Acyclic k-choosability on planar graphs

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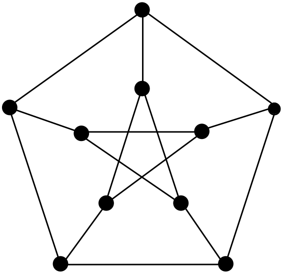
Outlines

- Definitions and some known results.
- Our main theorem.
- Conclusions and problems.

Proper coloring

Definition: A ***proper k-coloring*** of the vertices of a graph G is a mapping $\pi : V(G) \rightarrow \{1, \cdots, k\}$ such that $\forall uv \in E(G), \pi(u) \neq \pi(v)$.



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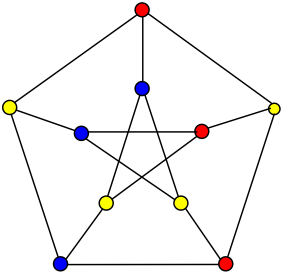

Acyclic coloring

- A proper vertex coloring of a graph G is ***acyclic*** if there is ***no bicolored cycle*** in G.

- A proper vertex coloring of a graph is ***acyclic*** if the graph induced by the union of every two color classes is ***a forest***.

- The ***acyclic chromatic number***, denoted by $\chi_a(G)$, of a graph G, is ***the smallest integer*** k such that G has an acyclic k-coloring.

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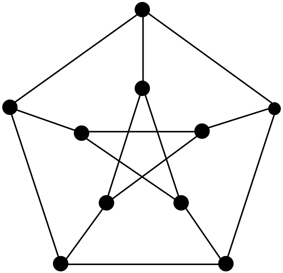
An example of Petersen graph

Question: $\chi_a(P_{10}) = ?$



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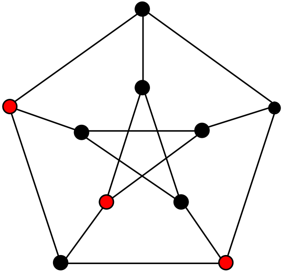

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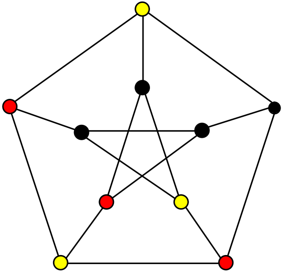

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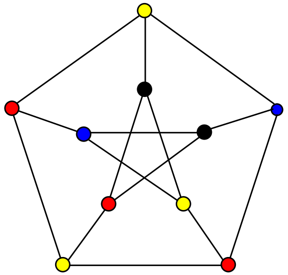

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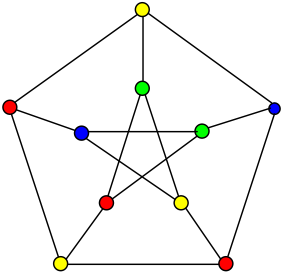
Question: $\chi_a(P_{10}) = 4$



Conjecture on acyclic coloring

Conjecture (Grünbaum, IJM, 1973)

Every planar graph is acyclically 5-colorable.

Let $\mathcal{P}$ denote the family of planar graphs.

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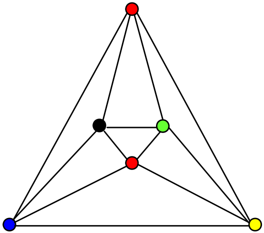
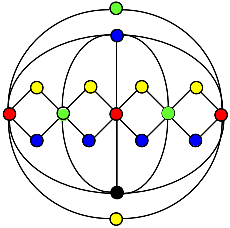


Grünbaum's example



Kostochka and Mel'nikov's example

Acyclic L-coloring

- L is a ***list assignment*** of a graph G if it assigns ***a list*** $L(v)$ ***of possible colors*** to each vertex $v \in V$.
  Denoted by $L = \{L(v) : v \in V\}$.

- A graph G is ***acyclically*** L***-list colorable*** if for a given list assignment L, there is an acyclic coloring π of the vertices such that $\pi(v) \in L(v)$.

- If G is acyclically L-list colorable for any list assignment L with $|L(v)| \geq k$ ***for all*** $v \in V$, then G is ***acyclically*** k***-choosable***.

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♠ **Conjecture***: **Every planar graph is acyclically 5-choosable.**

$\Longrightarrow$ Borodin's acyclic 5-color theorem (1979) and Thomassen's 5-choosability theorem (1994)

*Borodin, Flaass, Kostochka, Raspaud, Sopena, JGT, 2002.

Theorem

Every planar graph is acyclically 7-choosable.

Theorem (Wang and C., JGT, 2009)

*Every planar graph without **4-cycles** is acyclically 6-choosable.*

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Known results - Acyclic 5-choosability

Theorem (Montassier, Raspaud, Wang, JGT, 2007)

Every planar graph either without $\{4, 5\}$**-cycles** *or without* $\{4, 6\}$**-cycles** *is acyclically 5-choosable.*

Theorem (C., Wang, DM, 2008)

Every planar graph without **4-cycles** *and without* **two 3-cycles at distance less than 3** *is acyclically 5-choosable.*

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Every planar graph having neither **4-cycles** *nor* **chordal 6-cycles** *is acyclically 5-choosable.*

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Known results - Acyclic 4-choosability

Theorem

Planar graphs without $\{4, i, j\}$**-cycles** *with* $5 \leq i < j \leq 8$ *are acyclically 4-choosable.*

4	5	6	7	8	Reference
×	×	×			Montassier, Raspaud, Wang, 2006
×	×		×		
×	×			×	C., Raspaud, 2009
×		×	×		C., Raspaud, Wang, 2009
×		×		×	
×			×	×	C., Raspaud, Roussel, Zhu, 2009

Definition: The ***girth*** $g(G)$ of a graph G is the length of ***a shortest cycle in*** G.

Theorem (Borodin, Kostochka, Woodall, JLM, 1999)

Let G be a planar graph.
(1) If $g(G) \geq 7$ then $\chi_a(G) \leq 3$.
(2) If $g(G) \geq 5$ then $\chi_a(G) \leq 4$.

These two results are, respectively, improved by the following:

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Theorem (Hocquard, Montassier, IPL, 2009)

Every planar graph without cycles of **lengths 4 to 12** *is acyclically 3-choosable.*

Maximum average degree

Definition (Maximum average degree)

$$Mad(G) = max\{\frac{2|E(H)|}{|V(H)|} : H \subseteq G\}.$$

Observation

If G is a planar graph with girth g, then $Mad(G) < \frac{2 \cdot g}{g-2}$.

Theorem (Montassier, Ochem, Raspaud, JGT, 2005)

(1) Every graph G with $Mad(G) < \frac{8}{3}$ is acyclically 3-choosable;
(2) Every graph G with $Mad(G) < \frac{19}{6}$ is acyclically 4-choosable;
(3) Every graph G with $Mad(G) < \frac{24}{7}$ is acyclically 5-choosable.

By using relationship $Mad(G) < \frac{2 \cdot g}{g-2}$, then

Corollary

(1) Every planar graph G with $g(G) \geq 8$ is acyclically 3-choosable;
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Planar graphs without $\{4, 5, 8\}$-cycles are acyclically 4-choosable.

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- Show some reducible configurations of G.

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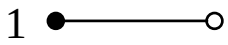
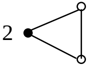
Lemma (Montassier, Raspaud, Wang, 2006)

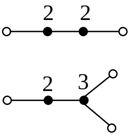
G does not contain the following twelve configurations.



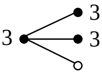
1-vertex.



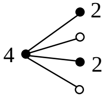
A 2-vertex is incident to a 3-face.



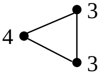
A 2-vertex is adjacent to a vertex of degree at most 3.



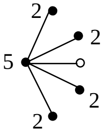
A 3-vertex is adjacent to at least two 3-vertices.



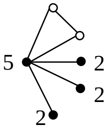
A 4-vertex is adjacent to at least two 2-vertices.



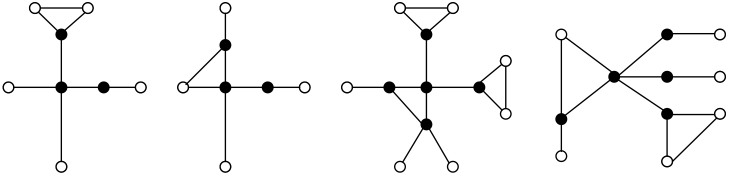
A 3-face incident to two 3-vertices and one 4-vertex.

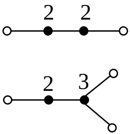


A 5-vertex is adjacent to at least four 2-vertices.



A 5-vertex is incident to one 3-face, adjacent to three 2-vertices.





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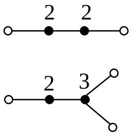

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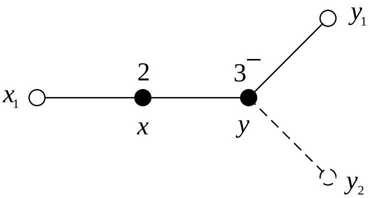

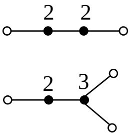

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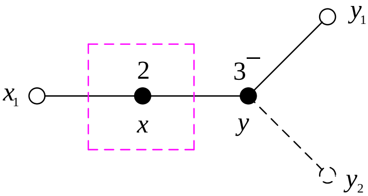

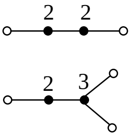

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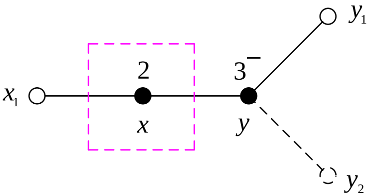

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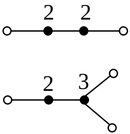

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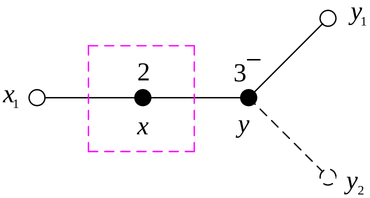

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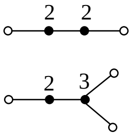
G' admits an acyclic 4-list-coloring c.



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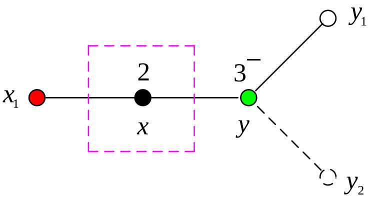
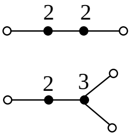


$G' = G - \{x\}$

G' admits an acyclic 4-list-coloring c.



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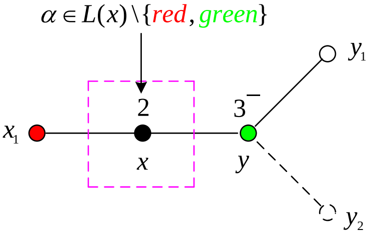

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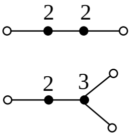

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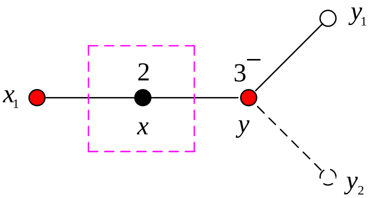

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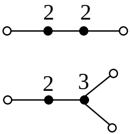

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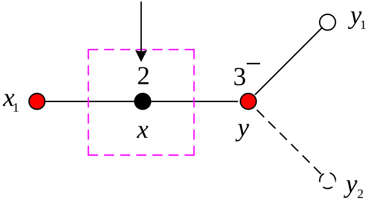
$\beta \in L(x) \setminus \{red, c(y_1), c(y_2)\}$

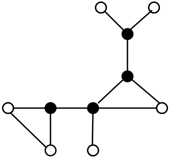


$G' = G - \{x\}$

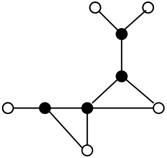
G' admits an acyclic 4-list-coloring c.

Lemma

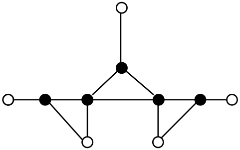
G does not contain $B1, B2, B3$ as a subgraph.



B1



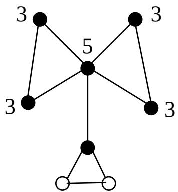
B2



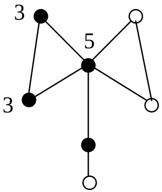
B3

Lemma

G does not contain $C1, C2$ as a subgraph.



C1



C2

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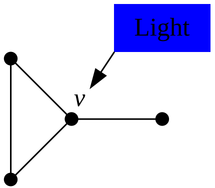
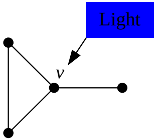
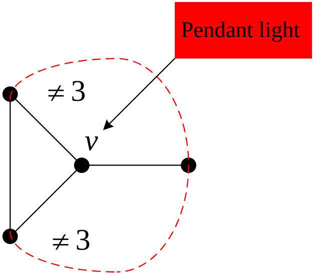
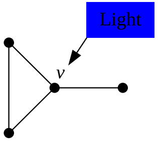
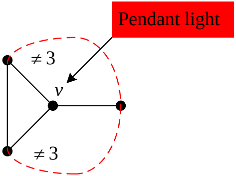
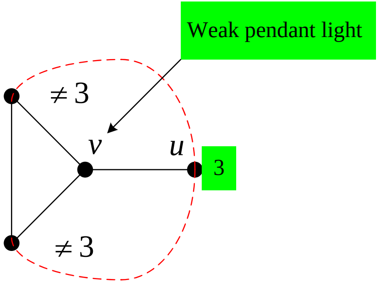

Light
v

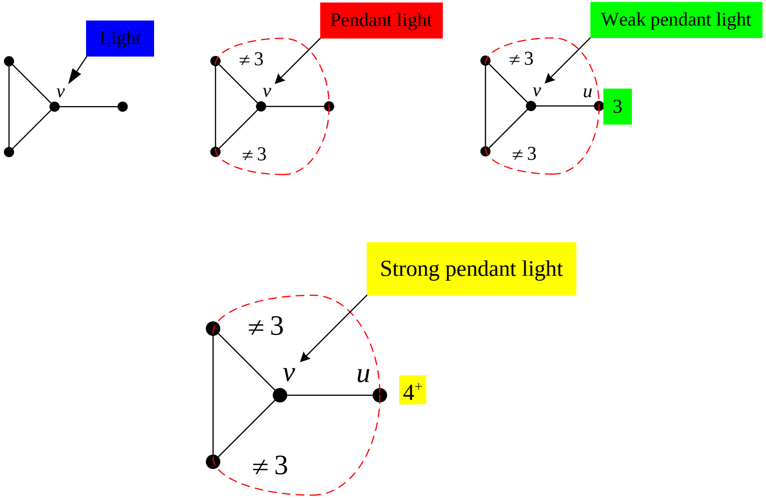

Light
v

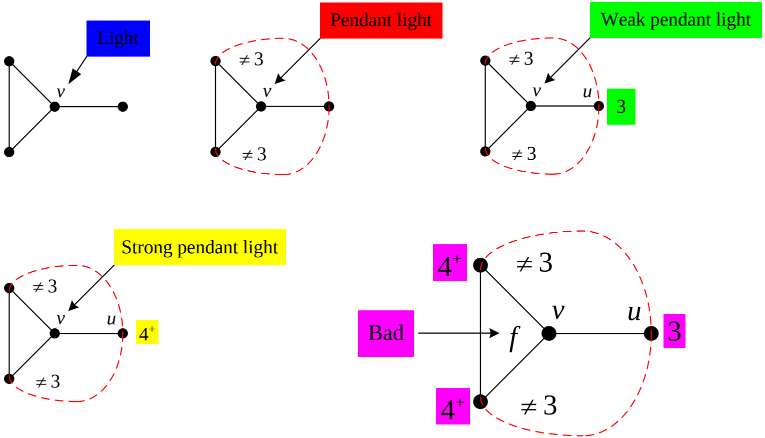

Pendant light
$\neq 3$
v
$\neq 3$

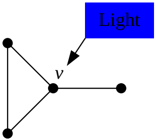

Light
v

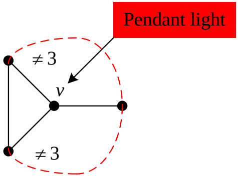

Pendant light
$\neq 3$
v
$\neq 3$

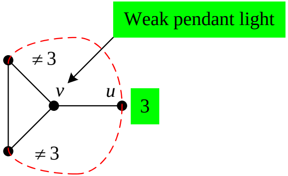

Weak pendant light
$\neq 3$
v
u
3
$\neq 3$

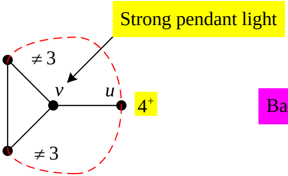

Light
v
Pendant light
≠ 3
v
≠ 3
Weak pendant light
≠ 3
v
u
3
≠ 3
Strong pendant light
≠ 3
v
u
4+
≠ 3

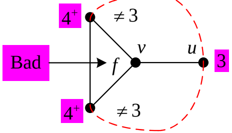

Light
v
Pendant light
≠ 3
v
≠ 3
Weak pendant light
≠ 3
v
u
3
≠ 3
Strong pendant light
≠ 3
v
u
4+
≠ 3
Bad
f
4+
≠ 3
v
u
3
4+
≠ 3

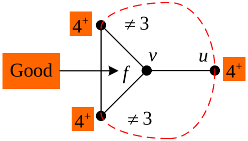

Light
v


Pendant light
$\neq 3$
v
$\neq 3$


Weak pendant light
$\neq 3$
v
u
3
$\neq 3$


Strong pendant light
$\neq 3$
v
u
4^+
$\neq 3$


4^+
$\neq 3$
Bad
f
v
u
3
4^+
$\neq 3$


4^+
$\neq 3$
Good
f
v
u
4^+
4^+
$\neq 3$

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- Choose a counterexample G with least number of vertices.
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- Use discharging argument to obtain a contradiction.

♠ We define a weight function:

$$\forall v \in V(G), \omega(v) = 2d(v) - 6;$$
$$\forall f \in F(G), \omega(f) = d(f) - 6.$$

By Euler's formula and handshake lemma, we derive an identity (4).

$$|V(G)| - |E(G)| + |F(G)| = 2 \qquad (1)$$

$$-6|V(G)| + 6|E(G)| - 6|F(G)| = -12 \qquad (2)$$

$$-6|V(G)| + 2\sum_{v\in V(G)} d(v) + \sum_{f\in F(G)} d(f) - 6|F(G)| = -12 \qquad (3)$$

$$\sum_{v\in V(G)} (2d(v) - 6) + \sum_{f\in F(G)} (d(f) - 6) = -12. \qquad (4)$$

Therefore

$$\sum_{x\in V(G)\cup F(G)} \omega(x) = -12.$$

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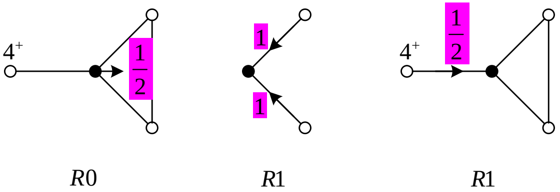
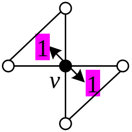
Discharging rules:

R0: Every strong pendant light 3-vertex sends $\frac{1}{2}$ to its incident 3-face.



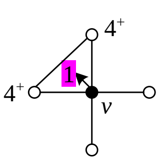
R1: Every 4^+-vertex gives 1 to its adjacent 2-vertex and $\frac{1}{2}$ to each pendant light 3-vertex.

R2: Denote v be a 4-vertex. Let f_1, f_2, f_3, and f_4 be the faces of G incident to v in a cyclic order such that $d(f_1)=3$.

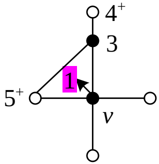


R2a: If $d(f_3)=3$, then $\tau(v \to f_1)=1$ and $\tau(v \to f_3)=1$.

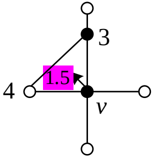
R2b1: If $d(f_3) \neq 3$, then $\tau(v \to f_1)=1$ when f_1 is a $(4,4^+,4^+)$-face or a good $(3,4,5^+)$-face.



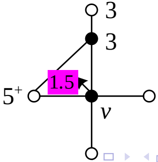
R2b1



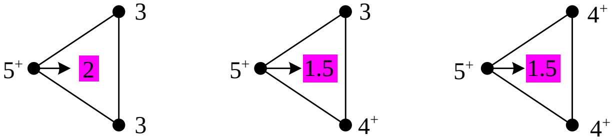
R2b2: If $d(f_3 \neq 3)$, then $\tau(v \to f_1)=1.5$ when f_1 is a $(3,4,4)$-face or a bad $(3,4,5^+)$-face.



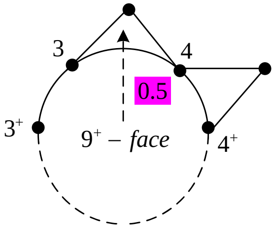
R2b2



R3: Every 5^+-vertex sends 2 to each incident $(3,3,5^+)$-face and 1.5 to each other incident 3-face.



R4: Every 9^+-face sends 0.5 to each of its sinks.



- G **does not contain 4, 5 and 8-faces.**
- **There is no** i**-face adjacent to two 3-faces with** $i = 3, 6, 7$**.**

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- Applying discharging rules R0 to R4, we obtain that:

$$\omega^*(x) \geq 0 \text{ for all } x \in V(G) \bigcup F(G).$$

- We derive the following obvious contradiction:

$$0 \leq \sum_{x \in V(G) \cup F(G)} \omega^*(x) = \sum_{x \in V(G) \cup F(G)} \omega(x) = -12.$$

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- Show some reducible configurations of G.
- Give some useful definitions.
- Use discharging argument to obtain a contradiction.
- Hence, no counterexample can exist.

♠ **Conjecture***: **Every planar graph is acyclically 5-choosable.**

*Borodin, Flaass, Kostochka, Raspaud, Sopena, JGT, 2002.

♠ **Weaker Conjecture:**
Every planar graph without 4-cycles is acyclically 5-choosable.

Let G be a planar graph having neither 4-cycles nor 3-cycles at distance less than d.

- $d = 0$ corresponds to the Weaker Conjecture.
- $d = \infty$ implies the case of $g(G) \geq 5$, which is shown to be acyclically 5-choosable by Montassier, Ochem, Raspaud in 2006.
- $d = 3$ is proved by C., Wang in 2008.

♠ **Question**: How about other integer d?

Thanks for your attention !