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Efficient Method for Magnitude Comparison in RNS Based on Two Pairs of Conjugate Moduli

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Outline

1. Motivation
2. Class of Moduli Sets
3. Method for comparing magnitude in RNS
4. Typical application: RNS motion estimator
5. Conclusions and future work

Motivation

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- Carry free arithmetic
  - Residue Number Systems (RNS) allows to parallelize +, -, *
- No general efficient method for comparison in RNS
  - to convert from residues to positional code: CRT requires modulo M operations and MRC is a sequential method!
  - [Miller 86, Dimauro 93, Wang 99]: computationally demanding, not suitable for hardware implementation
- To propose a method for comparing RNS numbers considering a representative class of moduli sets.

Class of Moduli Sets

- New class of multi-moduli sets that rely on pairs of conjugate moduli:

  $$S = \{m_1, m_1^*, \ldots, m_k, m_k^*\} = \{2^{n1}-1, 2^{n1}+1, \ldots, 2^{nk}-1, 2^{nk}+1\}$$

  – Only Mersenne rings and Fermat rings

- S is not a set of pairwise relatively prime, however modified CRT [Wang98] allows to obtain an integer from residues

- This class of multi-moduli leads to two-level residue number systems

- Important sub-class S': two pairs of balanced conjugate moduli sets

$$S' = \{m_1, m_1^*, m_2, m_2^*\} = \{2^n-1, 2^n+1, 2^{n+1}-1, 2^{n+1}+1\}$$

Class of Moduli Sets

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- For each of the two level we can use the modified CRT to compute RNS-to-binary

$$X = X_1 + m_1 \left\langle \left(\frac{m_1}{d}\right)^{-1} \frac{X_2 - X_1}{d} \right\rangle_{\frac{m_2}{d}}$$

  – d = GCD(m1,m2) ⇒ pairwise relatively prime CRTIII≡MRC

$$d = 1 \rightarrow X_1 = x_1^* + (2^n + 1)\left\langle 2^{n-1}(x_1 - x_1^*) \right\rangle_{2^{n-1}}$$

$$d = 3 \rightarrow X = X_2 + \left\langle \left(\frac{2^{2(n+1)} - 1}{3}\right)^{-1} \frac{X_1 - X_2}{3} \right\rangle_{\frac{2^{2n-1}}{3}}$$

- Very important for us is that the range is **odd**

$$M = \frac{(2^{2n} - 1)(2^{2n+2} - 1)}{3}$$

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Comparing magnitude in RNS

- Unsigned integer numbers (*A,B)* can be compared by subtraction:

$$C = \begin{cases} A - B & \text{for } A \geq B \\ M - A - B & \text{for } A < B \end{cases}$$

- Based on the well known mathematical axiom:
  - *the subtraction of two numbers with the same parity leads to an even number and the subtraction of two numbers with different parities leads to an odd number*

- and taking advantage that M is **odd** we can answer the question :
  - is A ≥ B or not?

Comparing magnitude in RNS

- PREPOSITIONS

- $A \geq B$ *iff:*
  - A and B have the same parity and C is an even number
  - A and B have different parities but C is an odd number.

- $A < B$ *iff:*
  - A and B have the same parity and C is an odd number
  - A and B have different parities but C is an even number.

So we have to compute the parity of A, B and C!

Comparing magnitude in RNS

- Problem: how to directly compute the parity of a RNS number?
  - without computing the number back to a traditional weighted system!

- The parity of an integer X in the range [0, M−1] represented on the $\{2^n-1, 2^n+1, 2^{n+1}-1, 2^{n+1}+1\}$ moduli set can be computed by:

$$\langle X \rangle_2 = \left\langle \langle X_2 \rangle_2 \oplus \left\langle \langle X_1 - X_2 \rangle_{2^{2n-1}} \right\rangle_2 \right\rangle_2$$

- by converting X1 and X2 in the 1st-level we also just need shift and one's complement addition

$$X_1 = x_1^* + (2^n + 1) \times \left\langle 2^{n-1}(x_1 - x_1^*) \right\rangle_{2^n - 1}$$

$$X_2 = x_2^* + (2^{n+1} + 1) \times \left\langle (2^n(x_2 - x_2^*) \right\rangle_{2^{n+1} - 1}$$

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Algorithm 1 Comparison of the numbers A, B represented in RNS $(a_1, a_1^*, a_2, a_2^*, b_1, b_1^*, b_2, b_2^*)$.

$c_1 = \langle a_1 - b_1 \rangle_{2^n-1}; c_1^* = \langle a_1^* - b_1^* \rangle_{2^n+1};$
$c_2 = \langle a_2 - b_2 \rangle_{2^{n+1}-1}; c_2^* = \langle a_2^* - b_2^*; \rangle_{2^{n+1}+1};$
$(A_1, A_2) =$ 1st-level-converter(a_1, a_1^*, a_2, a_2^*); {(15) and (16)}
$(B_1, B_2) =$ 1st-level-converter(b_1, b_1^*, b_2, b_2^*); {(15) and (16)}
$(C_1, C_2) =$ 1st-level-converter(c_1, c_1^*, c_2, c_2^*); {(15) and (16)}
$\overline{P_A} = LSB(\langle A_1 - A_2 \rangle_{2^{2n}-1}) \oplus LSB(A2);$ {'1' if X even}
$\overline{P_B} = LSB(\langle B_1 - B_2 \rangle_{2^{2n}-1}) \oplus LSB(B2);$
$\overline{P_C} = LSB(\langle C_1 - C_2 \rangle_{2^{2n}-1}) \oplus LSB(C2);$
if $P_A \oplus P_B \oplus P_C =$ '1' **then**
  $A \geq B$ is TRUE;
else
  $A < B$ is TRUE;
end if

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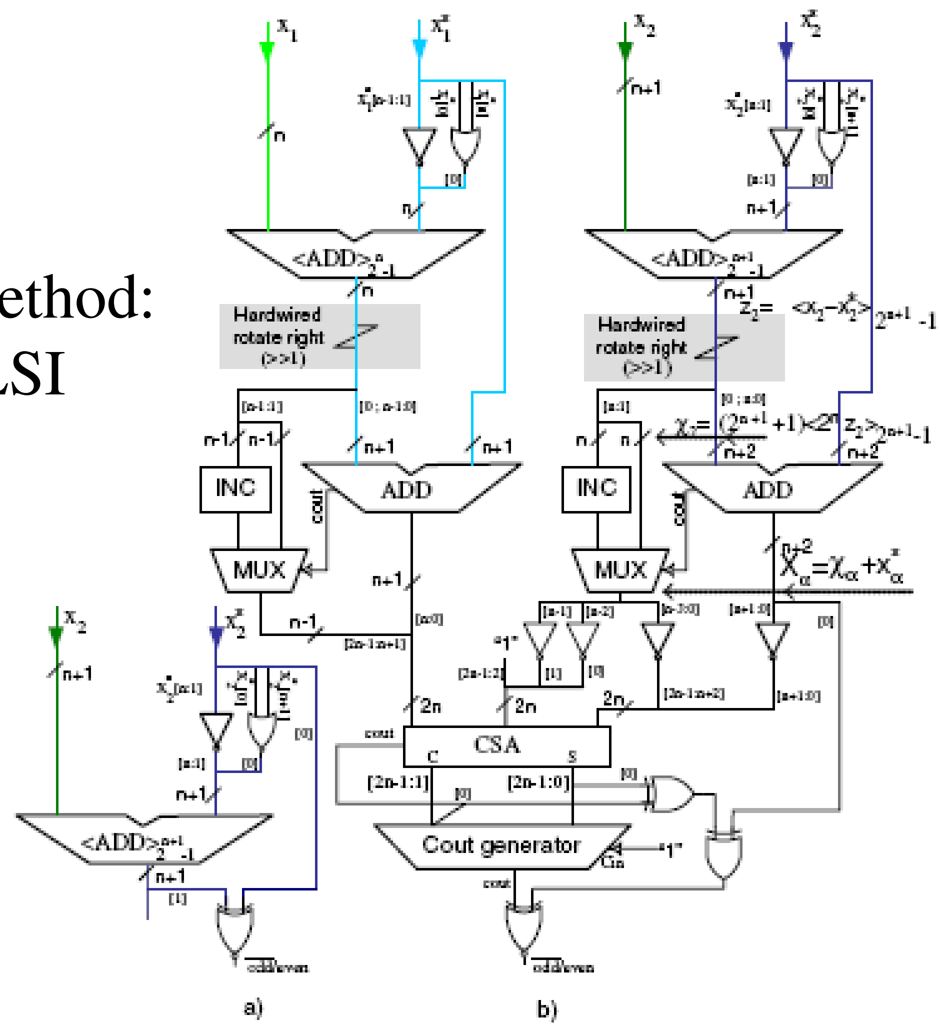
Comparing magnitude in RNS

Parity detection method: suitable for VLSI



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Comparing magnitude in RNS

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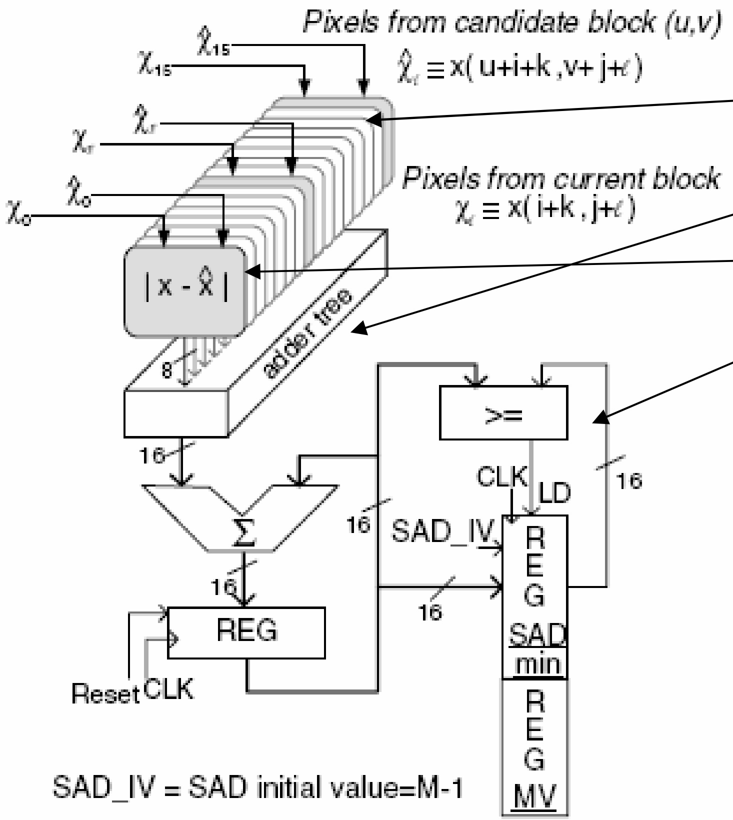
Maximum Operation Size (MOS)		
Algorithm	MOS (n,M)	MOS (n=4)
Miller	≅4M	≅86955
Dimauro	modulo(≅ 2^{3n}+2)	modulo(≅ 4098)
Wang	modulo(2^{2n}-1)	modulo(255)
Proposed	modulo(2^{n+1}+1)	modulo(33)

Typical application: RNS motion estimator

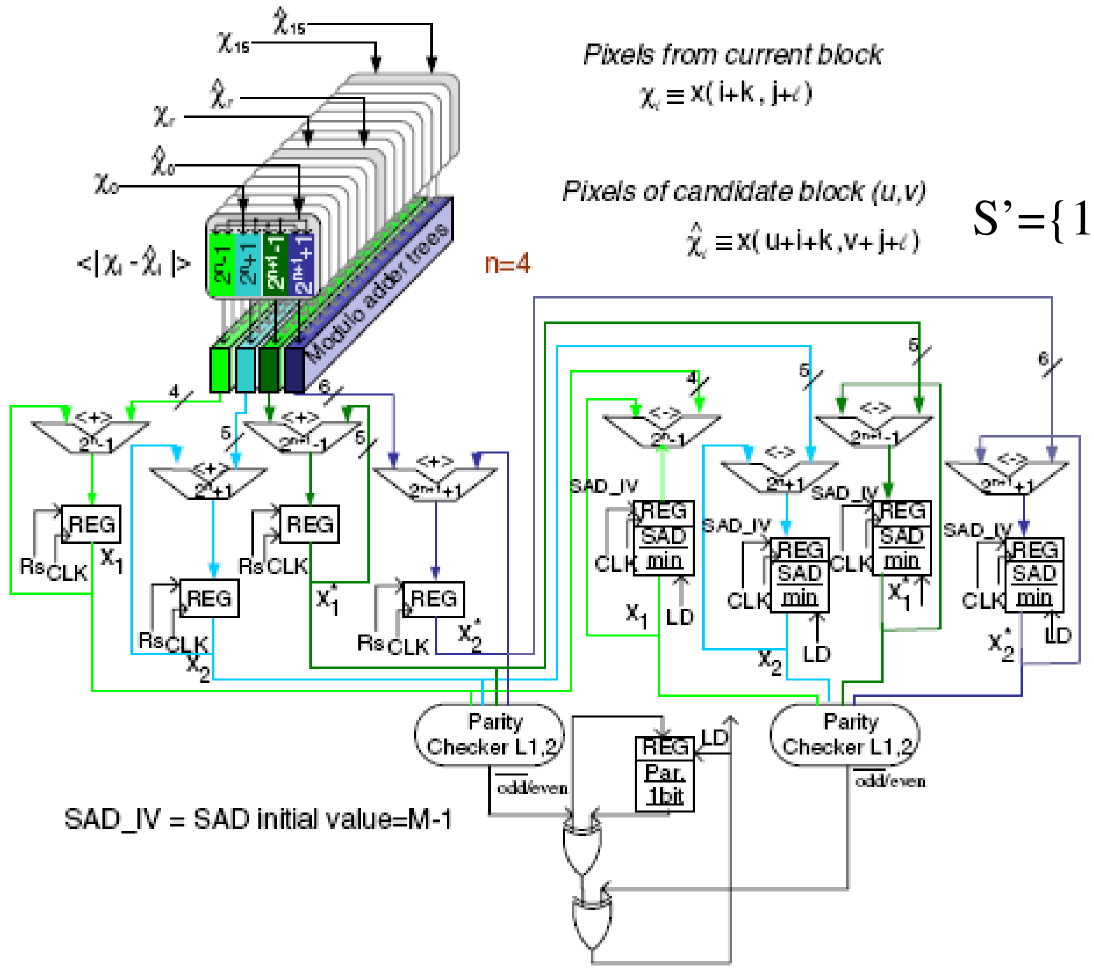
- Tradicional architecture



Subtractors and Adders

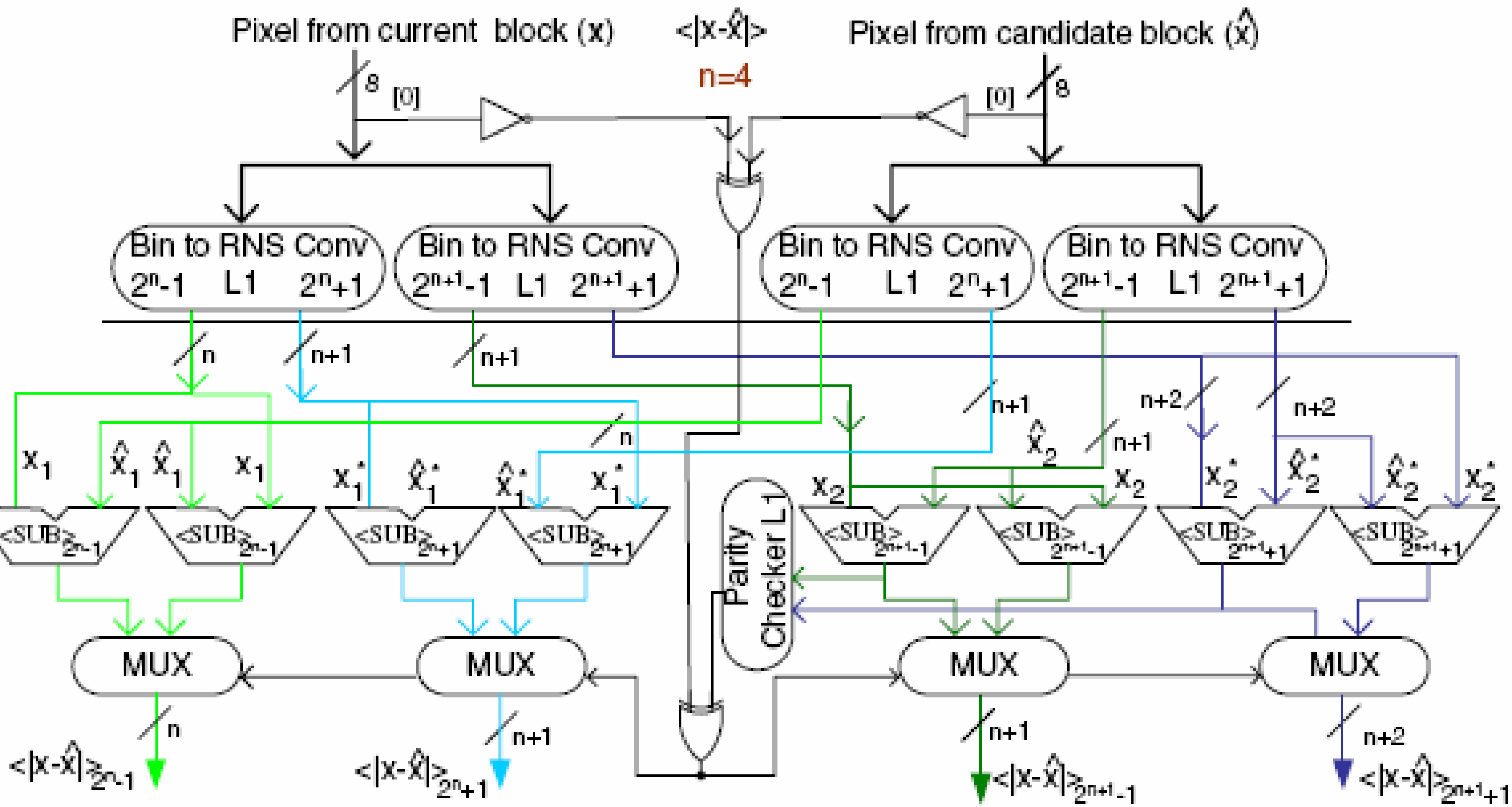
Comparators: proposed hardware

RNS motion estimator



S'={15,17,31,33}

RNS motion estimator



Experimental Results: RNS motion estimator

- SAD unit implemented in a FPGA with arithmetic units directly mapped on Look-Up-Tables (LUT)
- FPGA Xilinx VirtexII Pro (xc2vp50-7)
- Synthesis with ISE (8.2) tools

Slices (% total)	BRAMs (% total)	Freq. MHz	Latency Cycles	Throughput Blocks/s
246 (1%)	211 (90%)	254	12	1.5×10^7

Conclusions and future work

- New efficient method is proposed for magnitude comparison in RNS based on two pairs of conjugate moduli

- This is the first method leading to VLSI architectures with practical interest for comparing the magnitude of numbers in RNS
  - Efficient RNS minimum SAD unit was already implemented in FPGA
  - We are implementing a SAD unit on an ASIC (0.18μm CMOS)

- We are now extending the idea to other moduli sets, all with a common characteristic: M odd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