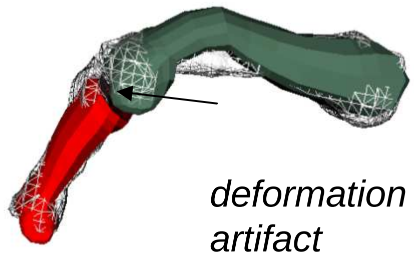


PCA on *8 hands*

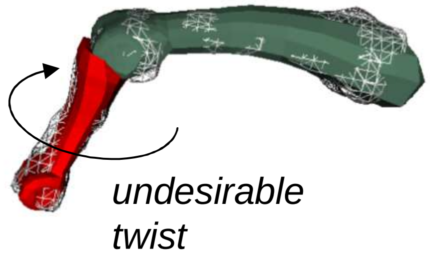
# Application specific DOFs *[gilles10]*




- Whole skeleton elastic registration

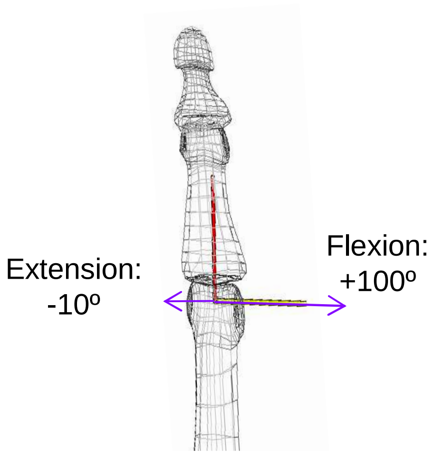


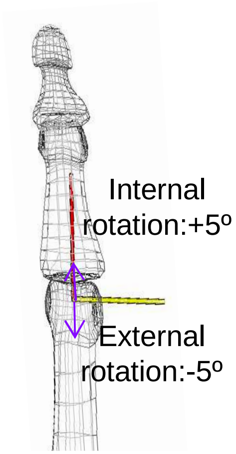

- Independent bone elastic registration

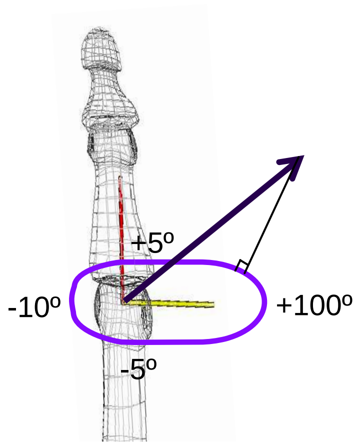



→ Joint limits

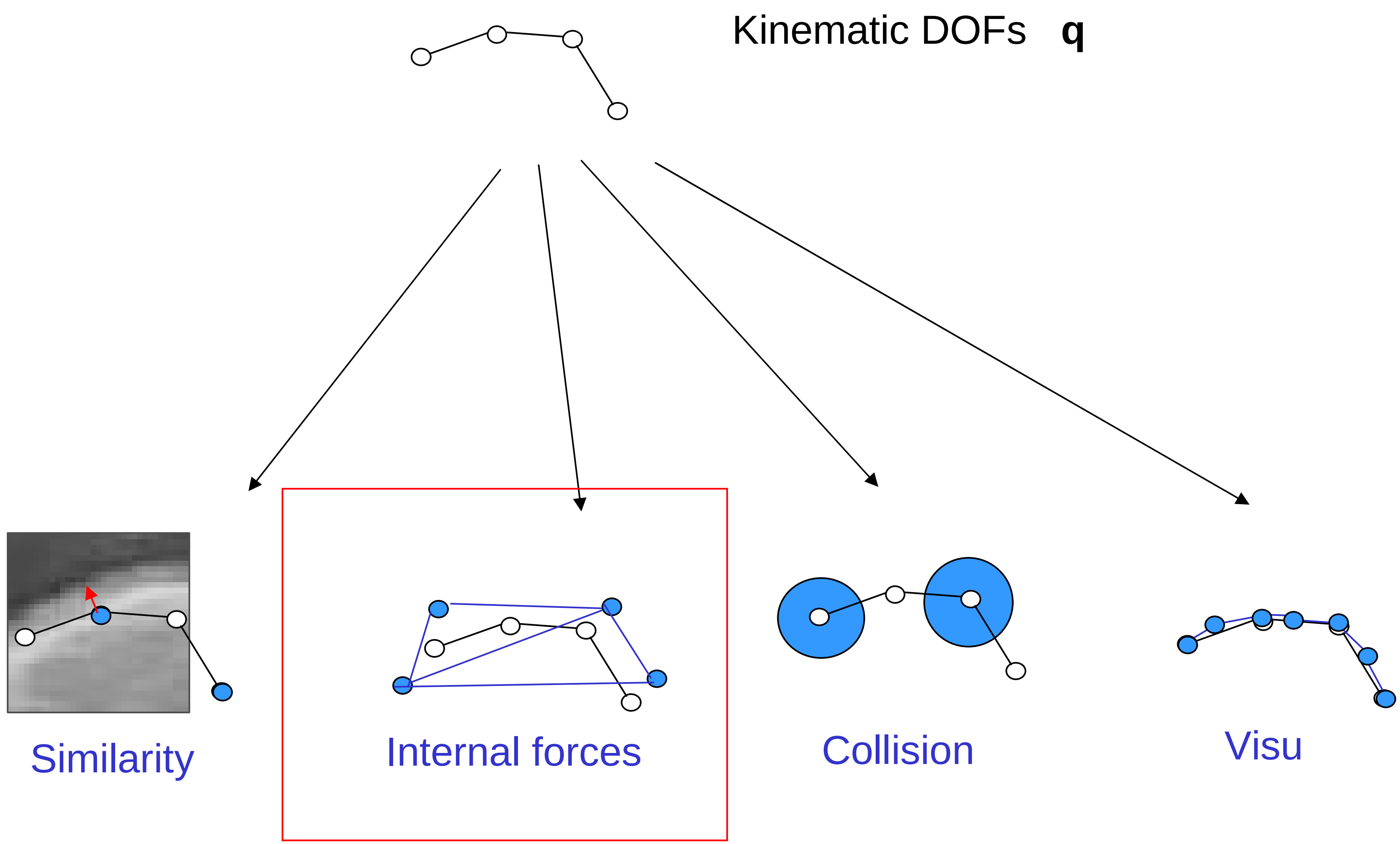
- Modeling from literature on anatomy
- Piecewise ellipsoids in the axis-angle and translation spaces
  - *Simple projections into allowed transformation space*



# Mapping -> separate problems



# Internal forces

– Smoothing: Enforce shape continuity via energy minimisation

Tikhonov differential stabilisers [terzopoulos87], [mcinerney95]

Curves: $E_{reg} = \int \sum_{1 \leq i \leq p} w_i(u) \| \frac{\partial^i \mathbf{C}(u)}{\partial u^i} \|^2 du$

Surfaces: $E_{reg} = \int \sum_{1 \leq i+j \leq p} \frac{(i+j)!}{i!j!} w_{ij}(u,v) \| \frac{\partial^{i+j} \mathbf{S}(u,v)}{\partial u^i \partial v^j} \|^2 dudv$

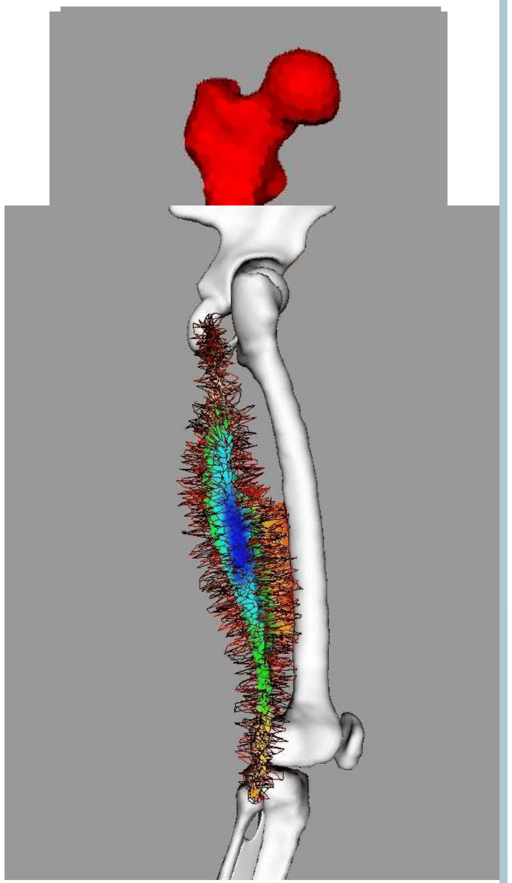
Volumes: $E_{reg} = \int \sum_{1 \leq i+j+k \leq p} \frac{(i+j+k)!}{i!j!k!} w_{ijk}(u,v,w) \| \frac{\partial^{i+j+k} \mathbf{V}(u,v,w)}{\partial u^i \partial v^j \partial w^k} \|^2 dudvdw$

- Elastic forces (=Laplacian smoothing)

→ curvature minimisation (1st order) [cohen91]

- Bending forces

→ curvature averaging (2nd order) [montagnat01]



Radial forces → thickness averaging [pizer03], [hamarneh04]

Anisotropic smoothing based on images [horn81], [deriche95]

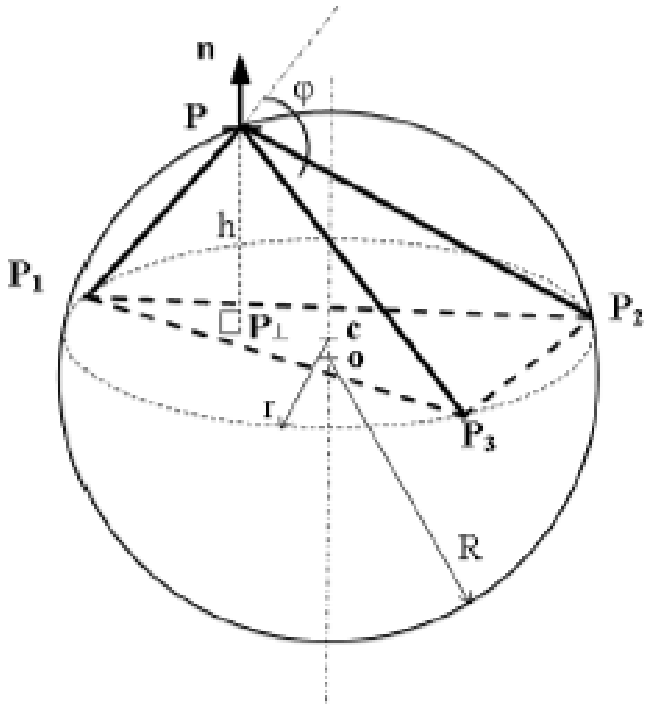
Can be temporal [terzopoulos87] [montagnat05]

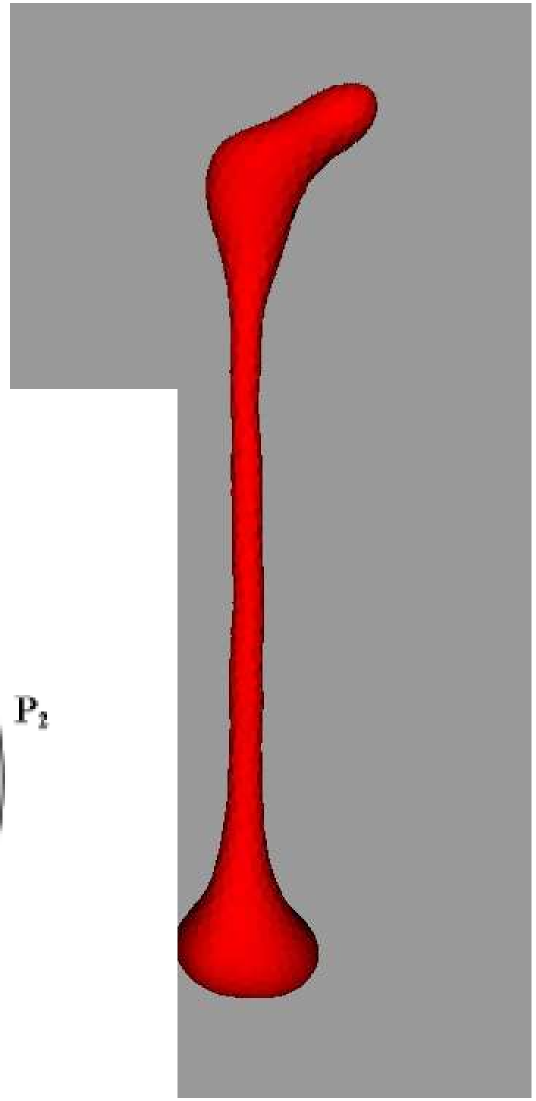
# Internal forces

– Shape memory

  • E.g. simplex surfaces

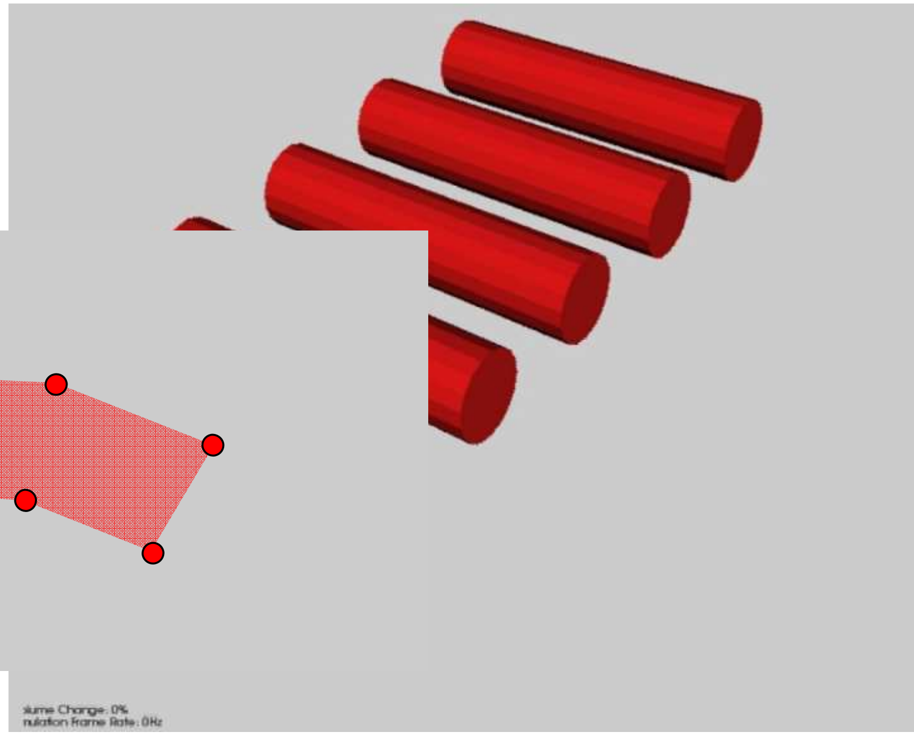
– Volume preservation

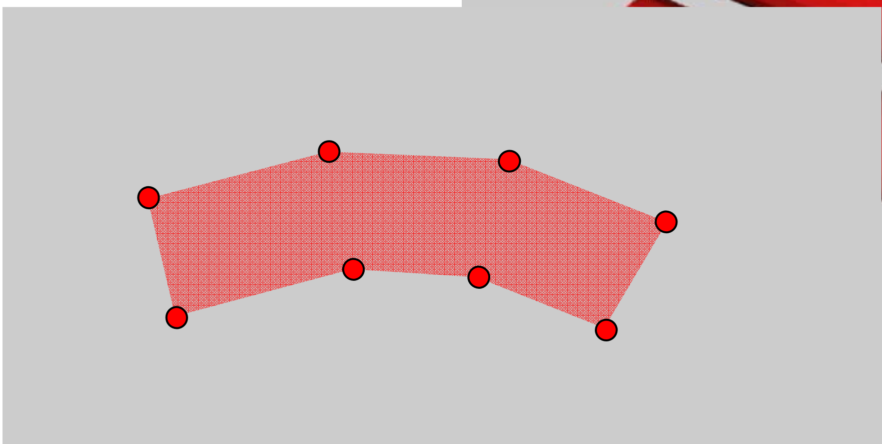


# Internal forces

- Shape memory
  - E.g. simplex surfaces
- Volume preservation
- Pseudo-elasticity:
  - E.g. shape matching



# Physically based regularization

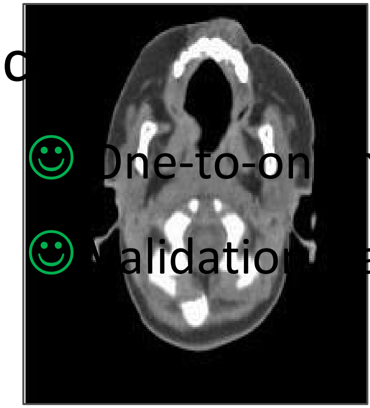
Discretization of continuum with mass-springs, FDM, FEM or FVM

Constitutive behavior: Linear elasticity (small displacements), hyperelastic, fluid

Minimisation of the strain energy [christensen96], [bro-nielsen96], [wang00], [veress06]

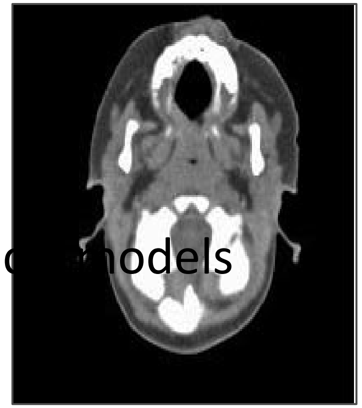
Collisions [park01]

Pros / c



☺ One-to-on mapping, no negative volume

☺ alidatio arametrization of biomechani odels

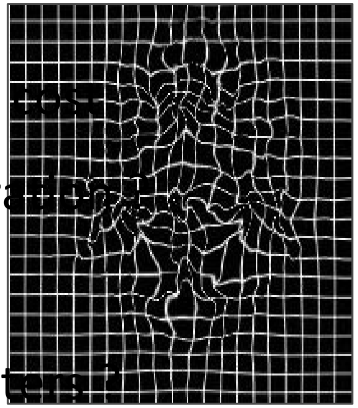



☹ High computational

☹ Inter-patient registr

☹ Image forces ?

☹ Mechanical paramet



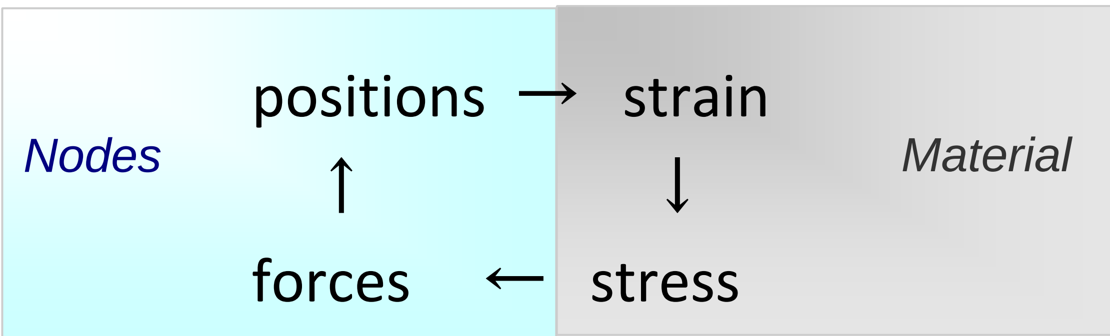
# What is needed for physically-based simulation ?

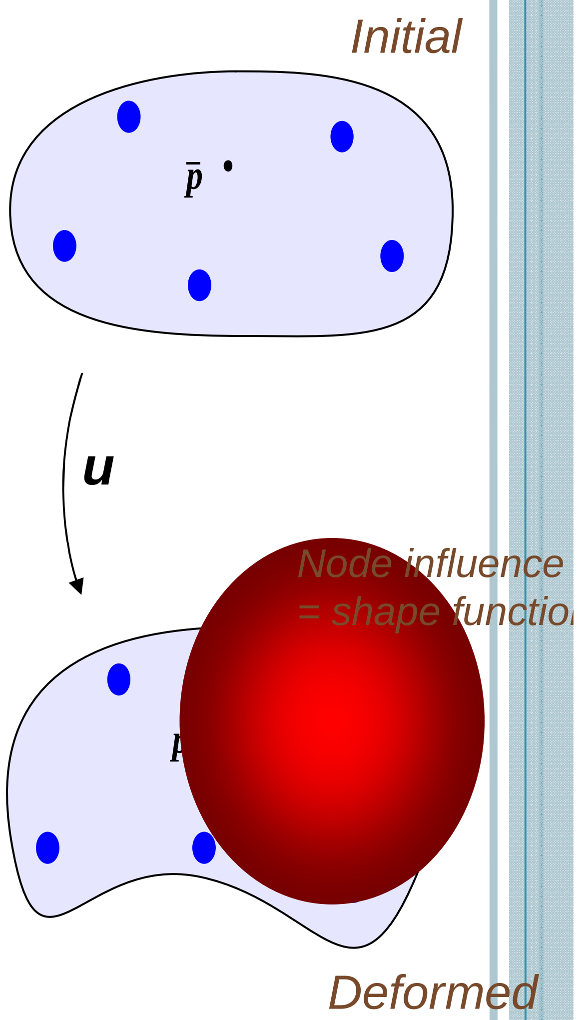
- Define control nodes

  = kinematic Degrees Of Freedom

- Interpolate a smooth displacement function

- Then, follow the classic continuum discretization:

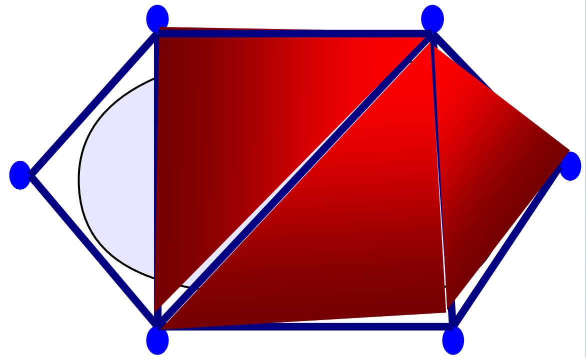


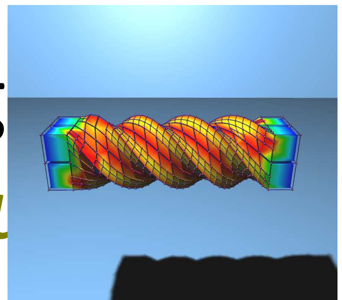
# Mesh-based methods

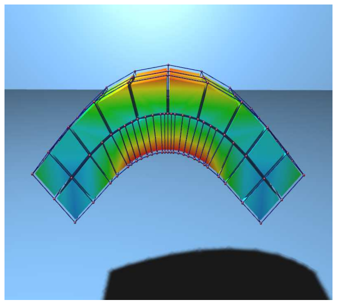

- Mass-spring networks *[Platt81]*

  F=k (L-Lo) (P2-P1)/||P2-P1||



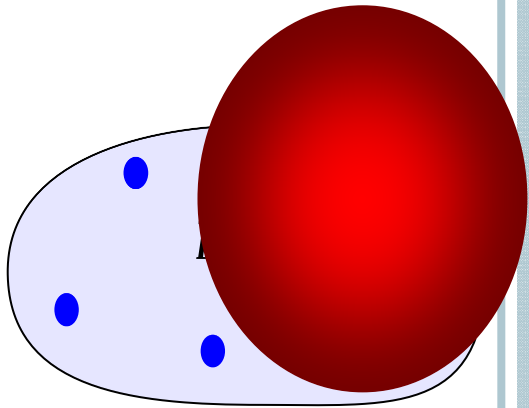
- Finite Element Methods *[Bathe96]*
  - Regular g
    *[Terzopou*





  - Corotational FEM

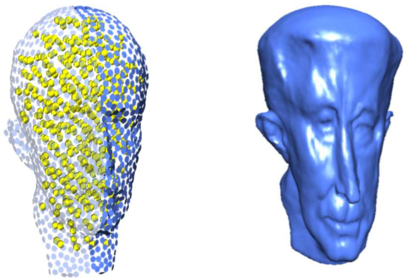
# Meshless methods

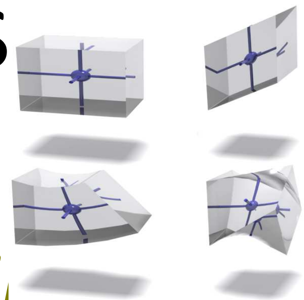


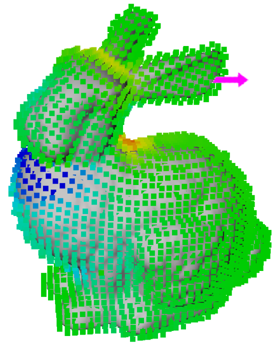
- Point based animation *[Müller04] [Gross07]*

- Moving Least Squares *[Fries03]*

- Generalized MLS







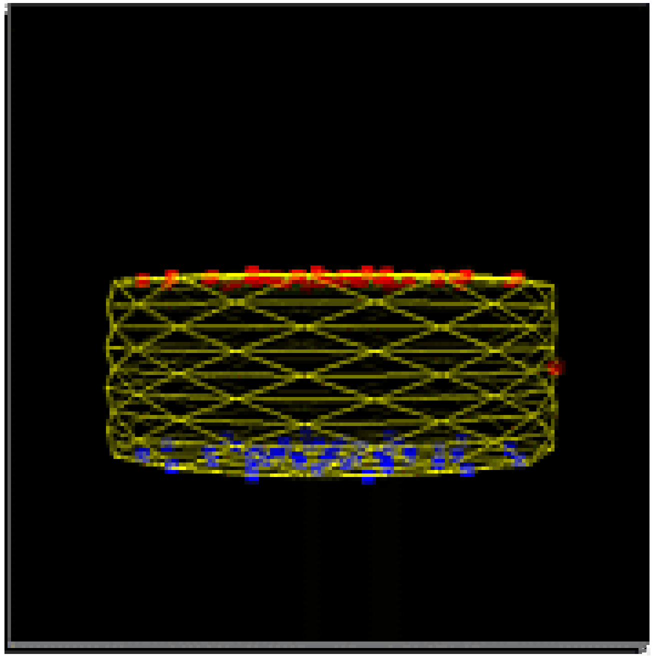
ed *[Gi*

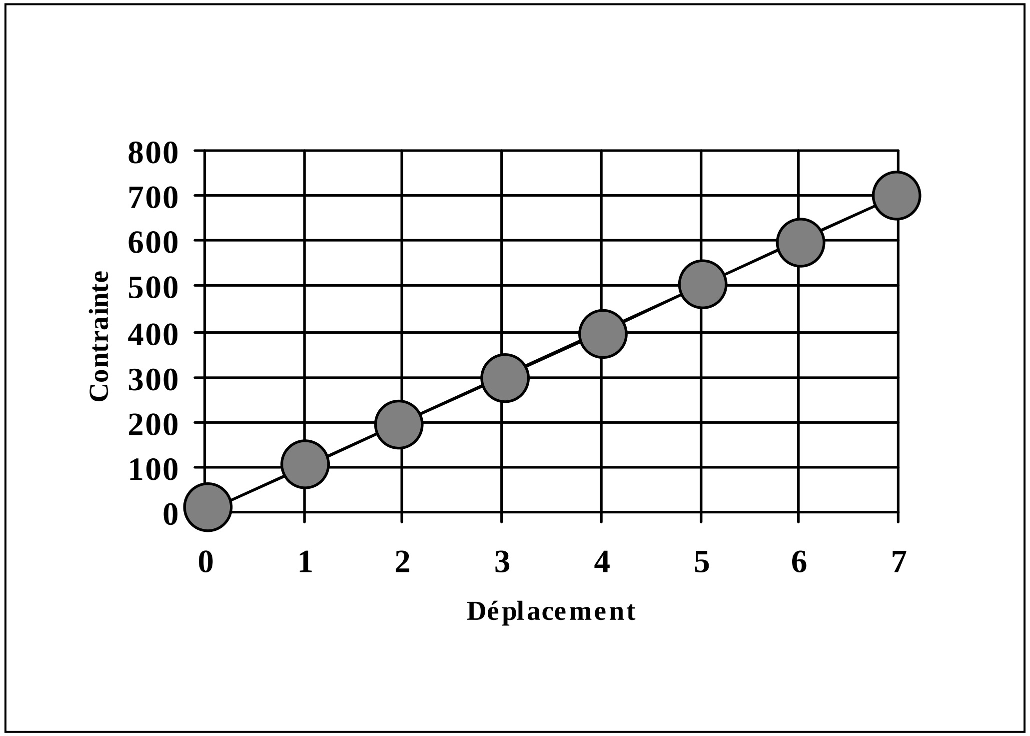
*Faure11]*

# Linear Elastic Material

Simplest Material behaviour

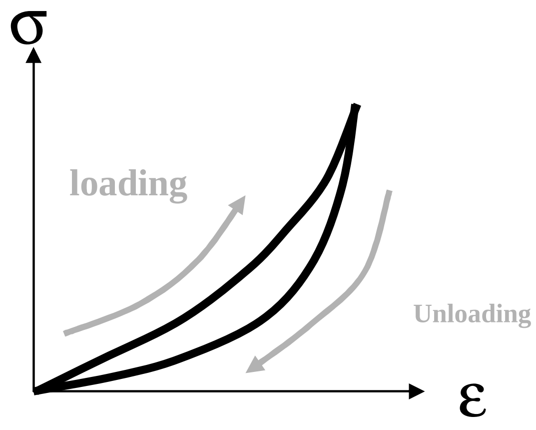
Only valid for small deformations (less than 5%)



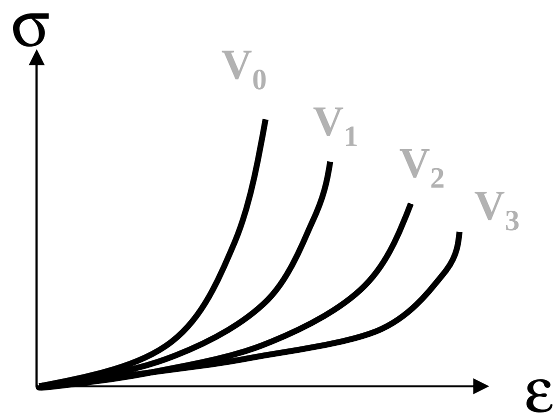


# Biological Tissue

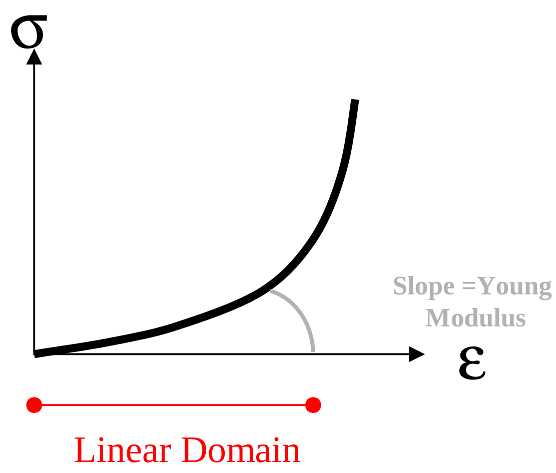
complex phenomena arises




**Hysteresis**

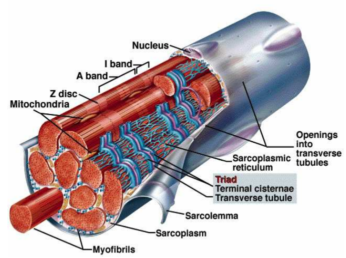



**Visco-elasticity**




**Non-Linearity**

**Anisotropy**



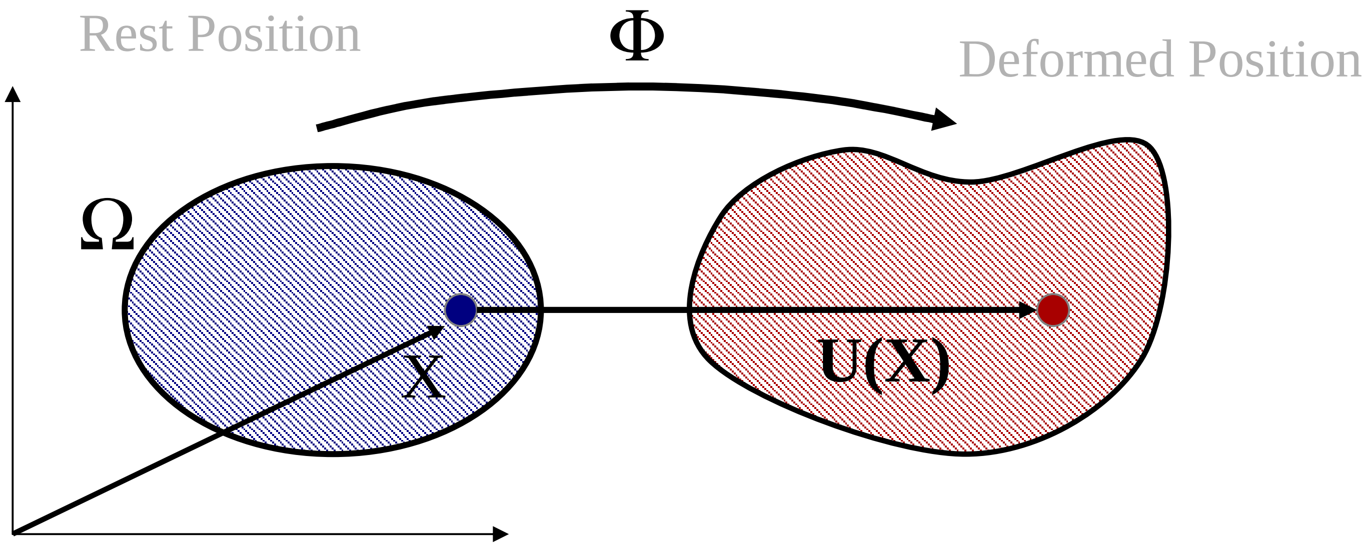
# Basics of Continuum Mechanics

Deformation Function

$$X \in \Omega \alpha \quad \phi(X) \in \Re^3$$

Displacement Function

$$U(X) = \phi(X) - X$$

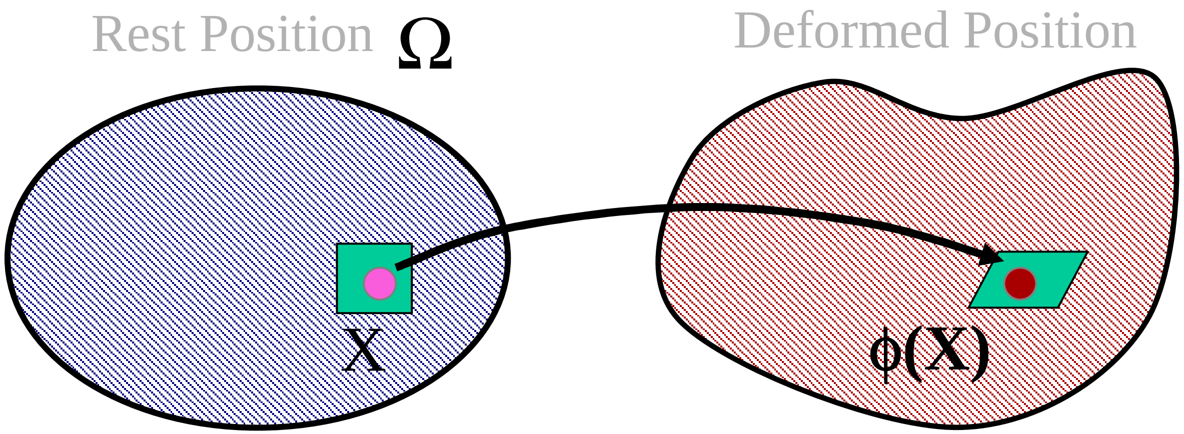


# Basics of Continuum Mechanics

The local deformation is captured by the deformation gradient :

$$F = \frac{\partial \phi}{\partial X}$$

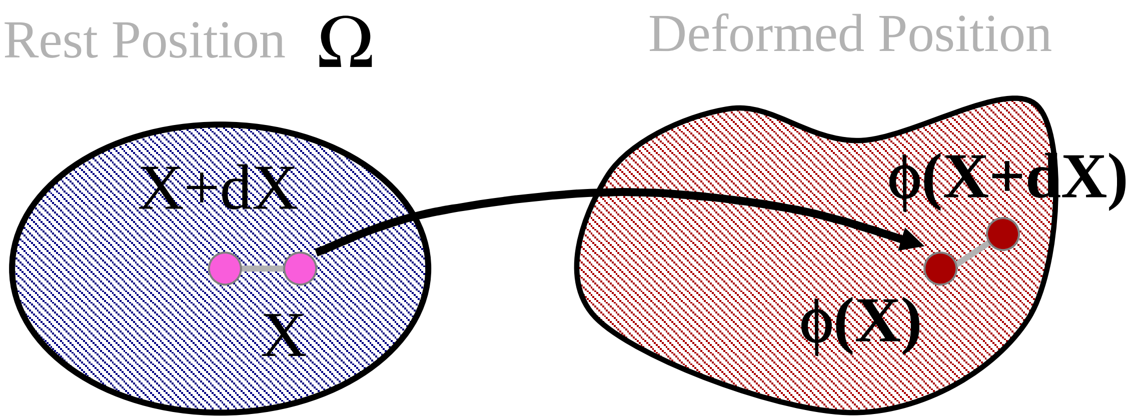
$$F_{ij} = \frac{\partial \phi_i}{\partial X_j} = \begin{bmatrix} \frac{\partial \phi_1}{\partial X_1} & \frac{\partial \phi_1}{\partial X_2} & \frac{\partial \phi_1}{\partial X_3} \\ \frac{\partial \phi_2}{\partial X_1} & \frac{\partial \phi_2}{\partial X_2} & \frac{\partial \phi_2}{\partial X_3} \\ \frac{\partial \phi_3}{\partial X_1} & \frac{\partial \phi_3}{\partial X_2} & \frac{\partial \phi_3}{\partial X_3} \end{bmatrix}$$




F(X) is the local affine transformation that maps the neighborhood of X into the neighborhood of φ(X)

# Basics of Continuum Mechanics

Distance between point may not be preserved




Distance between deformed points

Right Cauchy-green Deformation tensor

$$(ds)^2 = \|\phi(X+dX) - \phi(X)\|^2 \approx dX^T (\nabla\phi^T \nabla\phi) dX$$

$$C = \nabla\phi^T \nabla\phi$$ Measures the change of metric in the deformed body

# Basics of Continuum Mechanics

Example : Rigid Body motion entails no deformation

$$\phi(X) = RX + T$$

$$F(X) = \nabla\phi(X) = R \qquad C = R^T R = Id$$

Strain tensor captures the amount of deformation

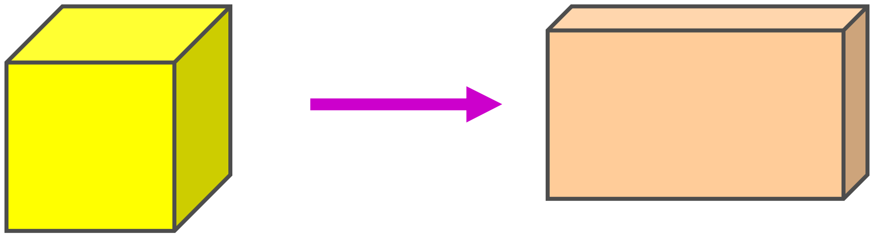
It is defined as the "distance between C and the Identity matrix"

$$E = \frac{1}{2}\left(\nabla\phi^T \nabla\phi - Id\right) = \frac{1}{2}(C - Id)$$

# Strain Tensor

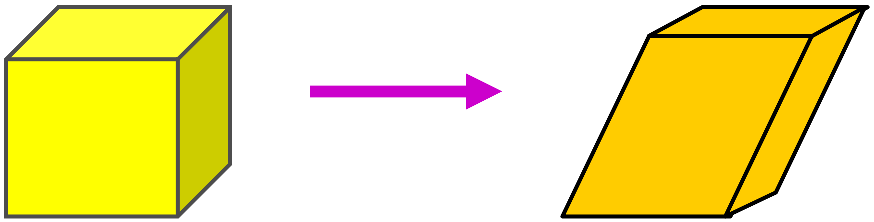
Diagonal Terms : $\varepsilon_i$

Capture the length variation along the 3 axis



Off-Diagonal Terms : $\gamma_i$

Capture the shear effect along the 3 axis



$$E = \begin{bmatrix} \varepsilon_x & \frac{1}{2}\gamma_{xy} & \frac{1}{2}\gamma_{xz} \\ \frac{1}{2}\gamma_{xy} & \varepsilon_y & \frac{1}{2}\gamma_{yz} \\ \frac{1}{2}\gamma_{xz} & \frac{1}{2}\gamma_{yz} & \varepsilon_z \end{bmatrix}$$

# Linearized Strain Tensor

Use displacement rather than deformation

$$\nabla \phi(X) = Id + \nabla U(X)$$

$$E = \frac{1}{2}\left(\nabla U + \nabla U^T + \nabla U^T \nabla U\right)$$

Assume small displacements

$$E_{Lin} = \frac{1}{2}\left(\nabla U + \nabla U^T\right)$$

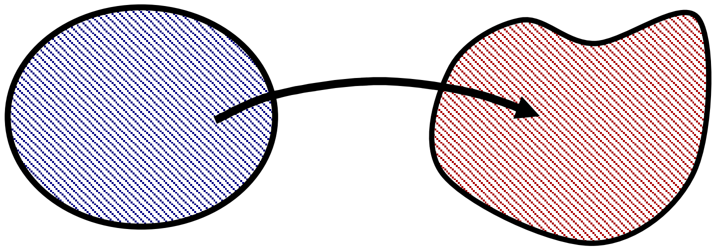
# Hyperelastic Energy

The energy required to deform a body is a function of the invariants of strain tensor E :

Trace E = = $I_1$

Trace E*E= $I_2$

Determinant of E = $I_3$




$$W(\phi) = \int_{\Omega} w(I_1, I_2, I_3)\, dX$$

Total Elastic Energy

# Linear Elasticity = Hooke's law

Isotropic Energy $$w(X) = \frac{\lambda}{2}(tr\,E_{Lin})^2 + \mu\, tr\, E_{Lin}^2$$

$(\lambda, \mu)$ **: Lamé coefficients**

$w(X)$ **: density of elastic energy**

Advantage :

Quadratic function of displacement

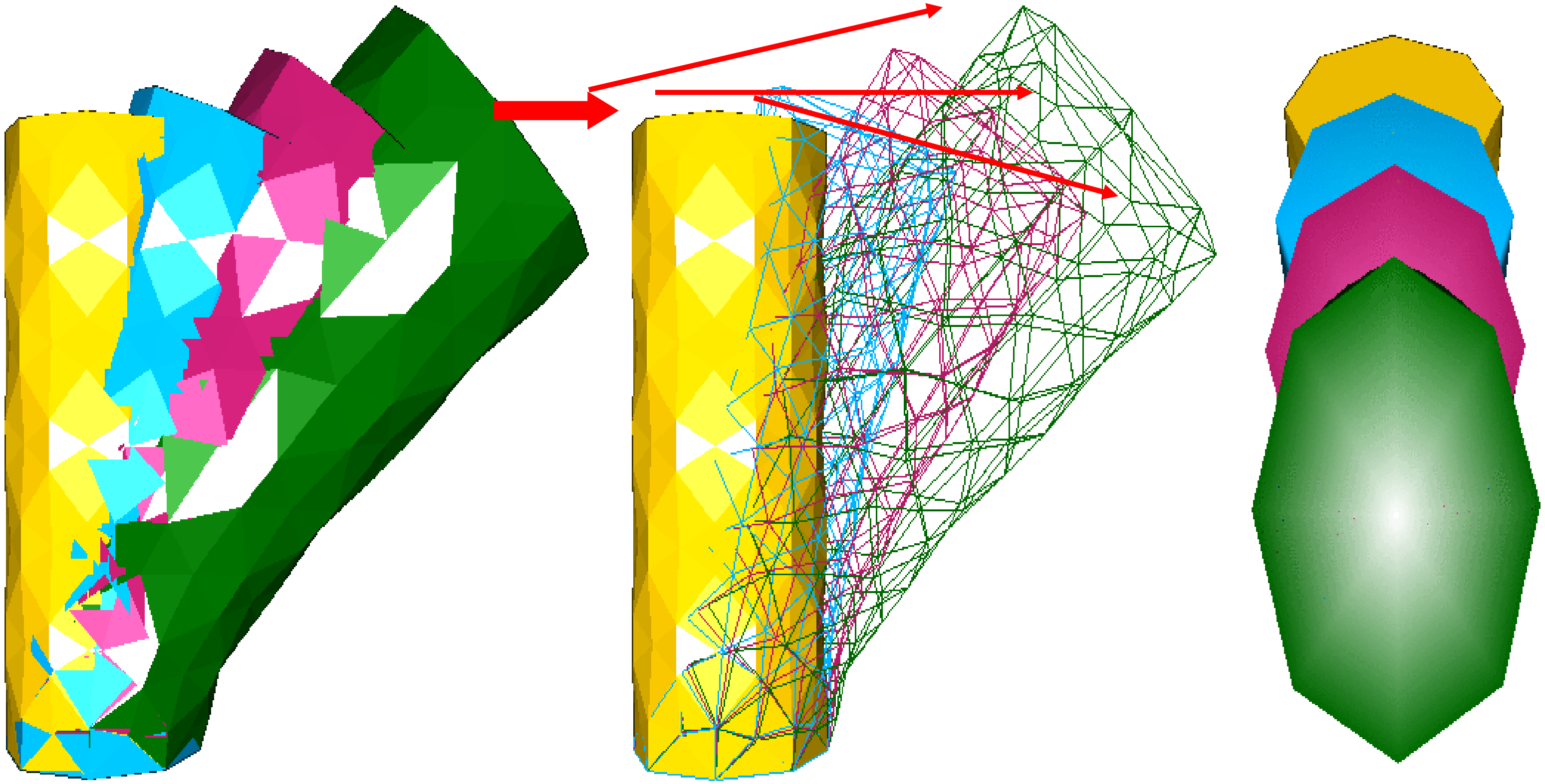
$$w = \frac{\lambda}{2}(div\,U)^2 + \mu\|\nabla U\|^2 - \frac{\mu}{2}\|rot\,U\|^2$$

Drawback :

Not invariant with respect to global rotation

# Shortcomings of linear elasticity

Non valid for « large rotations and displacements »



# St-Venant Kirchoff Elasticity

Isotropic Energy

$$w(X) = \frac{\lambda}{2}\left(tr\, E\right)^2 + \mu\, tr\, E^2$$

$(\lambda, \mu)$ **: Lamé coefficients**

Advantage :

- Generalize linear elasticity
- Invariant to global rotations

Drawback :

- Poor behavior in compression
- Quartic function of displacement

# Other Hyperelastic Material

- Neo-Hookean Model $$w(X)=\frac{\mu}{2}trE+f\left(I_3\right)$$
- Fung Isotropic Model $$w(X)=\frac{\mu}{2}e^{trE}+f\left(I_3\right)$$
- Fung Anisotropic Model $$w(X)=\frac{\mu}{2}e^{trE}+\frac{k_1}{k_2}\left(e^{k_2(I_4-1)}-1\right)+f\left(I_3\right)$$
- Veronda-Westman $$w(X)=c_1\left(e^{\gamma trE}\right)+c_2\,trE^2+f\left(I_3\right)$$
- Mooney-Rivlin : $$w(X)=c_{10}trE+c_{01}trE^2+f\left(I_3\right)$$

# Conclusion

Choice of regularization method and discretization :

- Deformation: global/local ? Large/small ?
  Mechanical ? Discontinuities ?
  Volume/surface/curve ?

# Outline

What is registered: **Registration features**

Registration criterion: **Similarity measure**

How to constrain the problem: **Regularisation**

How the registration is performed: **Evolution**

**Examples**

# Explicit resolution

- Project correspondences to the closest allowed transform

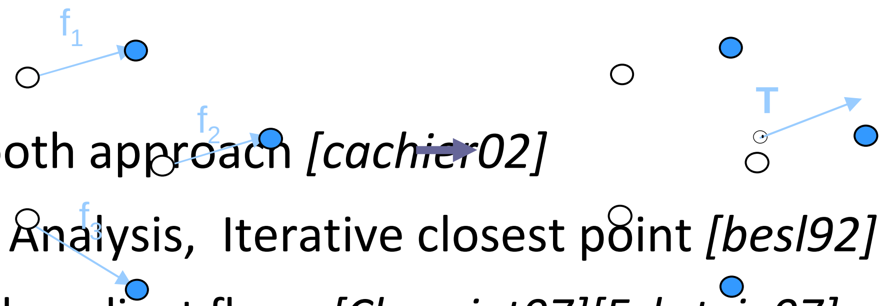
  – Analytical solution for simple transforms

  – Example: affine transform:

$$A^* = \Sigma_i (X_i - \mu_X)(Y_i - \mu_Y)^T \left[\Sigma_i (X_i - \mu_X)(X_i - \mu_X)^T\right]^{-1}$$

$$t^* = \mu_Y - A^* \mu_X$$

Used in :




– Pair & smooth approach *[cachier02]*

– Procrustes Analysis, Iterative closest point *[besl92]*

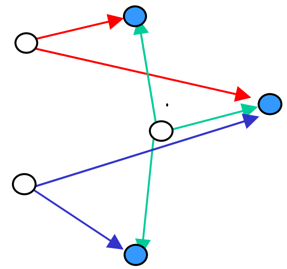
– Generalized gradient flows *[Charpiat07][Eckstein07]*

– Shape matching *[Mueller05][Rivers07][Gilles08]*

# Graph Matching approaches

Solve assignment problem:

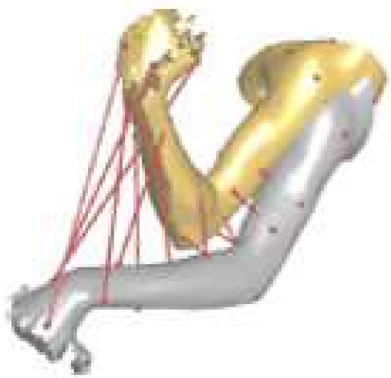
find map *T: pi* → *qj* st. *E(T(pi))* is minimal



– Linearization *[Jiang09]*

– Voting *[Lipman09]*

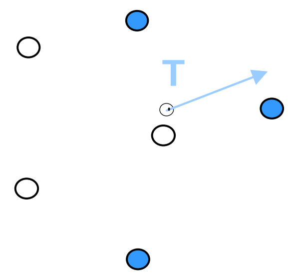
– Greedy algorithm *[Huang08]*



- Global correspondences
- Combined with a dense method

# Variational approaches

Minimize internal + external energy




- Global methods:

- Exhaustive or quasi-exhaustive methods (multigrid)
- Simulated annealing *[snyder92]*
  - Allow energy increase according to the temperature
- Genetic algorithm *[koza98]*
  - A fitness function is optimised through gene crossing
- Dynamic programming *[amini90]*

→ The global minimum is reached at the price of computations

# Variational approaches

Local methods = Oriented research

- Bracketing: simplex (amoeba) method [nelder65]
- Gradient descent

$$\rightarrow \; \delta \boldsymbol{P} = -\nabla \boldsymbol{E}(\boldsymbol{P}).dt$$ [thirion95]

- Powell's method → conjugate directions
- Newton (2nd order development)

$$\rightarrow \; \delta \boldsymbol{P} = -\nabla^2 \boldsymbol{E}(\boldsymbol{P})^{-1}.\; \nabla \boldsymbol{E}(\boldsymbol{P})$$ [vemuri97]

- Levenberg-Marquardt = Newton+ Gradient descent [Marquardt63]
- Newton-Raphson (1st order development)

$$\rightarrow \; \delta \boldsymbol{P} = -\| \nabla \boldsymbol{E}(\boldsymbol{P}) \|^{-2}.E(P).\, \nabla \boldsymbol{E}(\boldsymbol{P})$$ [müller06]

Bayesan framework [staib92], [wang00], [chen00]

- Maximisation of shape probability given the image

# Variational approaches

## Dynamic evolution

Discrete models = lumped mass particles submitted to forces

Newtonian evolution (1st order differential system):

$$\begin{cases} \delta \boldsymbol{P} = \boldsymbol{V}.dt \\ \delta \boldsymbol{V} = \boldsymbol{M}^{-1}\boldsymbol{F}dt \end{cases}$$

Explicit schemes:

- Euler: $\begin{cases} \delta \boldsymbol{P} = \boldsymbol{V}_t.dt \\ \delta \boldsymbol{V} = \boldsymbol{M}^{-1}\boldsymbol{F}_t dt \end{cases}$
- Runge-Kutta: several evaluations to better extrapolate the new state [press92]

  → Unstable for large time-step !!

Semi-Implicit schemes:

- Euler: $\begin{cases} \delta \boldsymbol{P} = \boldsymbol{V}_{t+dt}.dt \\ \delta \boldsymbol{V} = \boldsymbol{M}^{-1}\boldsymbol{F}_t dt \end{cases} \rightarrow \begin{cases} \boldsymbol{P}_{t+dt} = 2\boldsymbol{P}_t - \boldsymbol{P}_{t-dt} + \boldsymbol{M}^{-1}\boldsymbol{F}_t\, dt^2 \\ \boldsymbol{V}_{t+dt} = (\boldsymbol{P}_{t+dt} - \boldsymbol{P}_t)dt^{-1} \end{cases}$
- Verlet [teschner04]

# Variational approaches

Implicit schemes [terzopoulos87], [baraff98], [desbrun99], [volino01], [hauth01]

- First-order expansion of the force:

$$F_{t+dt} \approx F_t + \partial F/\partial P \; \delta P + \partial F/\partial V \; \delta V$$

- Euler implicit

$$\rightarrow \begin{cases} \delta P = V_{t+dt}.dt \\ \delta V = H^{-1} Y \end{cases} \qquad \begin{aligned} H &= I - M^{-1} \; \partial F/\partial V \, dt - M^{-1} \; \partial F/\partial P \, dt^2 \\ Y &= M^{-1} F_t + M^{-1} \; \partial F/\partial P \, V_t \, dt^2 \end{aligned}$$

- Backward differential formulas (BDF) : Use of previous states

→ Unconditionally stable for any time-step

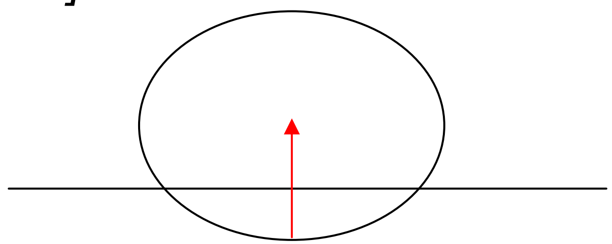
… But requires the inversion of a large sparse system

- Choleski decomposition + relaxation
- Iterative solvers: Conjugate gradient, Gauss Seidel
- Speed and accuracy can be improve through preconditioning (alteration of **H**)

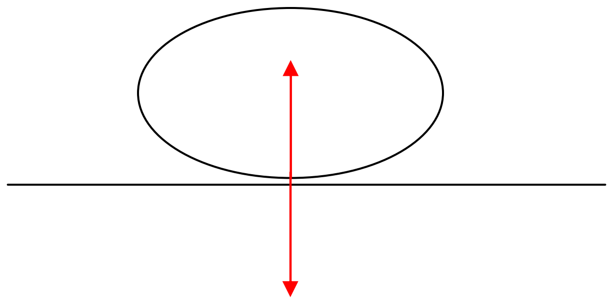
# Enforcing constraints

Non-penetration, articulations, range of motion, etc.

- Penalty methods
  - Acceleration-based : (stiff) springs *[Moore88]*
  - Velocity-based:  impulses *[Mirtich94][Weinstein06]*
  - Position-based  *[Gascuel94][Lee00]*

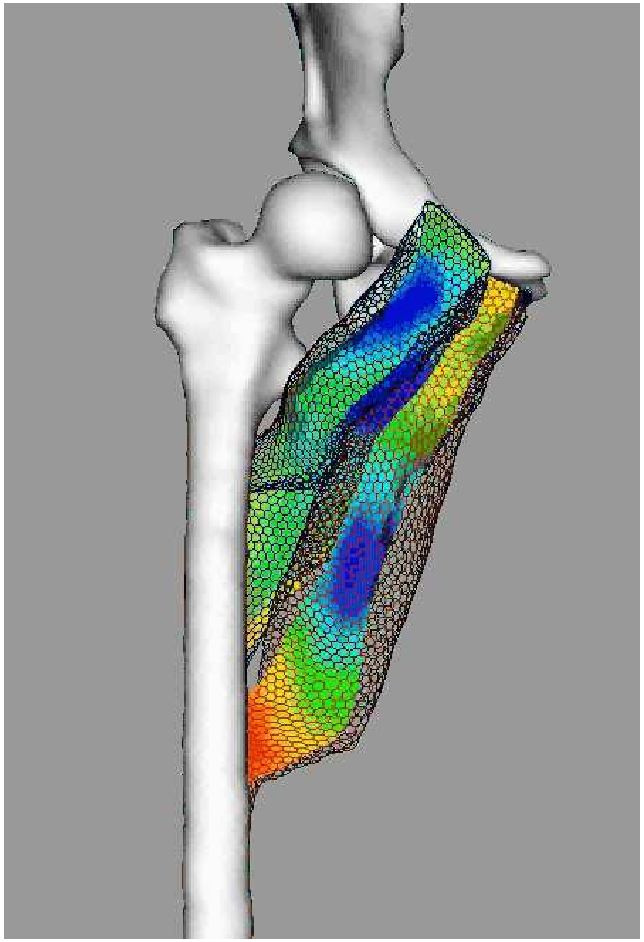


- Constrained dynamics *[Barraf94][Faure99]*
  - *Lagrange multipliers*



# Collision handling using medial axis

- *Exploit implicit representation*
- *Combined with BVH*
- *Correction of velocity and position*



# Joint constraints *[gilles10]*

Goal:

Estimate pose within joint limits
Minimize displacements from current positions

Requirements:

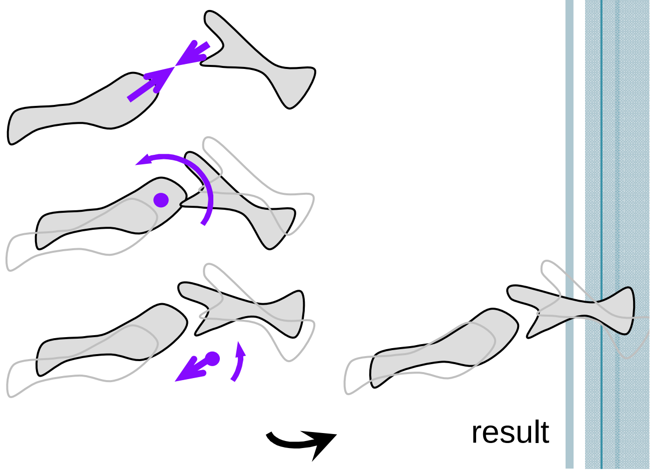
Handles loops
Joint limits = unilateral constraints

Position-based

- Goal: reach feasible pose while minimizing displacements
- Greedy algorithm (=Gauss Seidel):

*For each joint :*

- *Solve for translations (closed-form solution)*
- *Project to closest valid rotation*
- *Solve for the global rigid transform*



# Conclusion

Choice of evolution method :

- Energy: Analytic solution ?
  Smooth ?
  Inertia ?
  # DOFs ?
  Constraints ?

# Outline

What is registered: **Registration features**

Registration criterion: **Similarity measure**

How to constrain the problem: **Regularisation**

How the registration is performed: **Evolution**

**Examples**

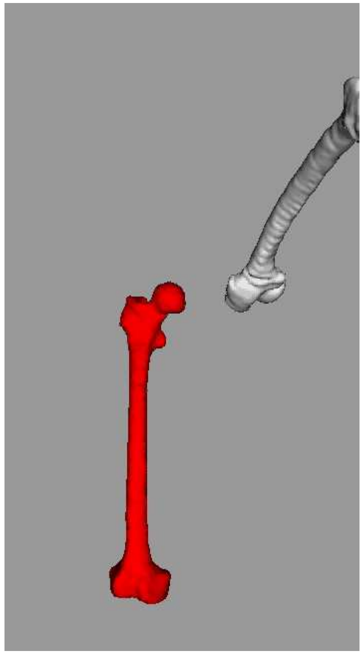
# Example: Iterative closest point

Pair and smooth approach

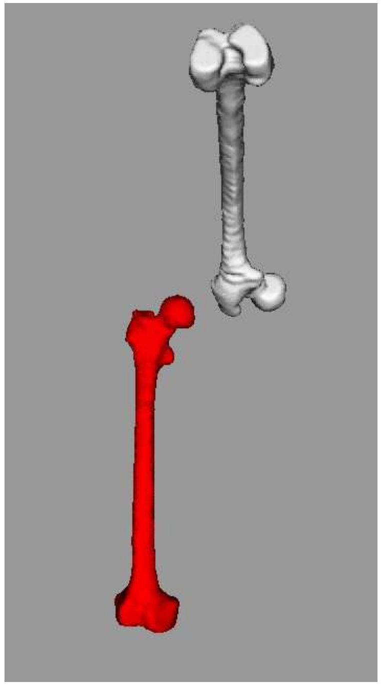
Explicit resolution

- rigid transforms

Closest Point similarity measure



Global minimum



Local minimum

# Example: Iterative closest point

Bone tracking

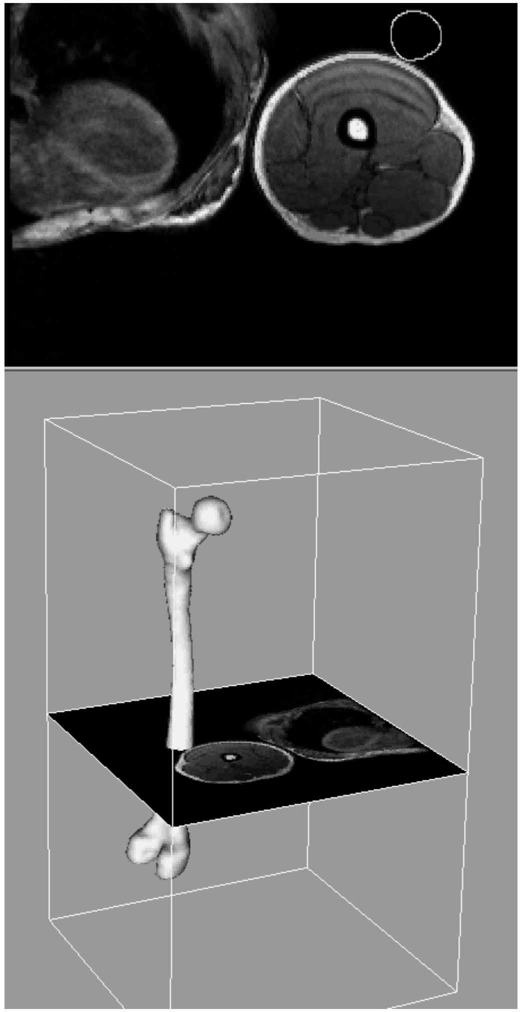
Pair and smooth approach

Explicit resolution

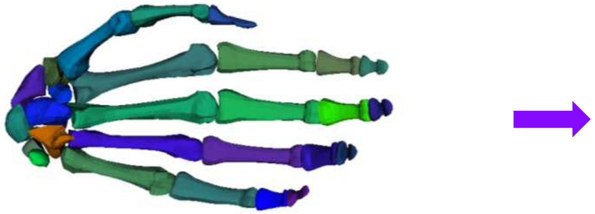
- rigid transforms

Iconic similarity measure

- Normalised cross-correlation



# Example: constrained ICP



Subject-specific model:

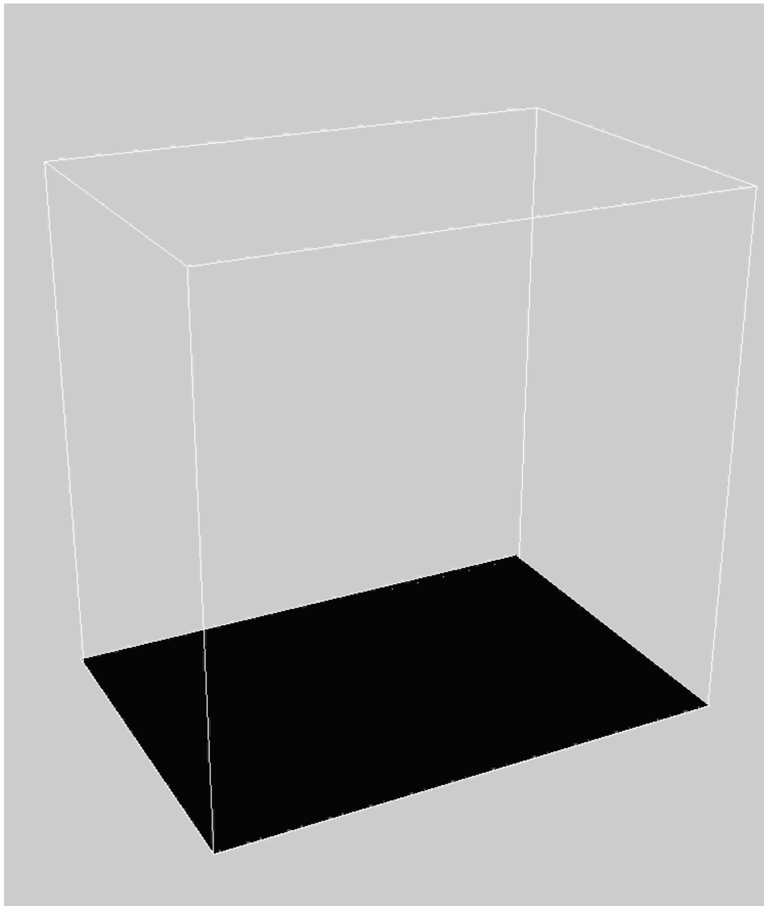
- *27 bones*
- *40 joints*
- *7k vertices*

Registration:

*3 min*

*50 iterations (elastic)*

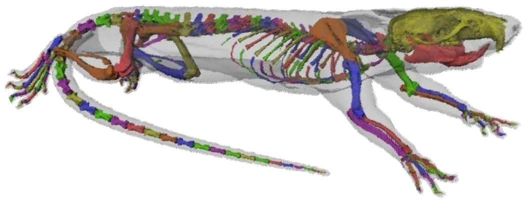
*500 iterations (plastic)*



MRI data

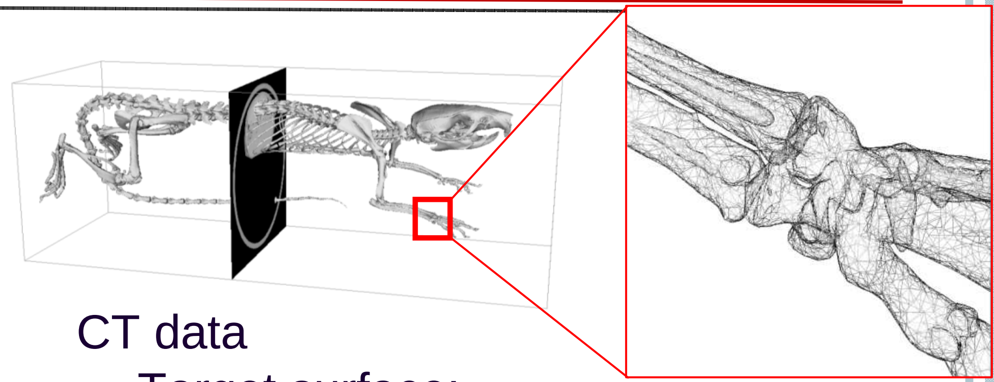
- *0.4 x 0.4 x 2mm*

# Surface registration : rat example



Template :

- *214 bones*
- *228 joints*
- *34k vertices*



CT data
→ Target surface:

- *50k vertices*

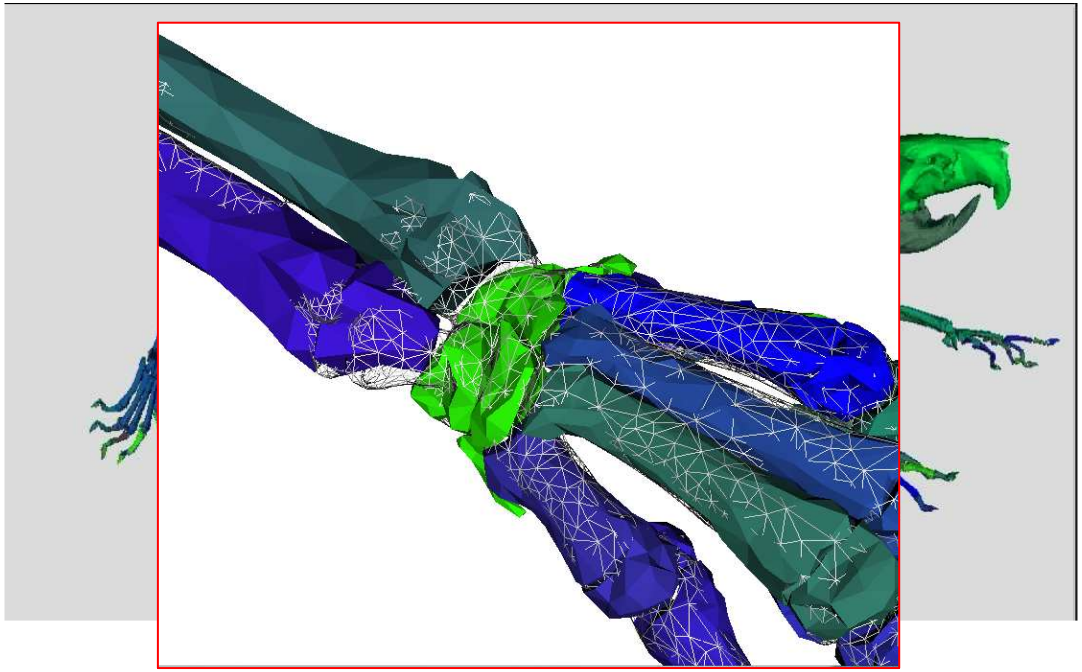
Registration:

*Shape matching (4 res)*

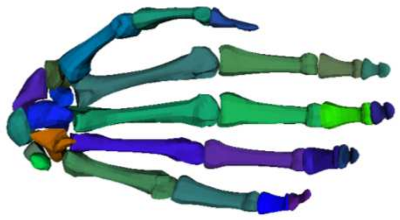
*2 min*

*60 iterations (elastic)*

*25 iterations (plastic)*

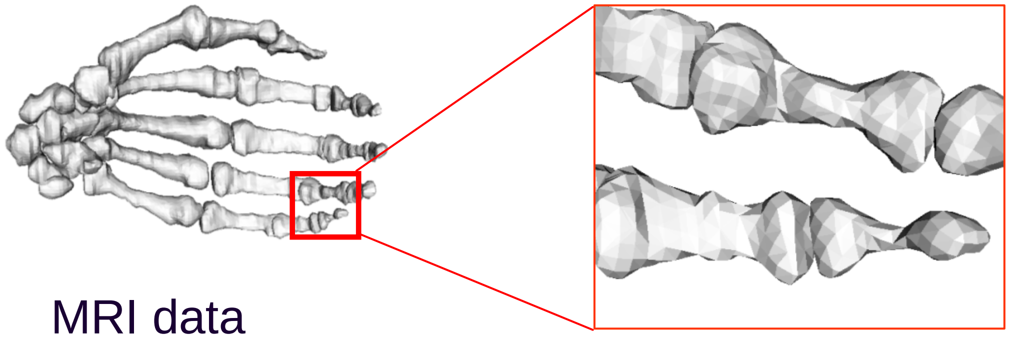


# Surface registration : hand example



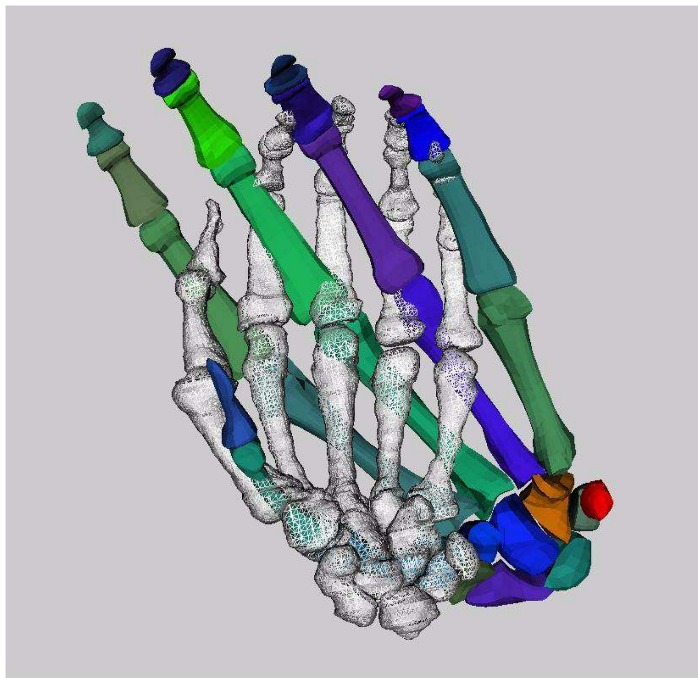
Template :

- *27 bones*
- *40 joints*
- *7k vertices*



MRI data
→ Target surface:

- *20k vertices*



Registration:

*Shape matching (4 res)*

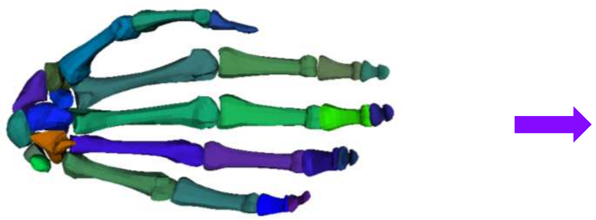
*8 PCA samples*

*3 min*

*211 iterations (elastic)*

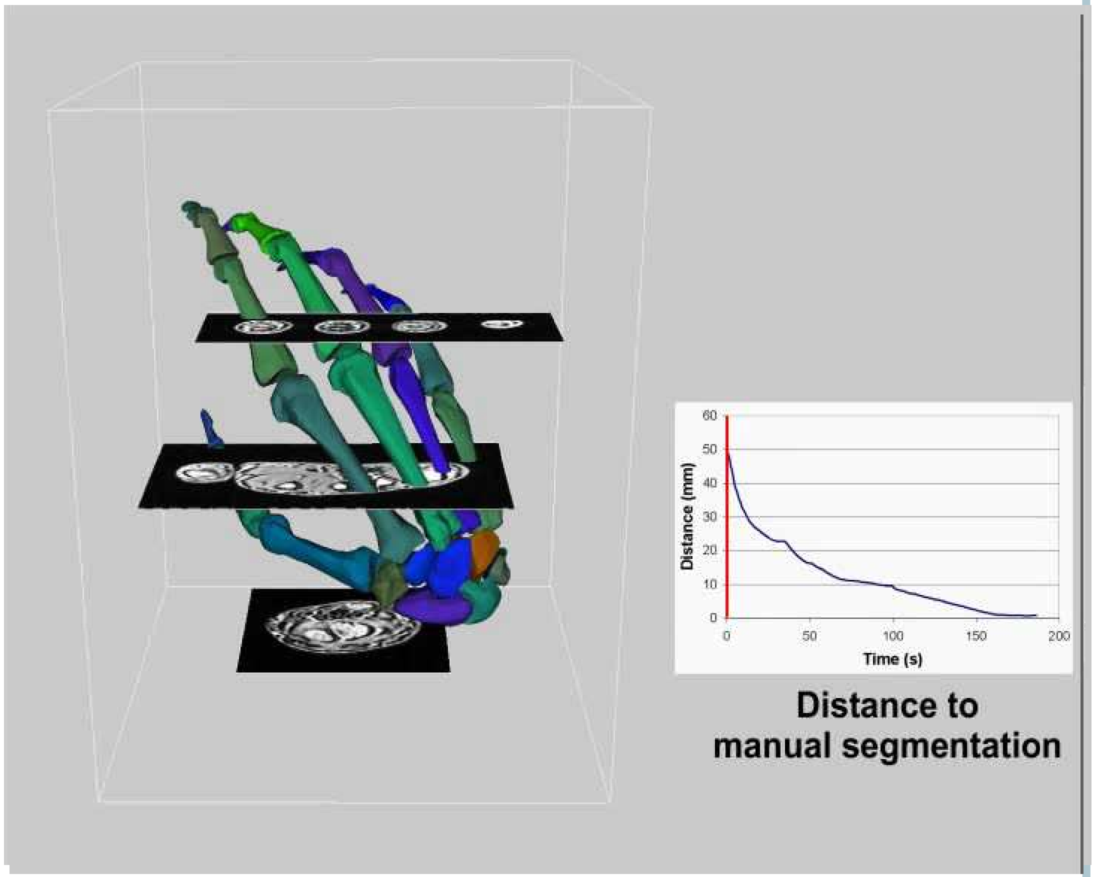
*58 iterations (plastic)*

# Image registration



Template :

- *27 bones*
- *40 joints*
- *7k vertices*




Registration:

*Shape matching (4 res)*

*8 PCA samples*

*3 min*

*490 iterations (elastic)*

*26 iterations (plastic)*

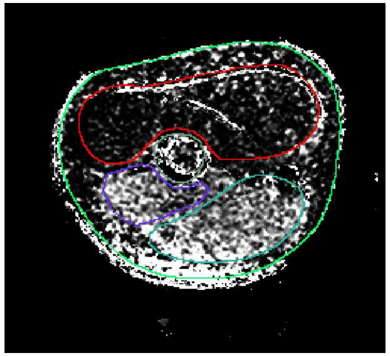
*Distance to manual segmentation = 0.8mm*

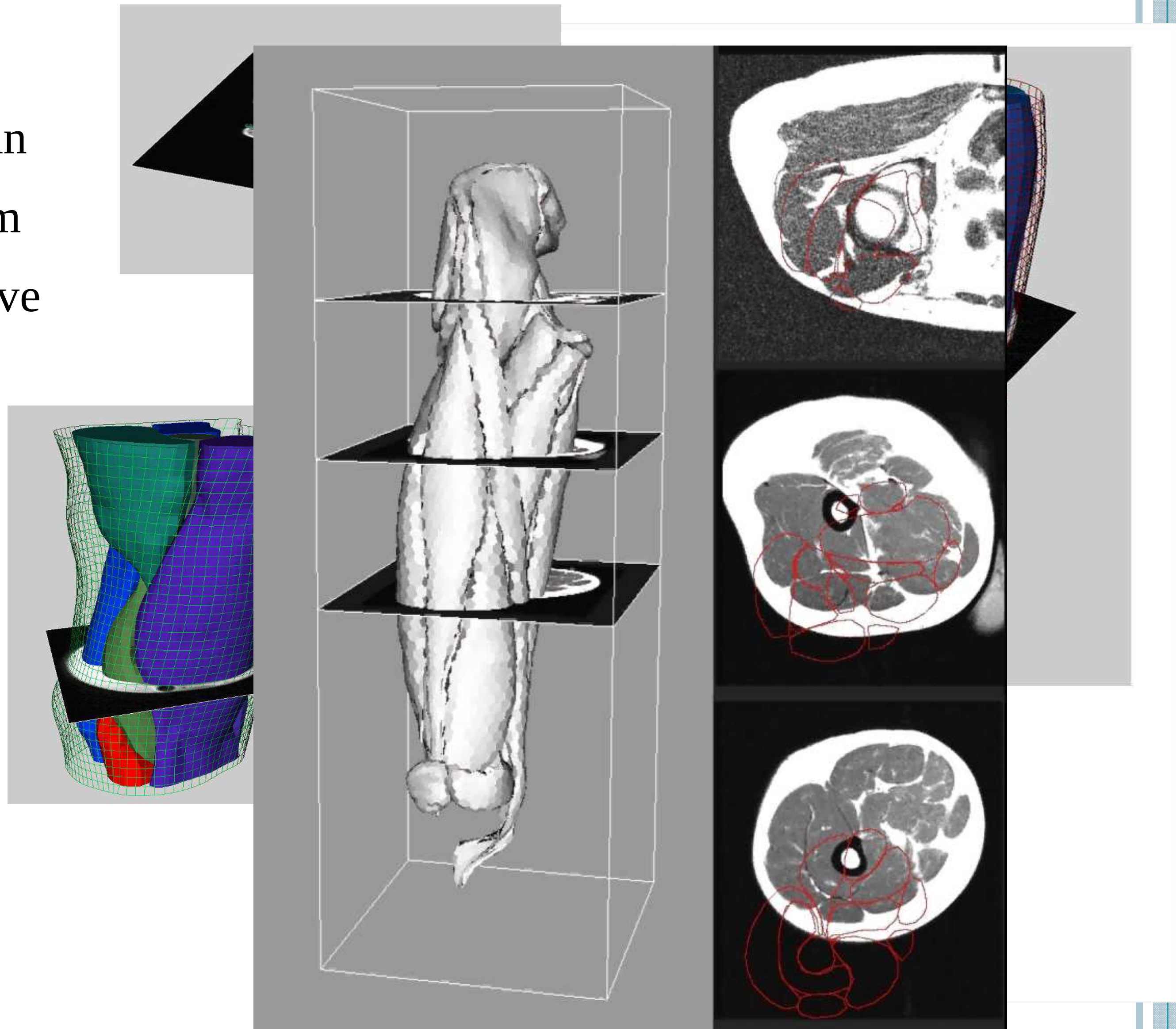
MRI data

- *0.3 x 0.3 x 1mm*

# Image registration

– Comp. time ~2min
– Accuracy ~1.5mm
– Possibly interactive

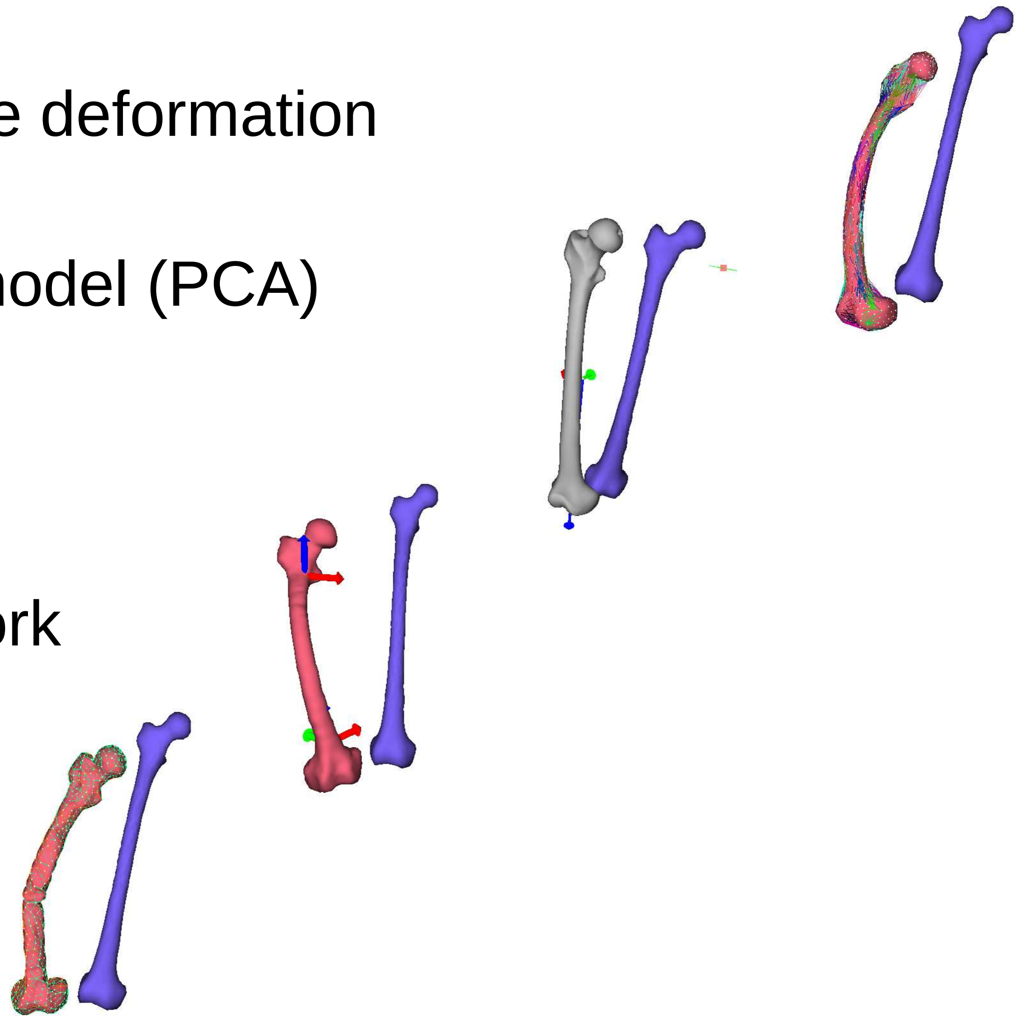

*Upper arm actuation map*



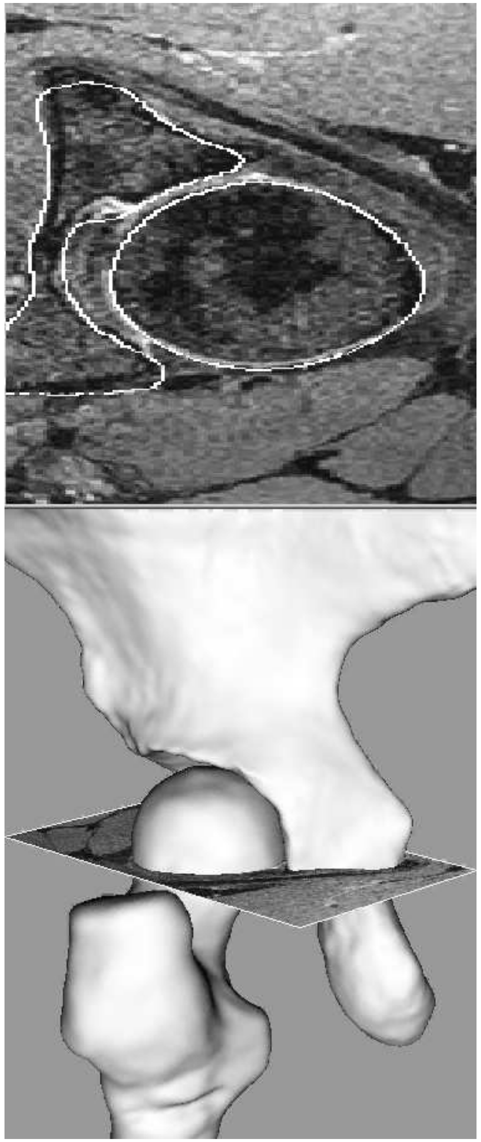
# Deformable ICP

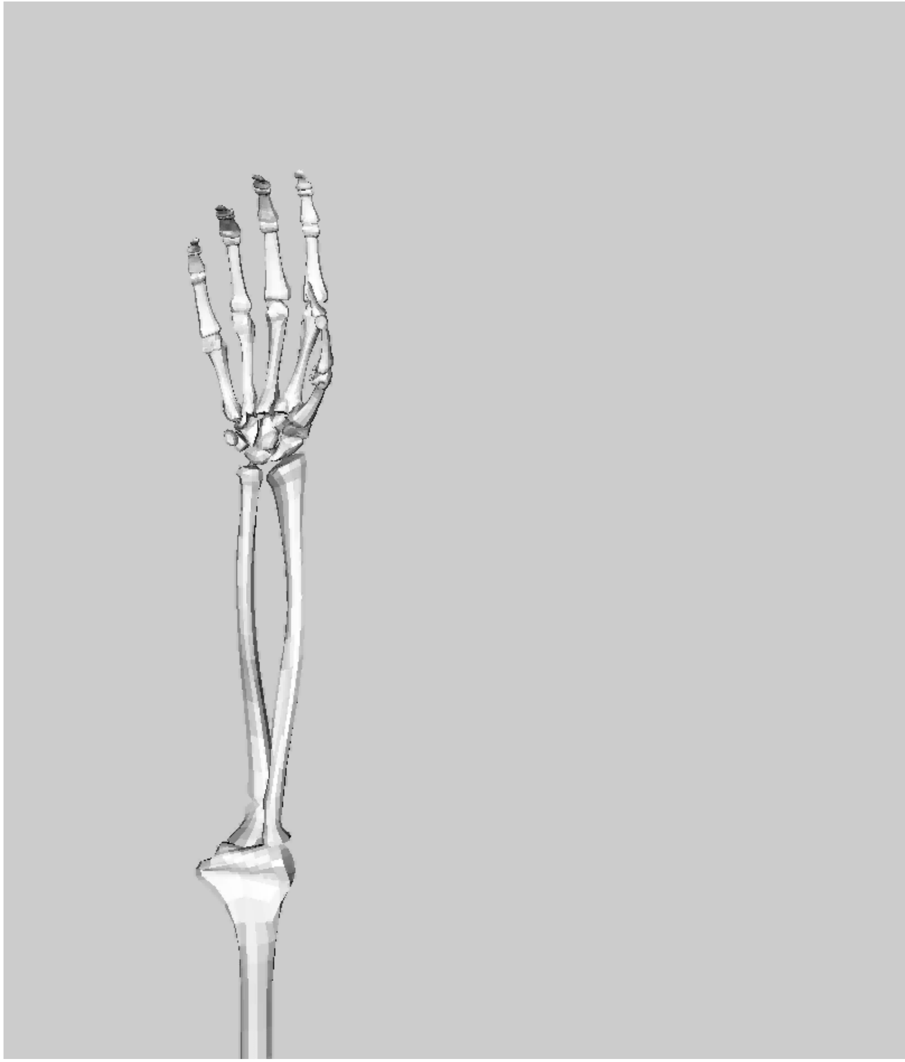
- Comparison of different deformation methods：
  - As rigid as possible deformation
  - Statistical shape model (PCA)
  - Frame-based
  - Mass-spring network
  - FEM

...

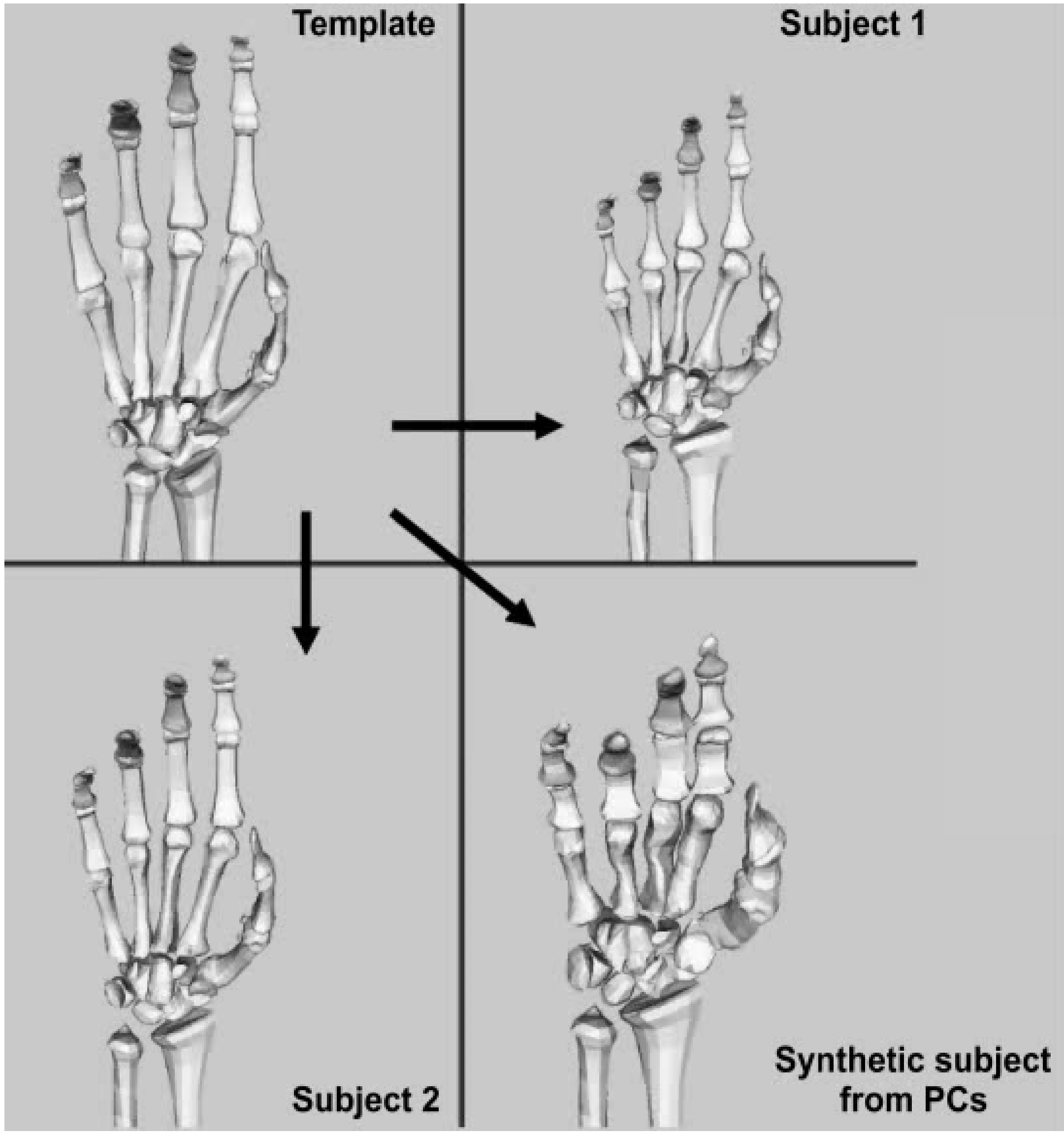


# Estimation



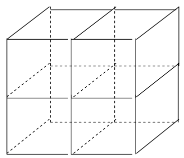


# Estimation



# Conclusion

## Deformable models for segmentation:



| Analysis | vs. | Prediction |
|---|---|---|
| Image-driven | | Physics-driven |
| Abstract models | | Anatomical models |
| Generic techniques | | Ad-hoc techniques |
| Modelling | | Simulation |
| Inter-patient registration | | Intra-patient registration |
| Low complexity | | High complexity |