Linear Time Split Decomposition of Undirected Graphs

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Joint work with
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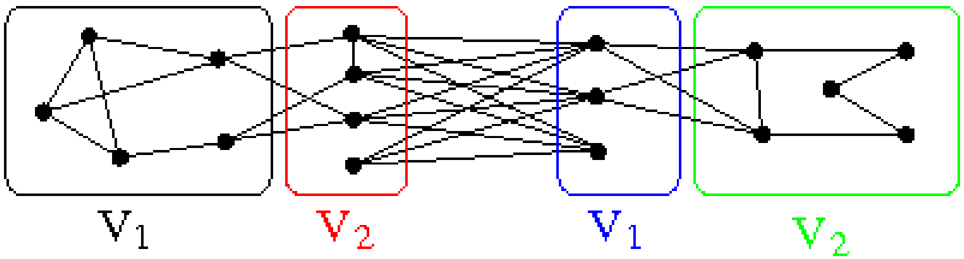
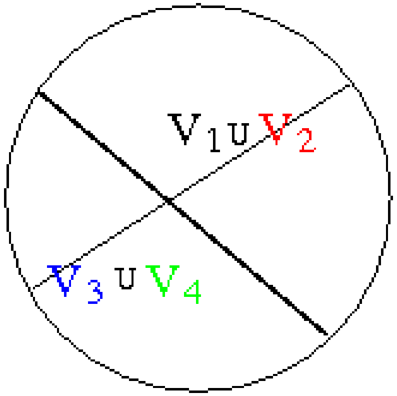
Cunningham 1972/1982



2 splits cross:



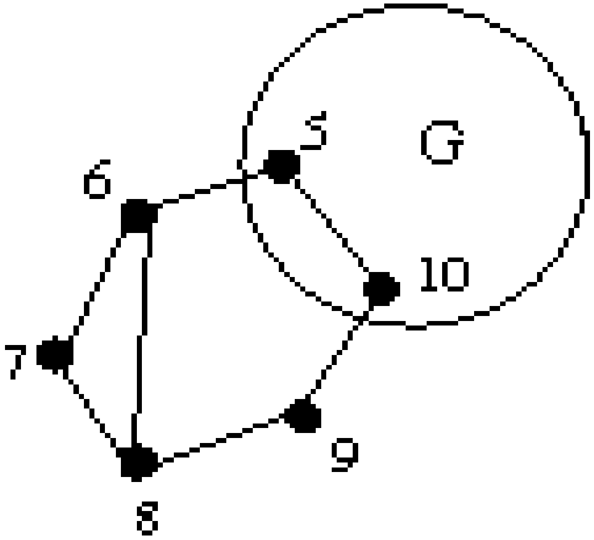
Strong split: does not cross any other split

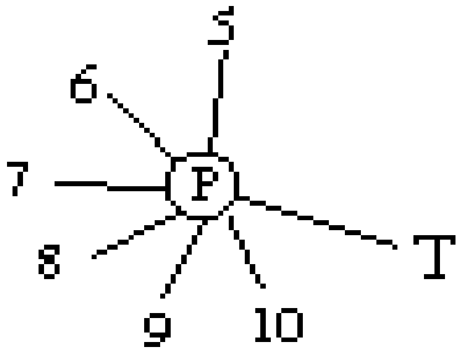
Stars Cliques Primes

Cunningham 1972/1982

Primes

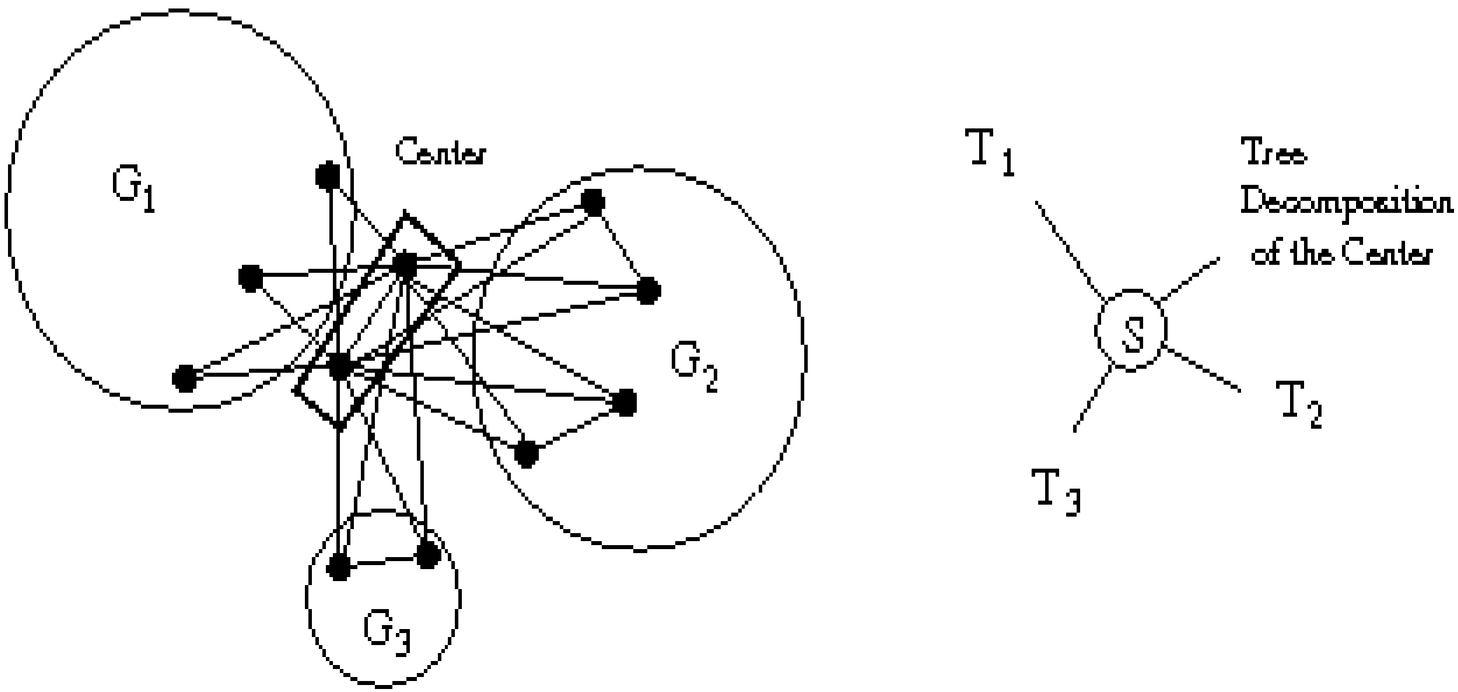
Non decompable subgraph





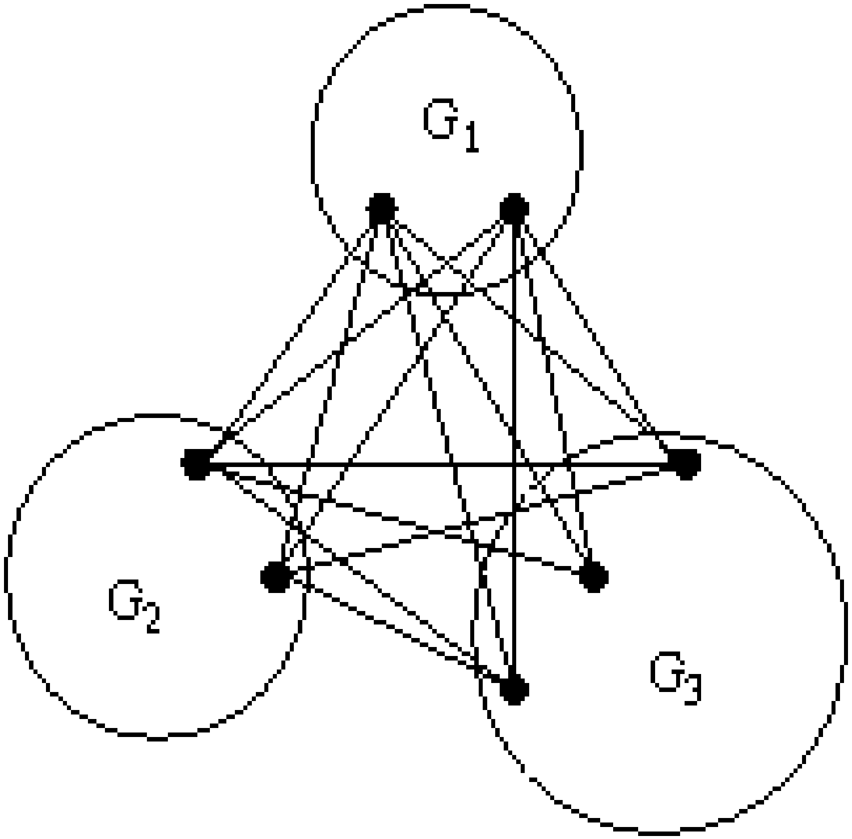
Cunningham 1972/1982

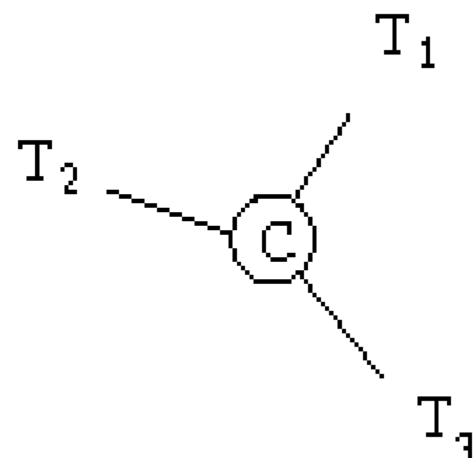
Stars



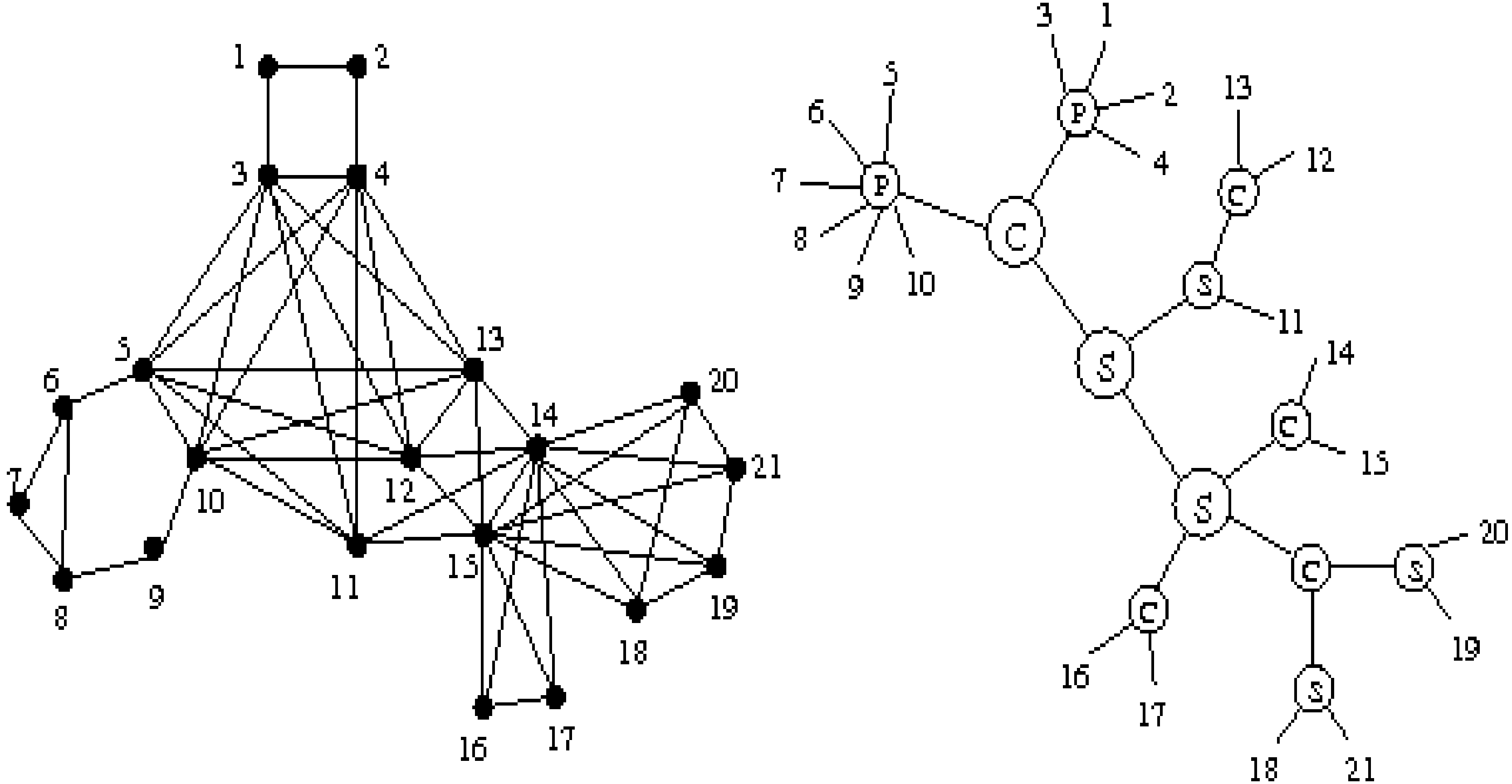
Cunningham 1972/1982

Cliques

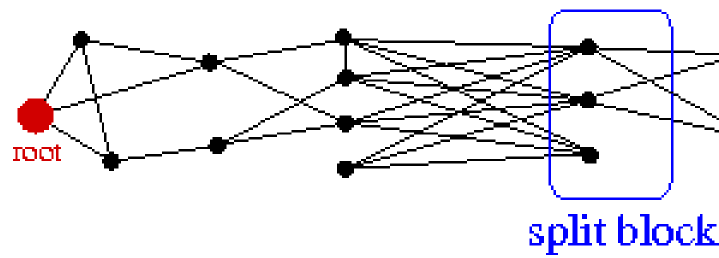


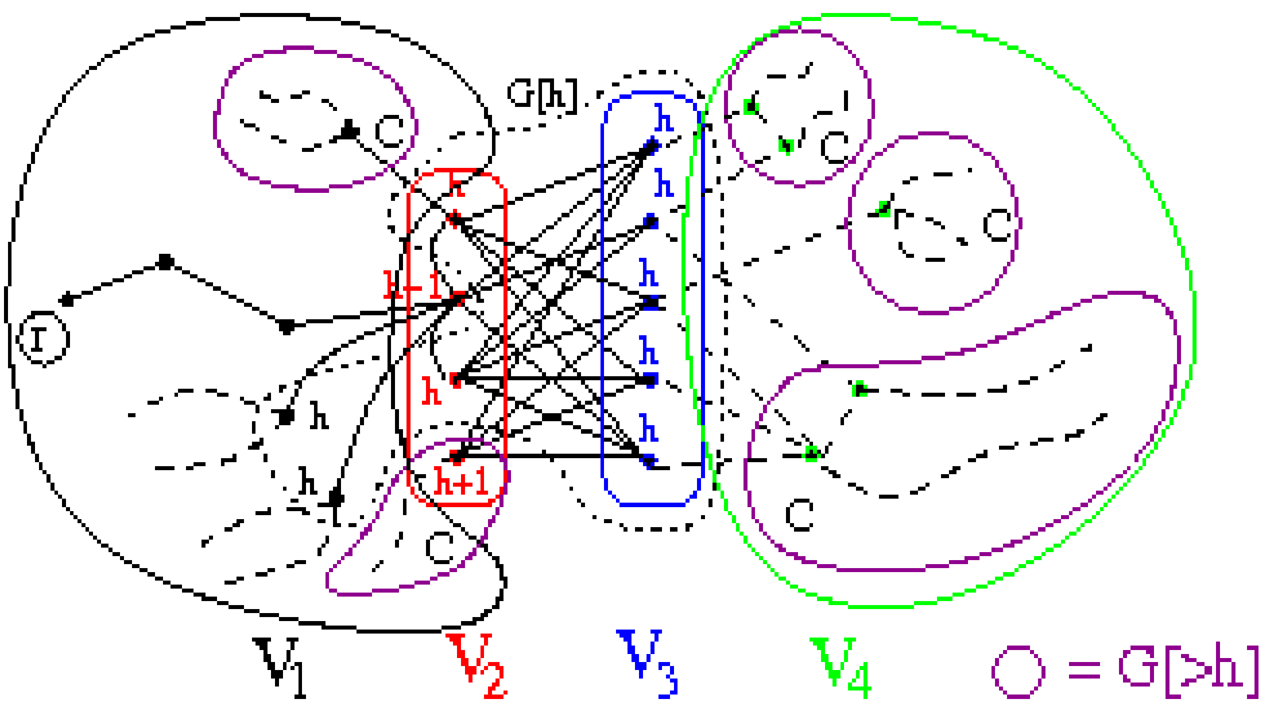


General example



Depth First Search (DFS)





Between layers

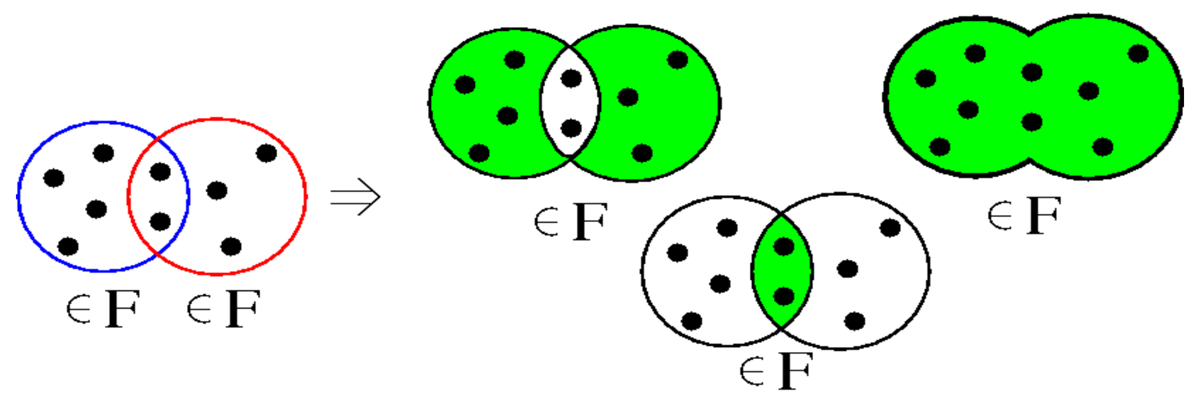
Stars
Cliques
Prime

Tree reconstruction

Each layer

identify all splits blocks

Partitive family

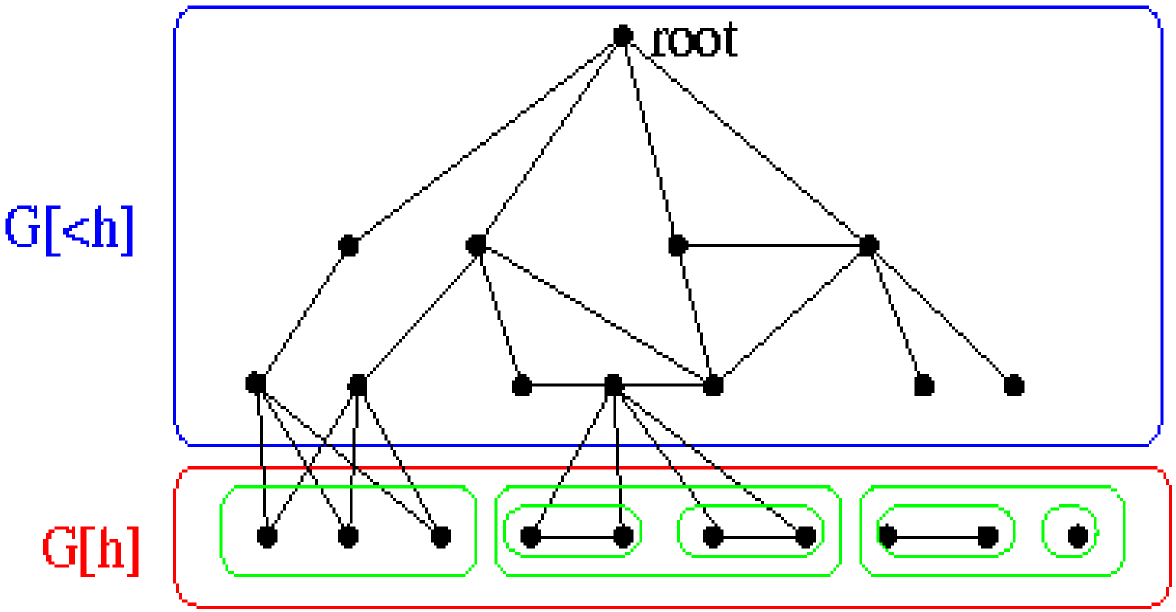


Tree structure

A layer, all split blocks.

Main theorem 1/6

M_h = modules of G[<=h] that are subset of layer h

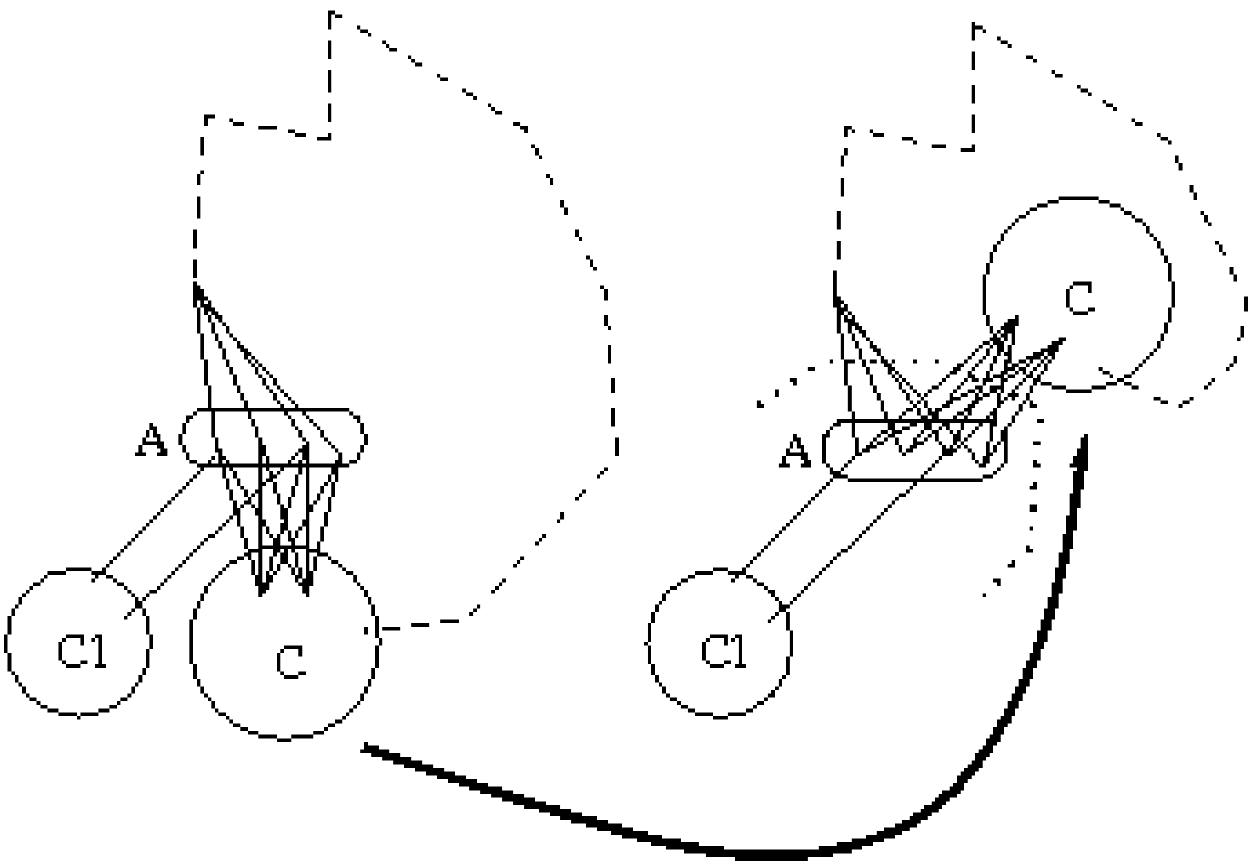


A layer, all split blocks.

Main theorem 2/6

M_h = modules of G[<=h] that are subset of layer h

Intuition: Neighborhood of C, component of G[>h]

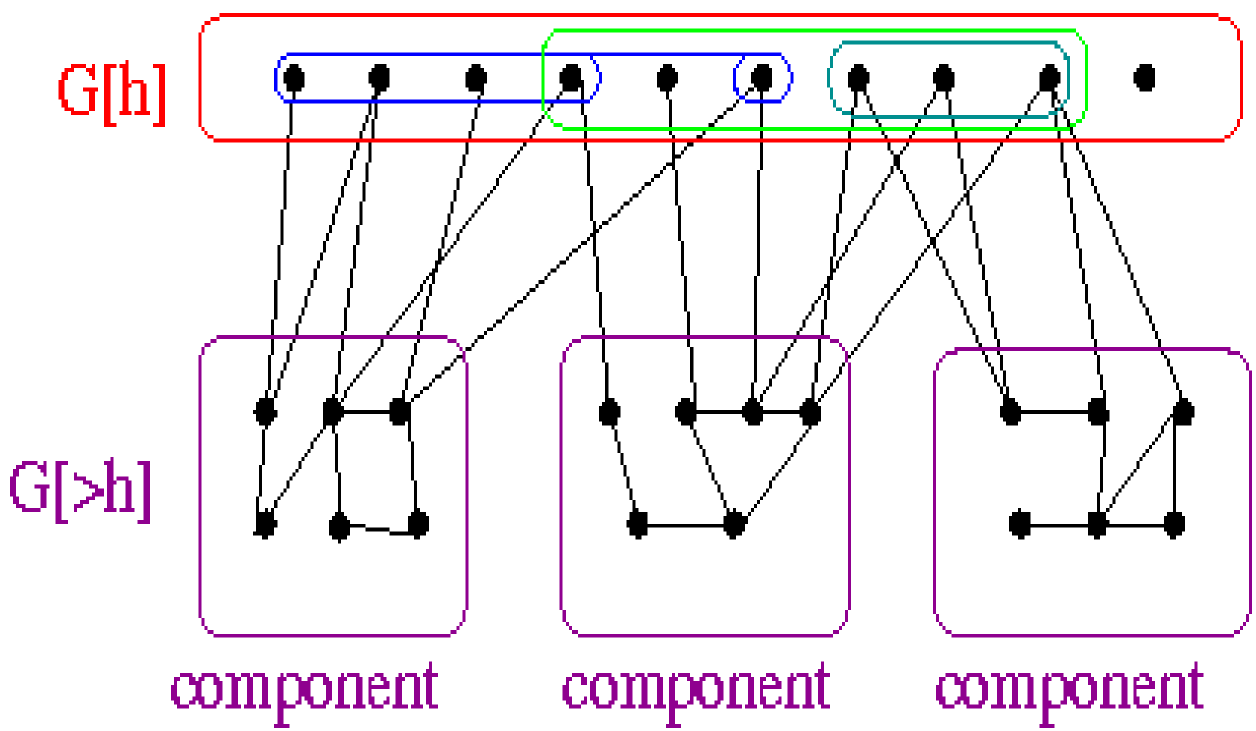


A layer, all split blocks.

Main theorem 3/6

M_h = modules of G[<=h] that are subset of layer h

Neighborhood of C, component of G[>h]



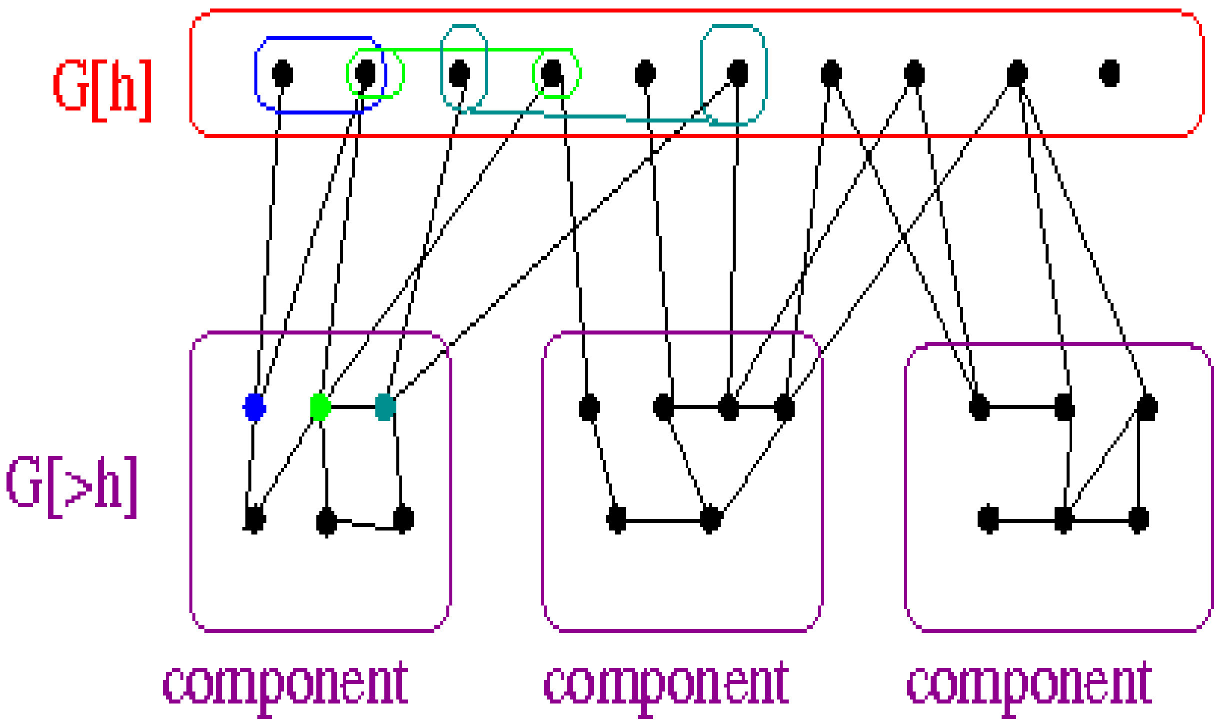
A layer, all split blocks.

Main theorem 4/6

M_h = modules of G[<=h] that are subset of layer h

Neighborhood of C, component of G[>h]

Neighborhood of x in [h+1]∩ layer h



A layer, all split blocks.

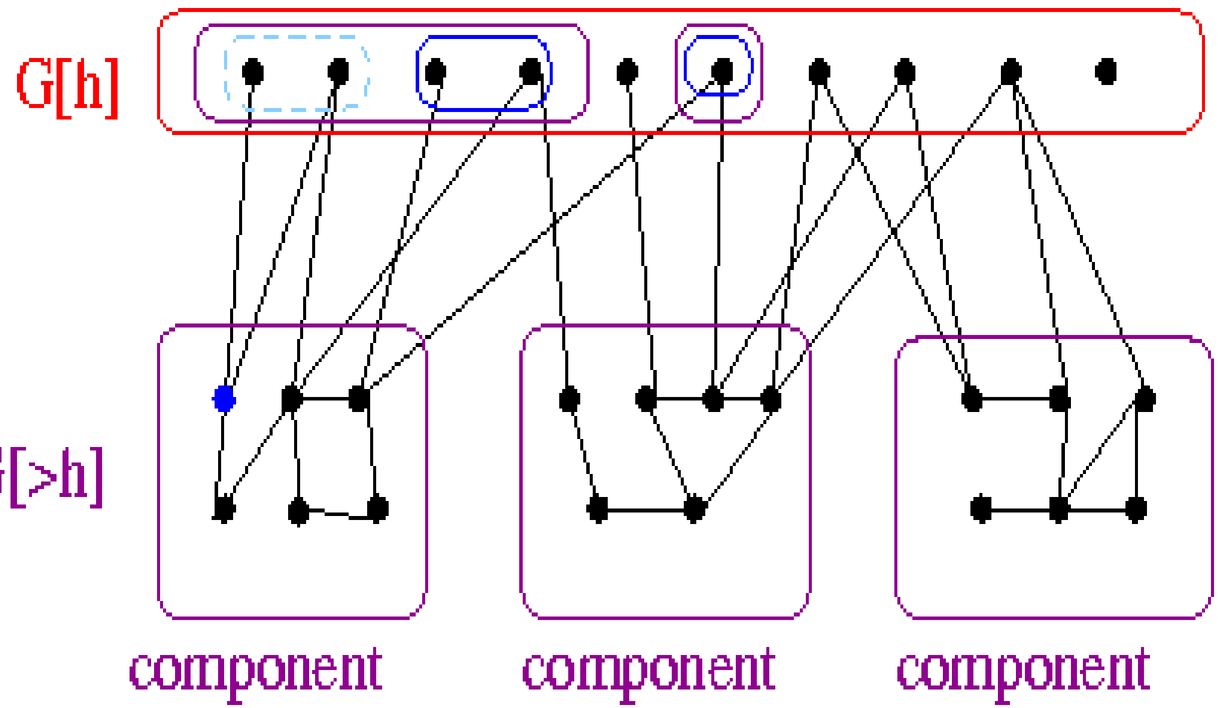
Main theorem 5/6

M_h = modules of G[<=h] that are subset of layer h

Neighborhood of C, component of G[>h]

Neighborhood of x in [h+1]$\cap$ layer h

Co-neighborhood of x in C: (N(C) \ N(x) $\cap$ [h])



A layer, all split blocks.

Main theorem 6/6

M_h = modules of G[<=h] that are subset of layer h

V_h:

- Neighborhood of C, component of G[>h]
- Neighborhood of x in [h+1] $\cap$ layer h
- Co-neighborhood of x in C: (N(C) \ N(x) $\cap$ [h])

$$B_h = M_h \cap V^{\perp}_h = (M^{\perp}_h \cup V_h)^{\perp}$$

B_h partitive familly

B_h representable as a tree

Computation

$B_h = (M^{\perp}_h \cup V_h)^{\perp}$ but

How to compute $M^{\perp}_h$?

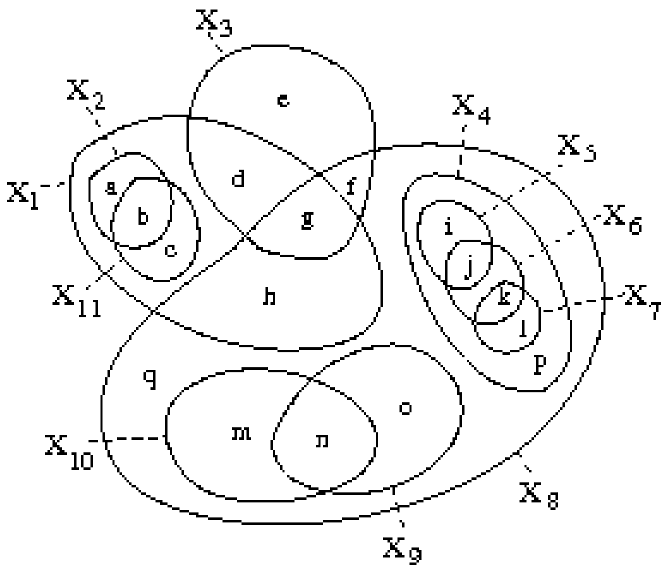
How to compute V_h ?

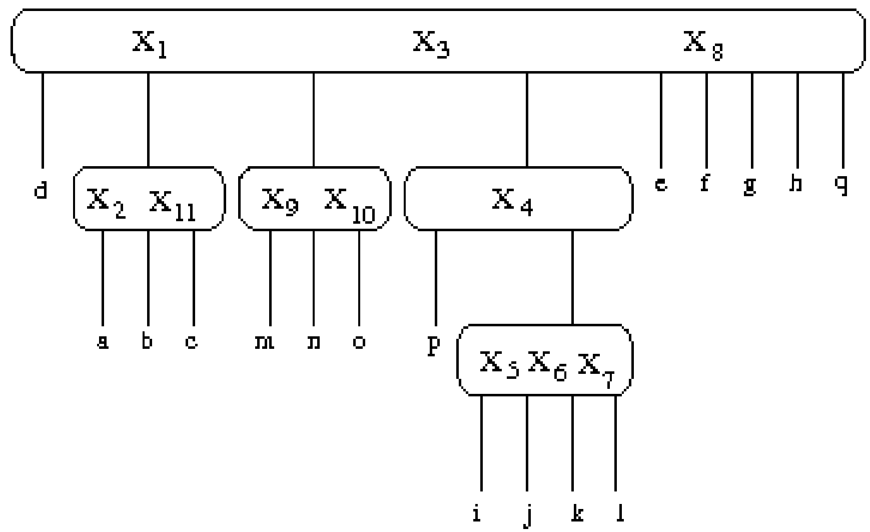
How to compute the main orthogonal ?

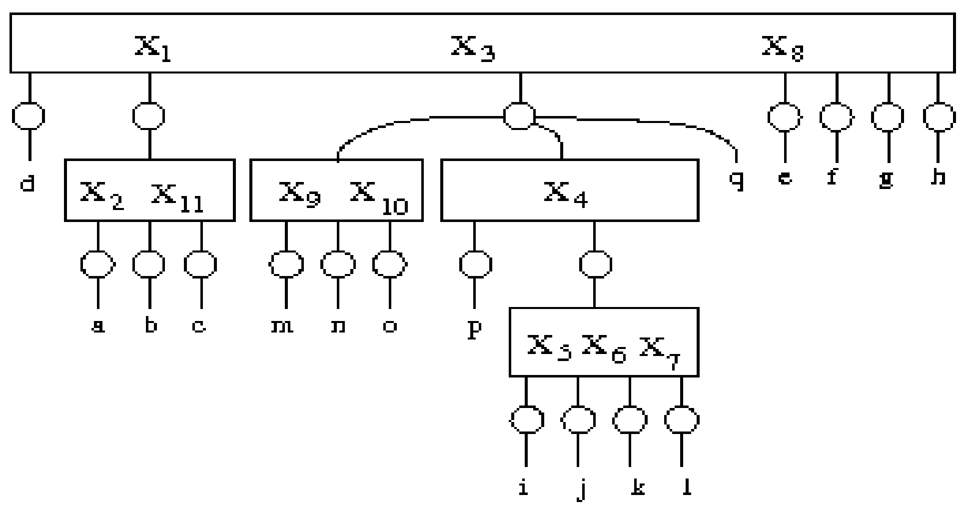
Several techniques !

A layer, all split blocks.

How to compute the main orthogonal ? F → $F^{\perp}$



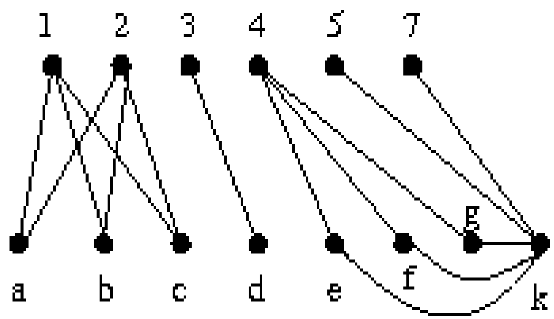


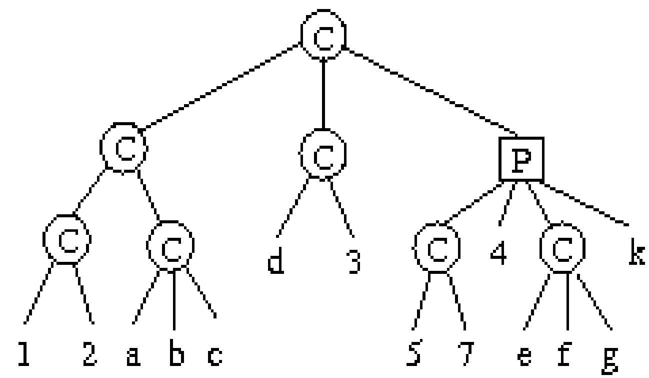


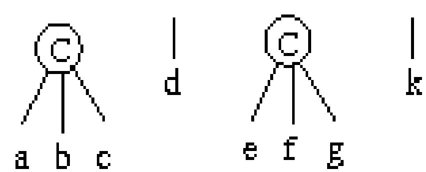
Warning: $\Omega(||F||)$

How to compute $M^{\perp}_{h}$? 1/2

1. Modular decomposition of G[h-1,h] + reduction to layer h





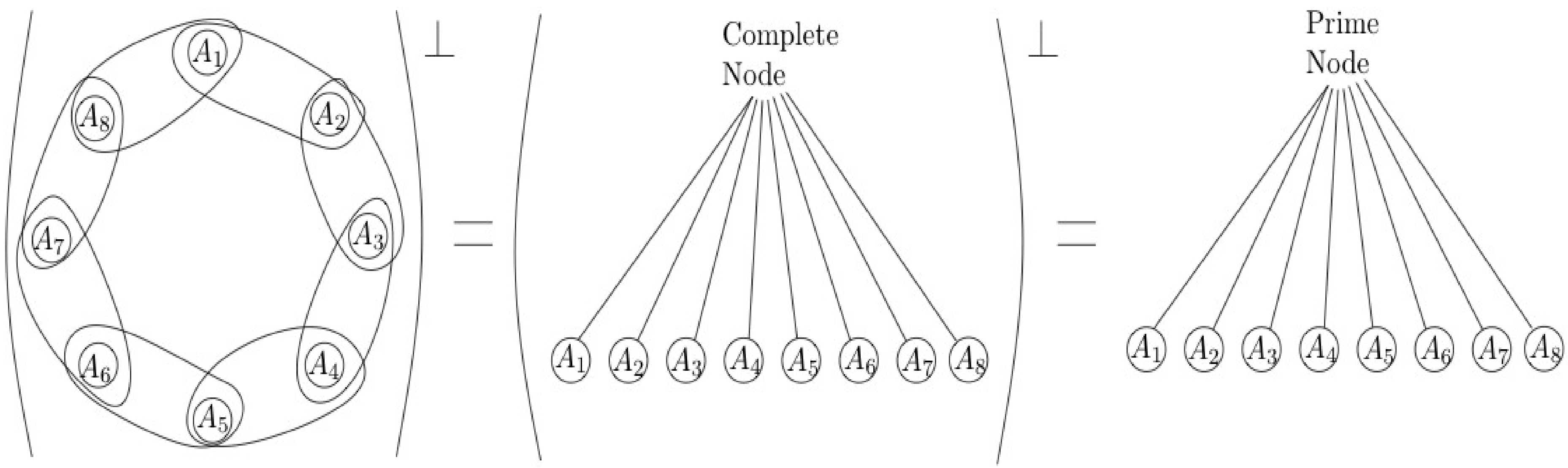


Pb: size of the family !

How to compute $M^{\perp}_h$? 2/2

2. Theorem: || strong modules || = O(n+m)

3. Trick



A layer, all split blocks.

How to compute V_h ?

V_h {
- Neighborhood of C, component of G[>h]
- Neighborhood of x in [h+1] $\cap$ layer h
- Co-neighborhood of x in C: (N(C) \ N(x) $\cap$ [h])

$$V_h \Rightarrow V_h^1 \cup V_h^2 \cup V_h^3 \cup .... \cup V_h^k$$

V_h^i = neighborhood of C_i
+ neighboorhood of its vertices $N(x)_{|C}$
+ co-neighborhood $N(x)_{|C}$

Second trick: partition refining on $N(C_i)$ using $N(x)$ as pivots

$\Rightarrow$ Partition B_1 .. B_r of $N(C_i)$

Overlapping a member of V_h^i $\Leftrightarrow$ overlapping $B_j \cup B_{j+1}$

Back to first trick ! family $W_h^i = \{N(C_i), B_1 \cup B_2, B_2 \cup B_{3}, ..., B_r \cup B_1\}$