Formation control for a class of output-feedback systems

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Abstract

We address the problem of designing decentralized output feedback laws to force the outputs of a class of strict feedback nonlinear systems (agents) to follow geometric paths while holding a desired formation pattern. To this effect we propose a general framework that takes into account, the topology of the communication links among the agents and the fact that velocities states of agents are unavailable. We provide a decentralized controller based on an observer backstepping approach. By using available exponential observers, stability of the interconnected observer-coordinated controller system is obtained. As a case study, the coordination output-feedback control design is performed for synchronizing a fleet of AUVs where only position measurements are available. Simulation results are presented to show the effectiveness of the approach.

Keywords: Formation control, synchronization, graph theory, surface vessel, backstepping technique,Output feedback, Lyapunov theory.

1 Introduction

Cooperating agents must be able to interact with each other using either explicit or implicit communication and frequently both. Explicit communication corresponds to a deliberate exchange of messages that is in general made through a wireless network. On the other hand implicit communication is derived through sensor observations that enable each vehicle to estimate the states and trajectories of its teammates. For example, each agent can observe relative state (position and orientation) of its neighbors (implicit communication), and through explicit communication exchange this information with the whole team in order to construct a complete configuration of the team.

A typical leaderfollower formation control approach (e.g., [1]) assumes only one group leader within the team. In this case, only the group leader has the knowledge of group trajectory information, which is either preprogrammed in the group leader or provided to the group leader by an external source. The formation is then built on the reaction of the other group members to the motion of the group leader. The fact that only a single group leader is involved in the team implies that the leaderfollower approach is simple to implement and understand, and the requirement on communication bandwidth is reduced. This is, however, a single point of massive failure type system because the loss of the group leader causes the entire group to fail. Another issue with the typical leaderfollower approach is the lack of inter-vehicle information feedback throughout the group. In order to overcome this type of single point of failure tendency, much research has been focusing on decentralized or distributed cooperative control strategies where vehicle control laws are coupled and each vehicle makes its own decision according to the states of its neighbors (e.g., [8],[2]-[4]). This allows the group to continue on to achieve an objective even in the presence of failure of any group member.

In spite of significant progress in these exciting areas, much work remains to be done to develop strategies capable of yielding robust performance of a fleet of vehicles in the presence of complex vehicle dynamics, unmeasured vehicles states, severe communication constraints, and partial vehicle failures. These difficulties are specially challenging in the field of marine robotics for two main reasons: i) the dynamics of marine vehicles are often complex and cannot be simply ignored or drastically simplified for control design purposes, and ii) controller may depend on states that are not measured. An observer is therefore designed to estimate the unknown states needed in the control law.

In this paper an output-feedback design method for coordinating a group of agents described by a class of strict feedback system is proposed. State-feedback control design is used for solving a coordination problem in [9]. This paper is motivated by the observer backstepping approach [10] from where damping terms are added to the controller to counteract the disturbances from the exponentially stable observer, and consequently ensure stability of the closed-loop system. The coordination technique used in [9, 11] is modified so as to force the geometric error converging asymptotically to zero. The controller is derived in two stages: first, a path following control law is used that drives each agent to its assigned path regardless the temporal speed profile adopted. Second, the derivative of each path parameter is used as an auxiliary controller to synchronize the agents positions, thus achieving the coordination scheme.

Notation $|x|$ denotes the standard Euclidean norm of a vector x in $\mathbb{R}^n$, and the induced matrix 2-norm of $A \in R^{n\times n}$ is denoted $\|A\|$. For a matrix $P = P^\top > 0$, let $p_m = \lambda_{min}(P)$ and $p_M = \lambda_{max}(P)$. We let $\mathcal{I} = \{1, \dots, n\}$, $x_{d_i}^{\gamma_i} = \frac{\partial z_{d_i}}{\partial \gamma_i}$ and $x_{d_i}^{\gamma_i^2} = \frac{\partial^2 z_{d_i}}{\partial^2 \gamma_i}$, $[a_i]_{i\in\mathcal{I}} := col(a_1 \dots a_n)$

1.1 Problem Statement

The objective of the proposed output-feedback design is to solve the coordination problem [9] where an observer has to be designed to estimates the unknown states. The problem is divided into two sub-problems. At the lower level, the path-following problem is solved for individual agent, each having access to a set of local measurements. The output of the observer shall converge to the measured states of the actual system. Finally coordination is achieved by synchronizing the so called coordination states through the use of the derivative of path parameters.

2 Coordinated path-following control system

This section proposes a coordinated path following control architecture for a group of n decoupled agents $\Sigma_i, i \in \mathcal{I}$ modeled by general systems of the form

$$\Sigma_i : \quad \begin{aligned} \dot{x}_{1i} &= x_{2i} \\ \dot{x}_{2i} &= f_i(y_i) + g_i(y_i)u_i \\ y_i &= x_{1i} \end{aligned} \tag{1}$$

where $x_i = [x_{1i}^\top, x_{2i}^\top]^\top \in \mathbb{R}^{2\times n_i}$ denotes the state of agent i, $u_i \in R^{m_i}$ its control input, $y_i \in \mathbb{R}^{n_i}$ its measured output. Assume that the functions f_i and g_i are smooth, and the matrix g_i is invertible. Since the state x_{2i} is unmeasured, an observer that can provide information about this unknown state must be designed. A reduced-order observer can be indeed designed. However, it is often noise-sensitive. Here we use the following full-order observer

$$\begin{aligned}
\dot{\hat{x}}_{1i} &= \hat{x}_{2i} + K_{1i}(x_{1i} - \hat{x}_{1i}) \\
\dot{\hat{x}}_{2i} &= f_i(y_i) + g_i(y_i)u_i + K_{2i}(x_{1i} - \hat{x}_{1i}) \\
y_i &= \hat{x}_{1i}
\end{aligned} \tag{2}$$

where K_{1i} and K_{2i} are positive matrix observer gains. By defining the observer error as

$$\tilde{x}_i = [(x_{1i} - \hat{x}_{1i})^\top, (x_{2i} - \hat{x}_{2i})^\top]^\top$$

and subtracting (2) from (1), we have

$$\tilde{x}_i = A_i \tilde{x}_i, \quad A_i = \begin{bmatrix} K_{1i} & \mathbf{I} \\ K_{2i} & \mathbf{0} \end{bmatrix} \tag{3}$$

It is direct to show that

$$|\tilde{x}_i(t)| \leq \varphi_i |\tilde{x}_i(t_0)| e^{-\sigma_i(t - t_{0i})} \tag{4}$$

for some positive constants φ_i and σ_i, which implies that (2) is a global exponential observer of (1). Therefore, in the following we will design the desired coordinated path following controllers u_i, based on the following system

$$\begin{aligned}
\dot{x}_{1i} &= \hat{x}_{2i} + \tilde{x}_{2i} \\
\dot{\hat{x}}_{2i} &= f_i(y_i) + g_i(y_i)u_i + K_{2i}(x_{1i} - \hat{x}_{1i})
\end{aligned} \tag{5}$$

2.1 Path-following Controller

A solution to the path-following problem was given in [10] and can be formulated as follows: Given an agent i and a desired path $y_{di}(\gamma_i)$, design feedback controller laws for u_i such that all the closed-loop signals are bounded, the position of the agent converges to and remains in desired path, and the vehicle travels at a desired speed assignment v_{ri}.
To prepare for the control law, we define the following variables

$$\begin{aligned}
e_{1i} &= x_{1i} - y_{di}(\gamma_i) \\
e_{2i} &= \hat{x}_{2i} - \alpha_{1i} \\
e_{\dot{\gamma}_i} &= \dot{\gamma}_i - v_{ri}(t)
\end{aligned} \tag{6}$$

where α_{1i} is a virtual control to be specified later. Following the design in [10], we proceed in two backstepping steps as follows:

Step I The time derivative of e_{1i} along the solutions of (5) gives

$$\begin{aligned}
\dot{e}_{1i} &= \hat{x}_{2i} + \tilde{x}_{2i} - y_{di}^{\gamma_i}\dot{\gamma}_i \\
&= \hat{x}_{2i} + \tilde{x}_{2i} - y_{di}^{\gamma_i}(e_{\dot{\gamma}_i} + v_{ri})
\end{aligned} \tag{7}$$

Define the following Lyapunov function

$$V_{1i} = e_{1i}^\top P_{1i} e_{1i} + \frac{1}{\delta_{1i}} \tilde{x}_i^\top \Gamma_i \tilde{x}_i \tag{8}$$

where $\delta_{1i} > 0$, P_{1i} and Γ_i are symmetric positive matrices. The time derivative of (8) is

$$\dot{V}_{1i} = 2e_{1i}^{\top} P_{1i}(e_{2i} + \alpha_{1i} + \tilde{x}_{2i} - y_{di}^{\gamma_i}(e_{\dot{\gamma}_i} + v_{ri})) - \frac{2}{\delta_{1i}} \tilde{x}_i^{\top} \Gamma_i A_i \tilde{x}_i \tag{9}$$

Select the virtual control law as

$$\alpha_{1i} = A_{1i} e_{ei} + y_{di}^{\gamma_i} v_{ri} + \alpha_{oi} \tag{10}$$

where α_{oi} is a damping term to be determined later, and A_{1i} satisfies $A_{1i}^{\top} P_i + P_i A_{1i} = -Q_i, Q_i > 0$. Equation (9) rewrites

$$\begin{aligned} \dot{V}_{1i} &= 2e_{1i}^{\top} P_{1i}(A_i e_{1i} + e_{2i} + \tilde{x}_{2i} - y_{di}^{\gamma_i} e_{\dot{\gamma}_i} + \alpha_{oi}) - \frac{1}{\delta_{1i}} \tilde{x}_i^{\top} \Gamma_i A_i \tilde{x}_i \\ &= -e_{1i}^{\top} Q_i e_{1i} + 2e_{1i}^{\top} P_{1i} e_{2i} + 2e_{1i}^{\top} P_{1i} \tilde{x}_{2i} + \mu_{1i} e_{\dot{\gamma}_i} \\ &\quad + 2e_{1i}^{\top} P_{1i} \alpha_{oi} - \frac{2}{\delta_{1i}} \tilde{x}_i^{\top} \Gamma_i A_i \tilde{x}_i \end{aligned} \tag{11}$$

by completing the squares, we obtain the following

$$\begin{aligned} \dot{V}_{1i} \le\ & -e_{1i}^{\top} Q_i e_{1i} + 2e_{1i}^{\top} P_{1i} e_{2i} + \mu_{1i} e_{\dot{\gamma}_i} \\ & + 2e_{1i}^{\top} P_{1i} \Big[\alpha_{oi} + \varepsilon_{1i} P_{1i} \Big] e_{1i} + \frac{1}{2\varepsilon_{1i}} \tilde{x}_{2i}^{\top} \tilde{x}_{2i} \\ & - \frac{2}{\delta_{1i}} \tilde{x}_i^{\top} \Gamma_i A_i \tilde{x}_i \end{aligned} \tag{12}$$

where $\mu_i = 2e_{1i}^{\top} P_{1i} y_{di}^{\gamma_i}$ and ε_{1i} is an arbitrary positive constant. Now pick $\alpha_{oi} = -\varepsilon_{1i} P_{1i}$ and chose Γ_i such that $A_i^{\top} \Gamma_i + \Gamma_i A_i = -I$, this gives

$$\begin{aligned} \dot{V}_{1i} &\le -e_{1i}^{\top} Q_i e_{1i} + 2e_{1i}^{\top} P_{1i} e_{2i} + \mu_{1i} e_{\dot{\gamma}_i} \\ &\quad + \frac{1}{2\varepsilon_{1i}} \tilde{x}_{2i}^{\top} \tilde{x}_{2i} - \frac{1}{\delta_{1i}} \tilde{x}_i^{\top} \tilde{x}_i \\ &\le -e_{1i}^{\top} Q_i e_{1i} + 2e_{1i}^{\top} P_{1i} e_{2i} + \mu_{1i} e_{\dot{\gamma}_i} \\ &\quad - \kappa_{1i} \tilde{x}_i^{\top} \tilde{x}_i \end{aligned} \tag{13}$$

where $2\kappa_{1i} = \frac{1}{\delta_{1i}} - \frac{1}{\varepsilon_{1i}} > 0$.

Step II The second step of the backstepping procedure will require the time derivative of the stabilizing function α_{1i}, to this end we define the following terms

$$\chi_i = \alpha_{1i}^{x_{1i}} \dot{x}_{1i} + \alpha_{1i}^{t}, \quad \varphi_{2i} = \alpha_{1i}^{\gamma_i} \tag{14}$$

The time derivative of e_{2i} along the solutions of the second equation of (2) and (10), gives

$$\dot{e}_{2i} = f_i(y_i) + g_i(y_i) u_i + K_{2i}(x_{1i} - \hat{x}_{1i}) - \chi_i - \varphi_{2i}(e_{\dot{\gamma}_i} + v_{ri}) \tag{15}$$

To ensure stability of e_{2i}, we define the second candidate Lyapunov function

$$V_{2i} = V_{1i} + \frac{1}{2} e_{2i}^{\top} P_{2i} e_{2i} + \frac{1}{\delta_{2i}} \tilde{x}_i^{\top} \Gamma_i \tilde{x}_i \tag{16}$$

where $\delta_{2i} > 0$. The time derivative of (16) is

$$\begin{aligned} \dot{V}_{2i} = &\ \dot{V}_{1i} + 2e_{ei}^{\top} P_{2i}(f_i + g_i u_i - \chi_i - \varphi_{2i} v_{ri}) \\ & - 2e_{ei}^{\top} P_{2i} \varphi_{2i} e_{\dot{\gamma}_i} + 2e_{ei}^{\top} P_{2i} K_{2i}(x_{1i} - \hat{x}_{1i}) \\ & - \frac{1}{\delta_{2i}} \tilde{x}_i^{\top} \tilde{x}_i \end{aligned} \tag{17}$$

Using inequality (13), equation (17) rewrites

$$\begin{aligned}
\dot{V}_{2i} &\leq \dot{V}_{1i} + 2e_{2i}^{\top}P_{2i}(f_i + g_i u_i - \chi_i - \varphi_{2i}v_{ri}) \\
&\quad -2e_{ei}^{\top}P_{2i}\varphi_{2i}e_{\dot{\gamma}_i} + 2e_{2i}^{\top}P_{2i}K_{2i}(x_{1i} - \hat{x}_{1i}) \\
&\quad -\frac{1}{\delta_{2i}}\tilde{x}_i^{\top}\tilde{x}_i \\
&\leq -e_i^{\top}Q_i e_i + 2e_{2i}^{\top}P_{2i}\Big(P_{2i}^{-1}P_{1i}e_{1i} + f_i + g_i u_i \\
&\quad -\chi_i - \varphi_{2i}v_{ri}\Big) + \Big(\mu_{1i} - 2e_{ei}^{\top}P_{2i}\varphi_{2i}\Big)e_{\dot{\gamma}_i} \\
&\quad +2e_{2i}^{\top}P_{2i}K_{2i}(x_{1i} - \hat{x}_{1i}) - (\kappa_{1i} + \frac{1}{\delta_{2i}})\tilde{x}_i^{\top}\tilde{x}_i
\end{aligned} \tag{18}$$

with $e_i = [e_{1i}, e_{2i}]^{\top}$ and $Q_i = diag\{Q_{1i}, Q_{2i}\}$. Again applying the Young's inequality to (18), and select the control input u_i as

$$u_i = g_i^{-1}(A_{2i}e_{2i} - f_i + \chi_i + \varphi_{2i}v_{ri} - P_{2i}^{-1}P_{1i}e_{1i} + \alpha_{di}) \tag{19}$$

where α_{1i} is a damping term defined as

$$\alpha_{di} = -\varepsilon_{2i}K_{2i}K_{2i}^{\top}P_{2i} \tag{20}$$

where ε_{2i} is an arbitrary positive constant. Inequality (18) re-writes

$$\dot{V}_{2i} \leq -e_i^{\top}Q_i e_i + \mu_{2i}e_{\dot{\gamma}_i} - \kappa_{2i}\tilde{x}_i^{\top}\tilde{x}_i, \kappa_{2i} = \kappa_{1i} + \frac{1}{\delta_{2i}} - \frac{1}{2\kappa_{2i}} > 0 \tag{21}$$

In this stage, in order to render (21) negative, the loop must be closed by speed assignment design,i.e. (choosing an appropriate update for $e_{\dot{\gamma}_i}$). We modify the design in [10] in order to solve the coordination path following problem. Set

$$v_{ri} = v_{\mathcal{L}}, \quad v_{\mathcal{L}} > 0 \tag{22}$$

where $v_{\mathcal{L}}$ is a desired speed profile assigned to the formation. To achieve a coordination scheme, we propose a decentralized feedback law for $\dot{\gamma}_i$ as a function of the information obtained from the neighboring agents, this will be developed in the following section.

3 Coordinated controller

This section details the development of the coordination controller subsystem. To this effect, we first recall some key concepts from algebraic graph theory.

It is natural to model information exchange among vehicles by directed or undirected graphs. A digraph (directed graph) consists of a pair $(\mathcal{N}, \mathcal{E})$, where $\mathcal{N}$ is a finite nonempty set of nodes, and $\mathcal{E} \in \mathcal{N} \times \mathcal{N}$ is a set of ordered pairs of nodes, called edges. An edge (i, j) in a digraph denotes that vehicle j can obtain information from vehicle i, but not necessarily vice versa. In contrast, the pairs of nodes in an undirected graph are unordered, where an edge (i, j) denotes that vehicles i and j can obtain information from one another. Note that an undirected graph can be considered a special case of a digraph, where an edge (i, j) in the undirected graph corresponds to edges (i, j) and (j, i) in the digraph. If there is an edge from node i to node j in a digraph, then i is the parent node, and j is the child node. A directed path is a sequence of edges of the form $(\mathcal{V}_{i_1}, \mathcal{V}_{i_2}), (\mathcal{V}_{i_2}, \mathcal{V}_{i_3}), \dots,$ where $\mathcal{V}_{i_f} \in N$, in a digraph. An undirected path in

an undirected graph is defined analogously. In a digraph, a cycle is a directed path that starts and ends at the same node. A digraph is strongly connected if there is a directed path from every node to every other node. An undirected graph is connected if there is a path between any distinct pair of nodes. A directed tree is a digraph, where every node has exactly one parent except for one node, called the root, which has no parent, and the root has a directed path to every other node. Note that in a directed tree, each edge has a natural orientation away from the root, and no cycle exists. In the case of undirected graphs, a tree is a graph in which every pair of nodes is connected by exactly one path. A directed spanning tree of a digraph is a directed tree formed by graph edges that connect all of the nodes of the graph. A graph has or contains a directed spanning tree if there exists a directed spanning tree being a subset of the graph. Note that the condition that a digraph has a directed spanning tree is equivalent to the case that there exists at least one node having a directed path to all of the other nodes. In the case of undirected graphs, having an undirected spanning tree is equivalent to being connected. However, in the case of directed graphs, having a directed spanning tree is a weaker condition than being strongly connected.
The adjacency matrix $A = [a_{ij}] \in \mathbb{R}^{n\times n}$ of a digraph is defined as $a_{ii} = 0$ and $a_{ij} > 0$ if $(j,i) \in \mathcal{E}$ where $i \neq j$. The adjacency matrix of an undirect graph is defined analogously except that $a_{ij} = a_{ji}, \forall i \neq j$, since $(j,i) \in \mathcal{E}$ implies $(i,j) \in \mathcal{E}$. Let the matrix $L = [l_{ij}] \in \mathbb{R}^{n\times n}$ be defined as $l_{ii} = \sum_{j\neq i} a_{ij}$ and $l_{ij} = -a_{ij}$, where $i \neq j$. The matrix L satisfies the following conditions:

$$l_{ij} \leq 0, i \neq j, \quad \sum_{j=1}^{n} l_{ij} = 0, \quad i = 1,\dots,n \tag{23}$$

For an undirected graph, L is called the Laplacian matrix [9], which is symmetric positive semi-definite. However, L for a digraph does not have this property.

Consider now the coordination control problem with a communication topology defined by a graph $(\mathcal{N},\mathcal{E})$. Using a Lyapunov-based design, we propose a decentralized feedback law for $\dot{\gamma}_i$. Consider the following Lyapunov function

$$V_1 = \sum_{i\in\mathcal{I}} V_{2i} \tag{24}$$

according to (21), the time derivative of (24) is given as

$$\dot{V}_1 \leq \mu^\top e_{\dot{\gamma}} - \sum_{i\in\mathcal{I}} e_i^\top Q_i e_i + \kappa_{2i}\tilde{x}_i^\top \tilde{x}_i \tag{25}$$

where $\mu = [\mu_{2i}]_{i\in\mathcal{I}}$ and $e_{\dot{\gamma}} = [e_{\dot{\gamma}_i}]_{i\in\mathcal{I}}$. We assume that communication between robots are bi-directional and there is no losses or time delay in communication. We let

$$e_{\dot{\gamma}} = \pi - \mathbf{K}_1^{-1}(L\gamma - \mu) \tag{26}$$

where L is the Laplacian matrix of the underlying communication graph as described above and π is an auxiliary state governed by

$$\dot{\pi} = -(\mathbf{K}_1 + \mathbf{K}_2)\pi + L\gamma + \mu \tag{27}$$

where $\gamma = [\gamma_i]_{i\in\mathcal{I}}$, $\mathbf{K}_1$ and $\mathbf{K}_2$ are diagonal positive definite matrices. The closed loop coordination system is given by

$$\begin{aligned} \dot{\gamma} &= v_{\mathcal{L}} + \pi - \mathbf{K}_1^{-1}L\gamma - \mathbf{K}_1^{-1}\mu \\ \dot{\pi} &= -(\mathbf{K}_1 + \mathbf{K}_2)\pi + L\gamma + \mu \end{aligned} \tag{28}$$

Using the backstepping procedure results for path following controller, the properties of the communication system described above, and applying (28) we conclude the following result.

Theorem 1. *The feedback laws u_i for each agent given by (19) together with (28) solve the coordinated path following problem if and only if the communication graph defined by the graph $(\mathcal{N}, \mathcal{E})$ is connected. In particular, the path following error, the coordination errors $|\gamma_i - \gamma_j|$, and the speed tracking errors $|\gamma_i - v_{\mathcal{L}}|$ converge asymptotically to 0 as $t \to \infty$.*

Proof. Consider the augmented Lyapunov function given by

$$V_2 = V_1 + 0.5(\xi^\top \xi + \pi^\top \pi) \tag{29}$$

where $\xi = \mathbf{M}^\top s$, with $\mathbf{M}$ being the incident matrix of the graph. The time derivative of (29) along the solutions of (28) gives

$$\begin{aligned} \dot{V}_2 &\leq -e_{\dot{\gamma}}^\top \mathbf{K}_1 e_{\dot{\gamma}} - \pi^\top \mathbf{K}_2 \pi - \sum_{i \in \mathcal{I}} e_i^\top Q_i e_i + \kappa_{2i} \tilde{x}_i^\top \tilde{x}_i \\ &\leq -e_{\dot{\gamma}}^\top \mathbf{K}_1 e_{\dot{\gamma}} - \pi^\top \mathbf{K}_2 \pi - \sum_{i \in \mathcal{I}} q_{mi} |e_i|^2 + \kappa_{2i} |\tilde{x}_i|^2 \end{aligned} \tag{30}$$

where we have used the property that $\mathbf{M}^\top \mathbf{1} = 0$, with $\mathbf{1} = [1]_{i \in \mathcal{I}}$. Using Barbalat Lemma [7] , we conclude that states $(e_{\dot{\gamma}}, \pi, e_i, \tilde{x}_i)$ are bounded and the following limits hold

$$\begin{aligned} \lim_{t \to \infty} (e_i, \tilde{x}_i)^\top &= [0_i]_{1 \times 2} \\ \lim_{t \to \infty} \pi &= [0_i]_{i \in \mathcal{I}} \\ \lim_{t \to \infty} e_{\dot{\gamma}} &= [0_i]_{i \in \mathcal{I}} \end{aligned} \tag{31}$$

and consequently we have that $\dot{\gamma}_i \to v_{ri}(t) = v_{\mathcal{L}}$, by construction it is straightforward to see that μ vanishes as $t \to \infty$, then since $\pi = e_{\dot{\gamma}} + \mathbf{K}_1^{-1}(\mathcal{L}\gamma - \mu)$, therefore $\mathcal{L}\gamma \to [0_i]_{i \in \mathcal{I}}$, in another words $|\gamma_i - \gamma_j| \to 0$ as $t \to \infty$, which completes the proof. □

4 Case study: formation of a fleet of AUVs

It is realized that multiple autonomous agents can be used to carry out more complicated jobs for single agent hard to finish. The recent advances in sensing, communication and computation enable the conduct of cooperative missions. Multiple, highly autonomous systems are envisioned because they are capable of higher performance, lower cost, better fault tolerance, reconfigurability and upgradability. So the problem of coordinate control of multi-agents is emergent, among which the formation control of the multiple autonomous system has been the hot topic in the areas of distributed system and computer science during the past few years. As the oil and gas industry moves production to greater depths, the need for more underwater autonomous control increases. One envisioned task is inspection of underwater pipelines see Figure. 1. A formation of AUVs can be utilized to construct 3D images of the pipeline or even take time-synchronized snapshots covering a large spatial area of the seabed This will increase the probability of discovering abnormalities in a pipeline.

Consider a simplified system consisting of n AUVs, each being described by the following dynamic equations

$$\begin{aligned} \dot{x}_i &= u_i \\ \dot{y}_i &= v_i \\ \dot{u}_i &= \tau_{xi} - 2u_i \\ \dot{v}_i &= \tau_{yi} - 2v_i \end{aligned} \tag{32}$$

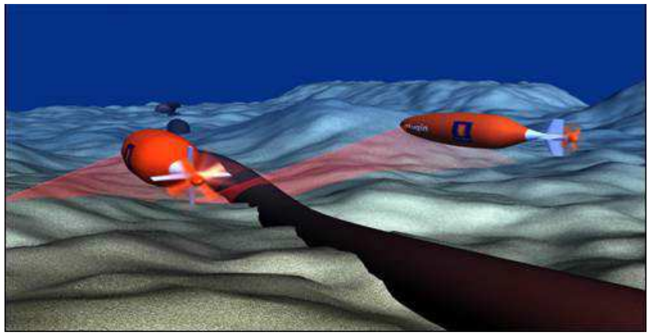


Figure 1: A formation of two AUVs

where x_i, y_i and u_i, v_i are the position and velocity of the i-th AUV in the direction of x and y axes respectively, τ_{xi} and τ_{yi} are the control input for each AUV to be designed. Rewrite system (31) in the form of system (1). Let :

$$p_i = \begin{bmatrix} x_i \\ y_i \end{bmatrix}, \quad q_i = \begin{bmatrix} u_i \\ v_i \end{bmatrix}, \quad F_i = \begin{bmatrix} \tau_{xi} \\ \tau_{yi} \end{bmatrix} \tag{33}$$

the dynamic equation of each AUV can be described more compactly as follows

$$\begin{aligned} \dot{p}_i &= q_i \\ \dot{q}_i &= -2q_i + F_i \end{aligned} \tag{34}$$

Assume that only position measurement are available so an observer is needed to reconstruct the velocity states. Proceeding like in section II, we propose an observer for (34) as follows

$$\begin{aligned} \dot{\hat{p}}_i &= \hat{q}_i + K_{1i}(p_i - \hat{p}_i) \\ \dot{\hat{q}}_i &= -2\hat{q}_i + F_i + K_{2i}(p_i - \hat{p}_i) \\ y_i &= \hat{p}_i \end{aligned} \tag{35}$$

Define the observer error $\tilde{x}_i = [p_i - \hat{p}_i, q_i - \hat{q}_i]^\top$, then subtracting (35) from (34) yields

$$\tilde{x}_i = A_i \tilde{x}_i, \quad A_i = \begin{bmatrix} -K_{1i} & \mathbf{I} \\ -K_{2i} & -2\mathbf{I} \end{bmatrix} \tag{36}$$

The observer error vector $\tilde{x}_i$ is exponentially staple if and only if K_{1i} and K_{2i} are chosen such that the matrix A_i is Hurwitz.

We next, propose to coordinate three AUVs to keep a formation pattern that consists of having them aligned along a common vertical line. To describe the communication between the AUVs in the formation, we use an undirected graph $\mathcal{G} = (\mathcal{N}, \mathcal{E})$ (see Figure. 2 for an example of a communication graph).

Applying the three steps design of the coordinated path following previously detailed in sections II and III, to system (34) with unmeasured states q_i, we obtain the following control laws

$$\begin{aligned} u_i &= (A_{2i}e_{2i} + 2\hat{q}_i + \chi_i + \varphi_{2i}v_{\mathcal{L}} - P_{2i}^{-1}P_{1i}e_{1i} + \alpha_{di}) \\ \dot{\gamma} &= v_{\mathcal{L}} + \pi - \mathbf{K}_1^{-1}L\gamma - \mathbf{K}_1^{-1}\mu \\ \dot{\pi} &= -(\mathbf{K}_1 + \mathbf{K}_2)\pi + L\gamma + \mu \end{aligned} \tag{37}$$

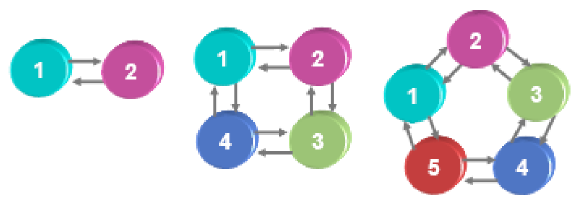


Figure 2: A bidirectional communication topology

The resulting e_i-dynamics is

$$\begin{aligned} \dot{e}_i &= \mathcal{A}_i e_i + \varphi_i e_{\gamma_i} + \mathcal{B}_i \tilde{x}_i \\ \dot{\tilde{x}} &= A_i \tilde{x}_i \\ e_{\dot{\gamma}_i} &= \dot{\gamma}_i - v_{\mathcal{L}} \end{aligned} \tag{38}$$

where

$$\mathcal{A}_i = \begin{bmatrix} A_{1i} - \varepsilon_{1i} P_{1i} & \mathbf{I} \\ -P_{2i}^{-1} P_{1i} & A_{2i} - \varepsilon_{2i} K_{2i} K_{2i}^{\top} P_{2i} \end{bmatrix}$$

$$\varphi_i = \begin{bmatrix} \varphi_{1i} \\ \varphi_{2i} \end{bmatrix}, \quad \mathcal{B} = \begin{bmatrix} 0 & \mathbf{I} \\ K_{2i} & 0 \end{bmatrix}$$

The architecture for the coordinated path-following control system proposed for the fleet of AUVs is shown in Figure. 3. It consists of three interconnected subsystems: i) The navigation system uses the position measurements to estimate the unavailable states, and feed this to the path following controller (**PFC**). ii) The Path Following Controller **PFC** a dynamical system whose inputs are a desired path y_{di}, a desired speed profile $v_{ri} = v_{\mathcal{L}}$ that is common to all AUV. Its output is the AUV's input u_i, computed so as to make it follow the path at the assigned speed. iii) The Coordinated Path Following Controller **CPFC** a dynamical system whose inputs are the generated desired path, speed profile for the AUV the generalized path-variable γ_j from the neighboring AUV of the group. Its output is the an updated law for the generalized variable γ_i for the actual AUV.

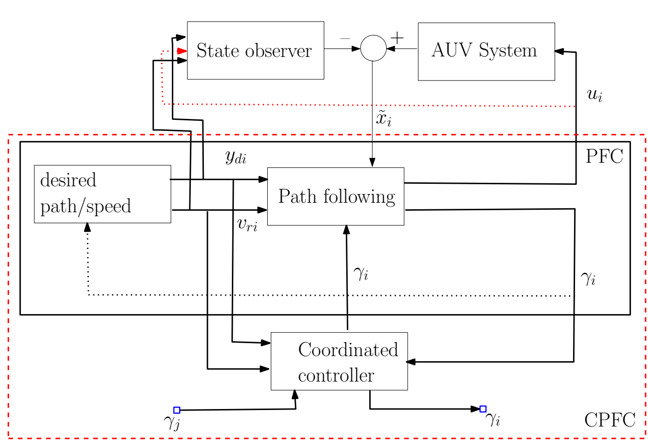


Figure 3: Coordinated path-following control system architecture

5 Simulation results

This section contains the results of simulations that illustrate the performance obtained with the coordinated path following control laws developed in the paper. In the simulations, the AUV are restricted to communicate on the way specified by the following graph.

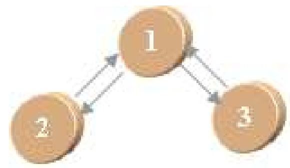


Figure 4: Communication topology of three AUV

The Laplacien Matrix for graph of Figure.4 is given by

$$L = \begin{bmatrix} 2 & -1 & -1 \\ -1 & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix}$$

which means that AUV 1 is allowed to communicate with AUV 2 and 3, but the last two do not communicate between themselves directly. It is required that AUVs keep a formation pattern that consists of having them aligned along a common vertical line. Specifically, the desired paths are parameterized as $x_{pi} = 20\tanh(\gamma_i) + \xi_i$ and $y_{pi} = \gamma_i$, with $\xi_{i\in\{1,2,3\}} \in \{10m, 0m, -10m\}$ is the offset between each AUVs trajectories. Figure 5 illustrates the transient behavior of the

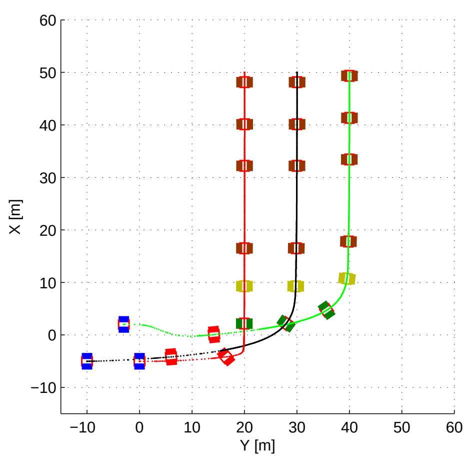
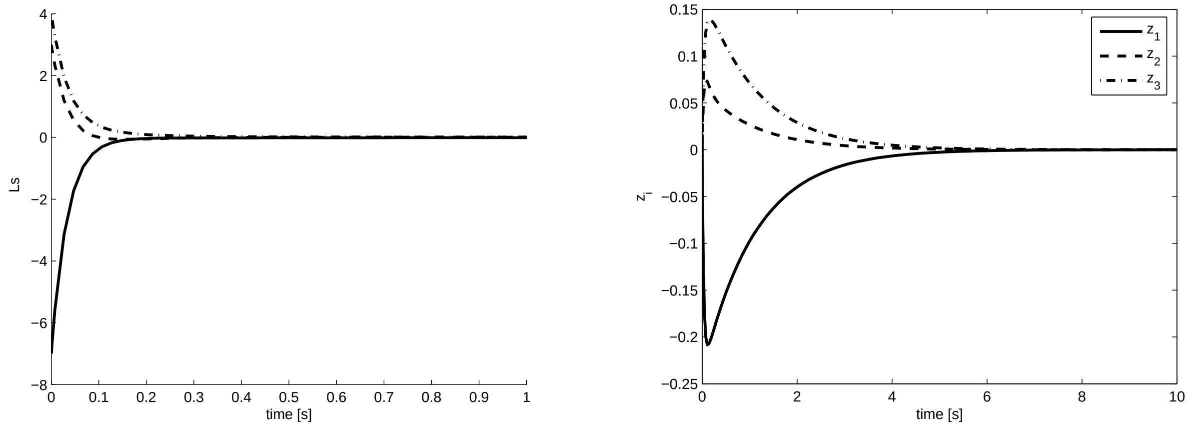


Figure 5: In-line formation of 3 AUVs

formation AUVs as they assemble and maintain a vertical line formation. Figure 6-(a) and Figure 6-(b) show the coordination errors $\gamma_1 - \gamma_2, \gamma_1 - \gamma_3$ and $\gamma_2 - \gamma_3$. Figure 7 plots the exponential convergence of the estimated velocity of a single AUV to its desired value.

6 Conclusion

A coordinating output-feedback control design method is proposed for a general class in strict feedback form. The decentralized solution adopted for coordinating agents does not require the



(a) Coordination errors γ_{ij} (b) Coordination errors z_i

Figure 6: Coordination error

concept of a leader and applies to a very general class of paths. The theoretical tools used for coordinating path following control brought together the backsteping technique and graph theory. As a case study, a coordination control law for formation of a simplified model of AUVs has been designed, and simulated to demonstrate and validate the theoretical results.

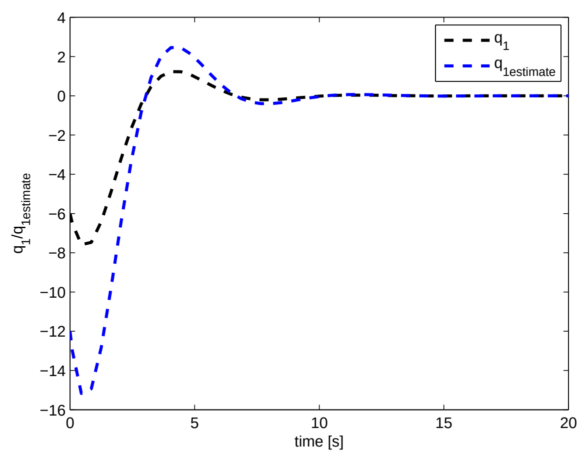


Figure 7: Convergence of the estimated state $\hat{q}_1$ to its real value q_1

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