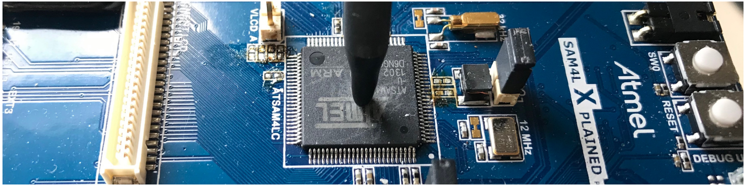

A Nearly Tight Proof of Duc *et al.*'s Conjectured Security Bound for Masked Implementations

Loïc Masure Olivier Rioul François-Xavier Standaert

CARDIS 2022, Birmingham, November 7th
https://ia.cr/2022/600

European Research Council
Established by the European Commission
GROUPE CRYPTO
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If You Are Interested...

Same result obtained independently by Akira Ito, Rei Ueno, Naofumi Homma.
To be presented at CCS 2022 (`https://ia.cr/2022/576`)

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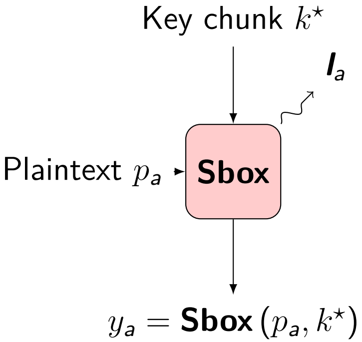
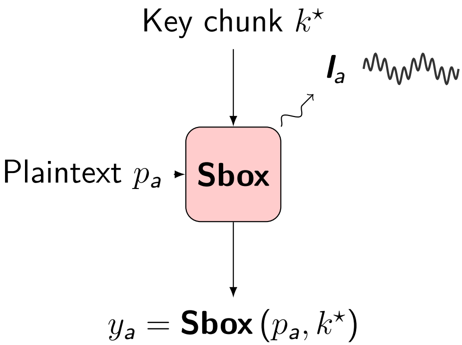
How does an Side-Chanel Analysis (SCA) work



How does an SCA work



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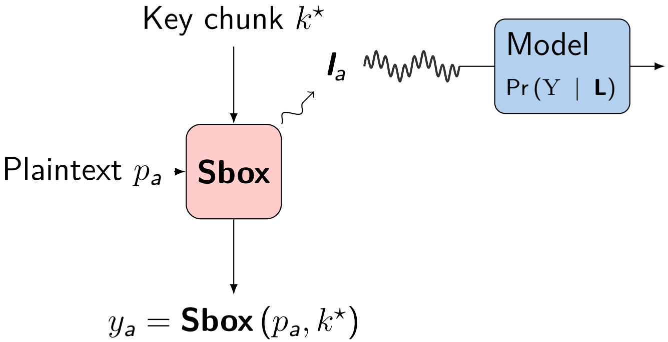

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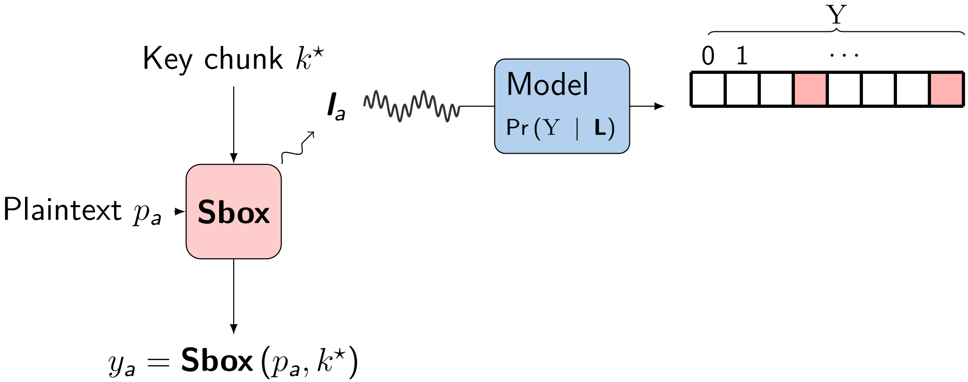

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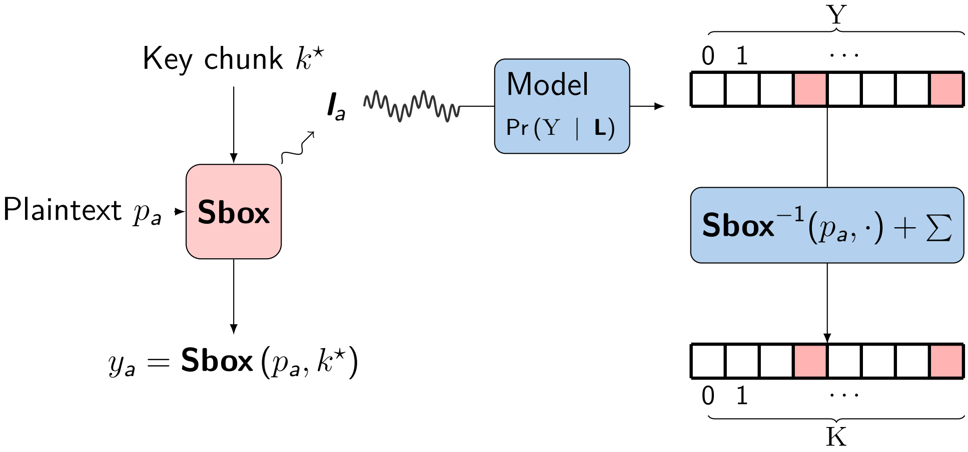

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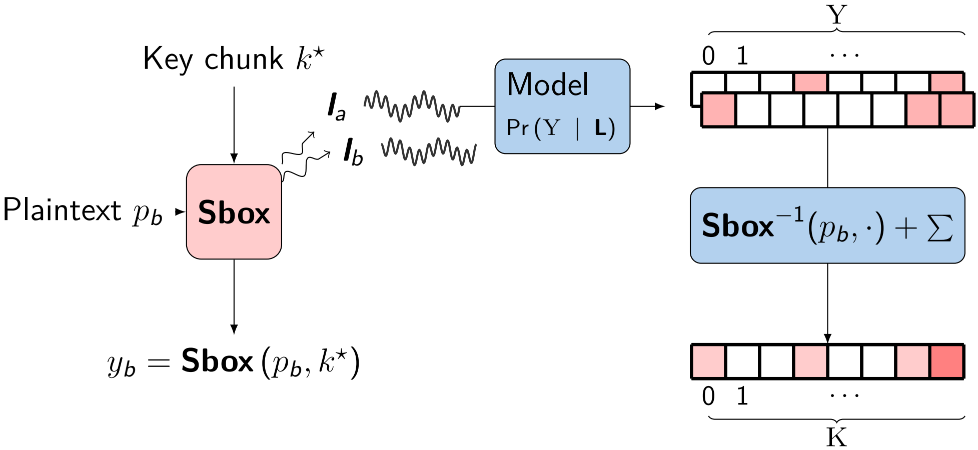

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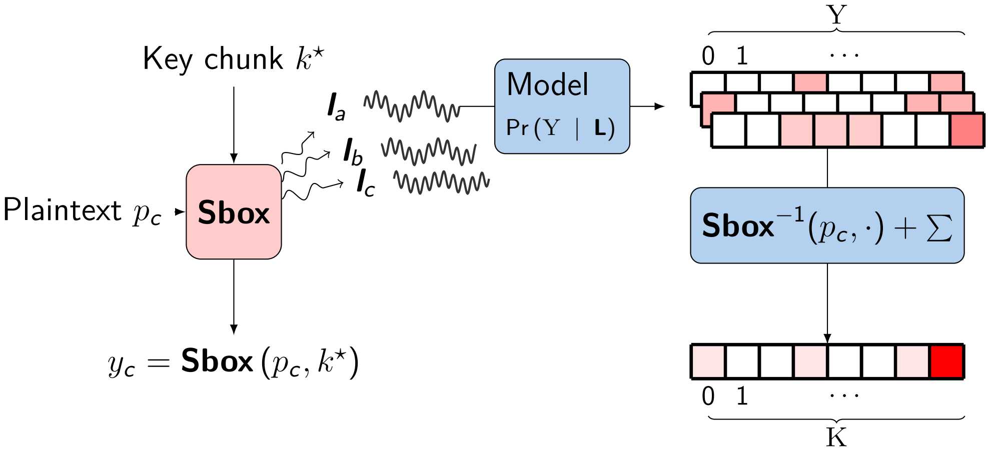

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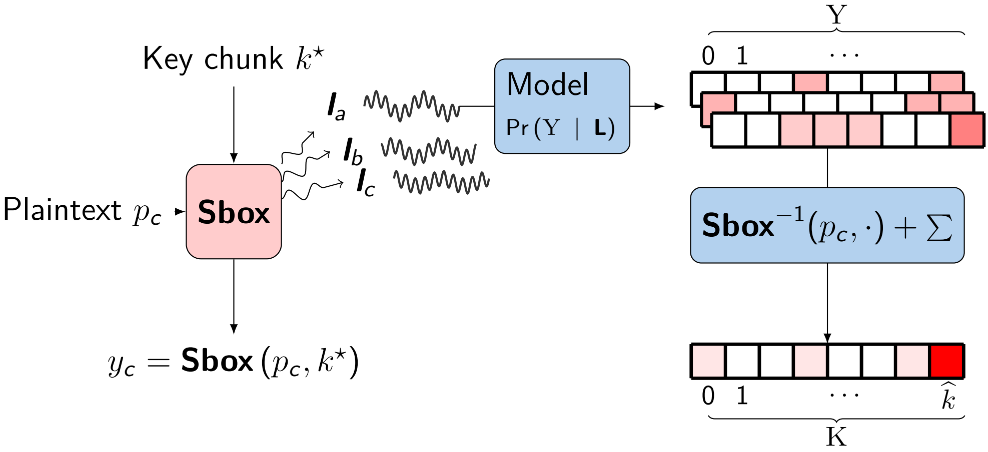

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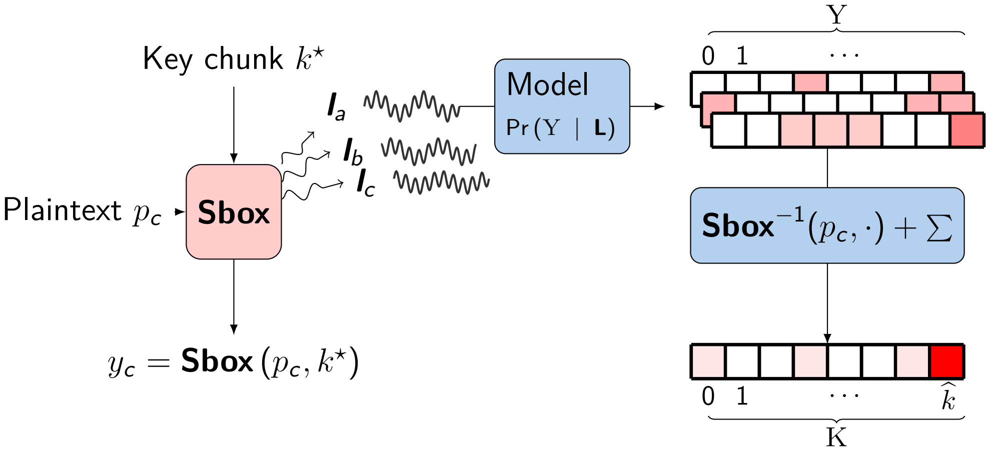
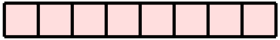


Successful attack iff $\widehat{k} = k^\star$

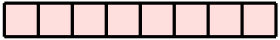
From scores to Metrics

If, the adversary gets:



From scores to Metrics

If, the adversary gets:



Sensitive computation unpredictable

SCA not more powerful than cryptanalysis

Device fully secure

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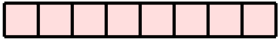

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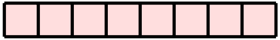

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Exact prediction of the sensitive computation
Success rate of 100% with *one* trace
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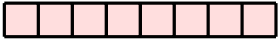

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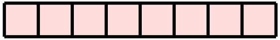

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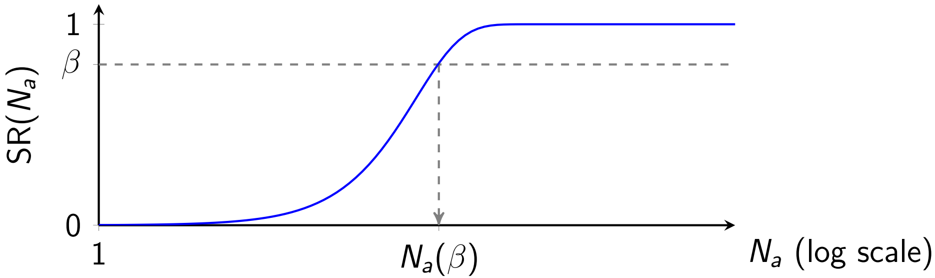
In general, the adversary gets:

How does this translate into SCA security metrics ?

Concrete SCA Metrics: the Success Rate (SR)



SR: probability to succeed the attack within N_a queries to the target

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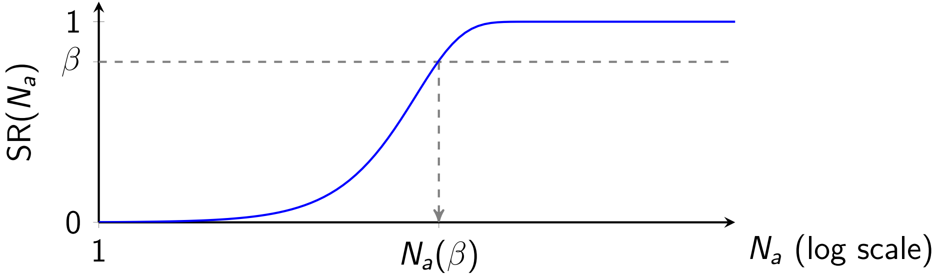

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Secured device with prob. $\geq 1 - \beta$, $\implies$ refresh secret every $N_a(\beta)$ use ✓

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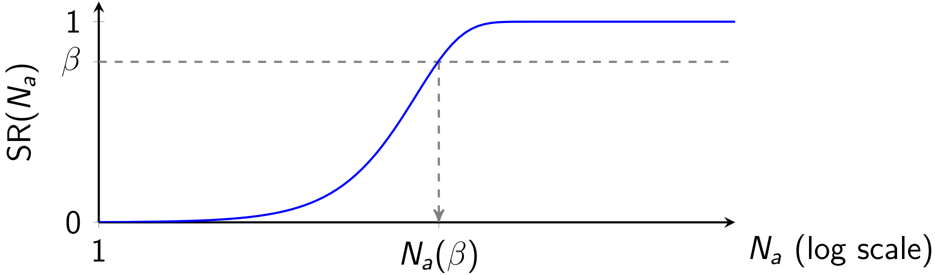

SR: probability to succeed the attack within N_a queries to the target

Secured device with prob. $\geq 1 - \beta$, $\implies$ refresh secret every $N_a(\beta)$ use ✓

Naive est. of $N_a(\beta)$ is expensive: complexity depends on $N_a(\beta)$ itself ✗

Circumventing the Drawbacks of the Success Rate (SR)

Can we find surrogate metrics characterizing $N_a(\beta)$?

[1] Mangard, Oswald, and Popp, *Power analysis attacks - revealing the secrets of smart cards*
[2] Chérisey et al., "Best Information is Most Successful: Mutual Information and Success Rate in Side-Channel Analysis"

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CPA [1]

Using correlation coeff.

$$N_a(\beta) \approx \frac{f(\beta)}{\rho^2}$$

Easy to estimate ρ ✓

Only for univariate, linear ✗

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General case [2]

Using the Mutual Information (MI),

$$N_a(\beta) \geq \frac{f(\beta)}{\mathrm{MI}\left(\mathrm{Y}; \mathbf{L}\right)}$$

MI generalizes ρ ✓
MI hard to estimate ✗

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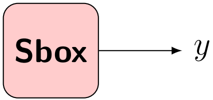
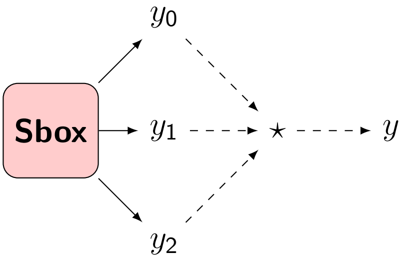
Content

How to protect against SCA: Masking



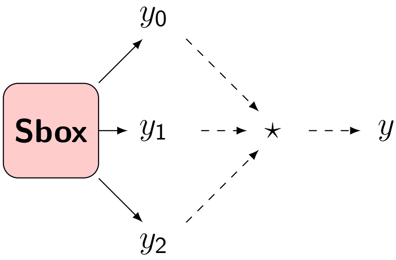
(a) Unprotected



(b) Masking with $d + 1 = 3$ shares

Each share y_i drawn uniformly, such that $y = y_0 \star \ldots \star y_d$

Masking = Convolutions



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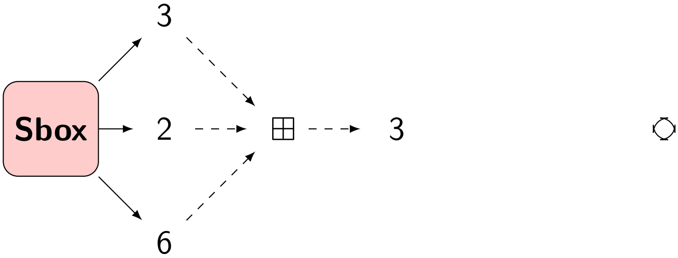

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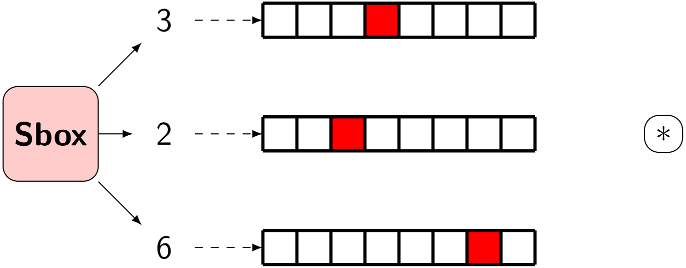

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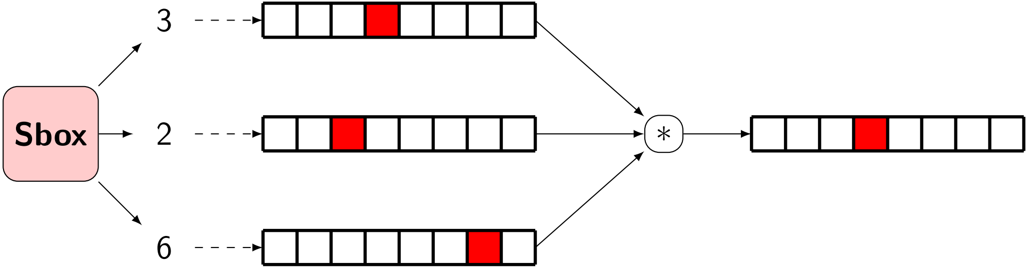

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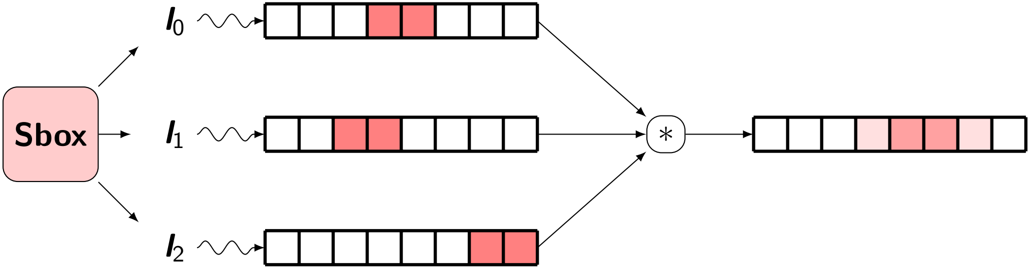

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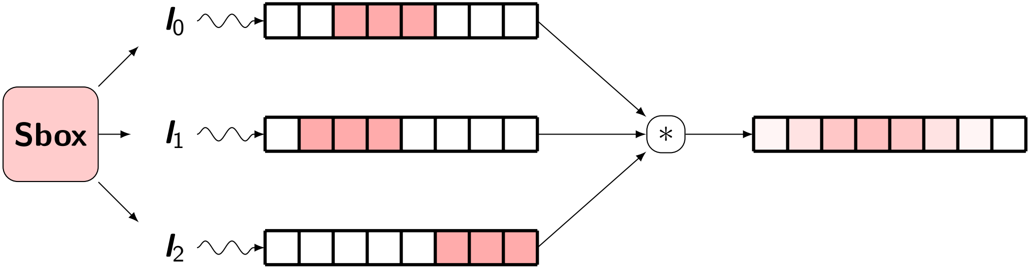

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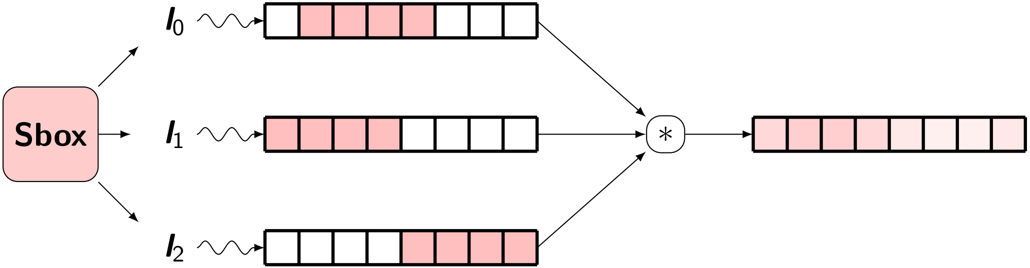


Masking amplifies the noise . . . exponentially with #shares

MI *very* hard to compute naively with masking

Curse of dimensionality increases with #shares

Higher #shares $\implies$ lower MI $\implies$ harder est.

Content

Duc et al's Conjecture [3]

"Can we infer the whole MI using the MI on each share (much easier) ?"

[3] Duc, Faust, and Standaert, "Making Masking Security Proofs Concrete (Or How to Evaluate the Security of Any Leaking Device), Extended Version"

Duc et al's Conjecture [3]

"Can we infer the whole MI using the MI on each share (much easier) ?"

$$N_a(\beta) \approx \frac{f(\beta)}{\prod_{i=0}^{d} \left(\mathsf{MI}\left(\mathrm{Y}_i; \mathrm{L}_i\right)/\tau\right)^r}$$

τ: noise amplification threshold

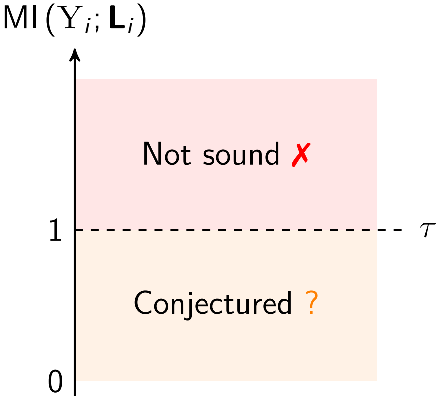
$d \cdot r$: "*effective*" security parameter

Duc *et al.* conjectured:

$\tau = 1$,

$r = 1$,

$f(\beta) \perp |\mathcal{Y}|$,



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τ: noise amplification threshold

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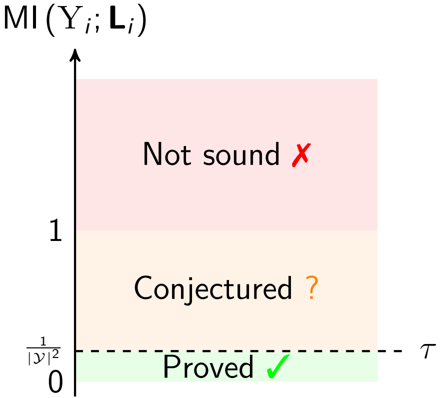
Duc *et al.* only proved:

- $\tau = 0.7/|\mathcal{Y}|^2$,
- $r = 1/2$,
- $f(\beta) \perp |\mathcal{Y}|$,



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In our paper we prove:

$\tau = 0.72,$

$r = 1,$

$f(\beta) \propto \log(|\mathcal{Y}|)/|\mathcal{Y}|,$

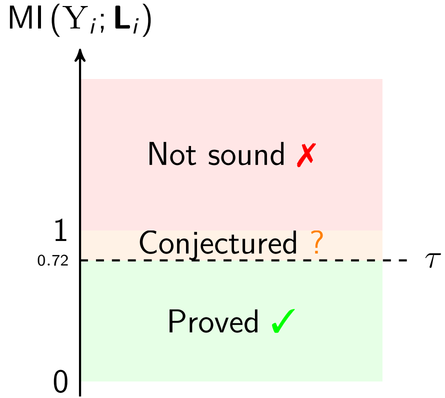

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Illustration on Simulations

Bitslice masking: $|\mathcal{Y}| = 2$, Leakage model: $\mathbf{L}_i = \mathrm{hw}(Y_i) + \mathrm{Noise}(0, \sigma^2)$

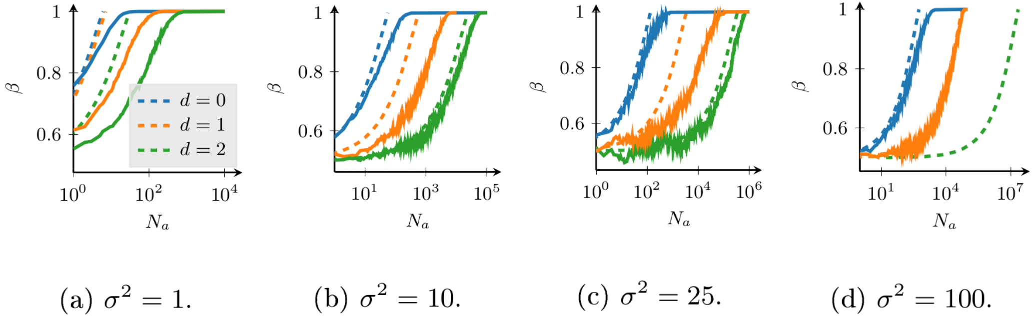
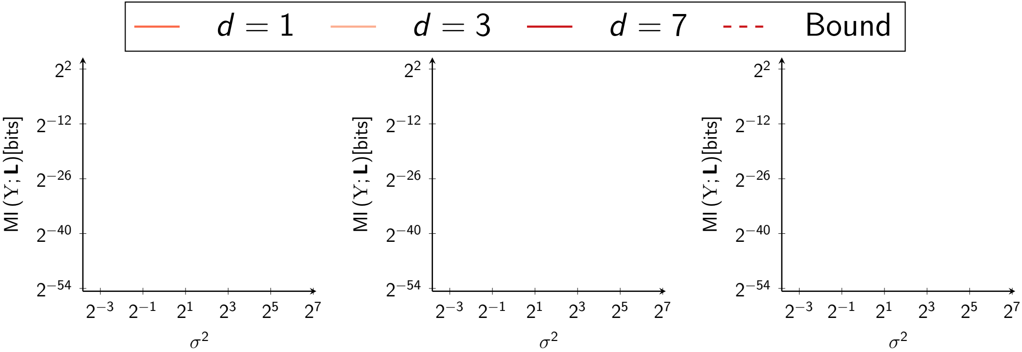


Figure: Success rate of concrete bit recoveries and MI-based upper bounds.

Content

Is Our Nearly-Tight Proof Actually Tight ?

Leakage model: $\mathbf{L}_i = \mathrm{hw}(\mathrm{Y}_i) + \text{ Noise}(0, \sigma^2)$. MI estimated with Monte-Carlo (MC) methods



(a) $|\mathcal{Y}| = 2$ (bit-slice). (b) $|\mathcal{Y}| = 16$. (c) $|\mathcal{Y}| = 256$.

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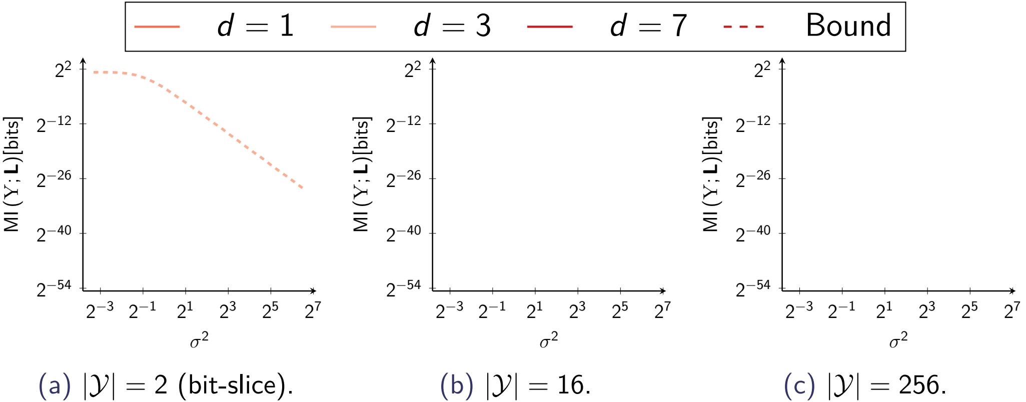

Figure: MI (plain) and new MI upper bound (dashed) for different field sizes.

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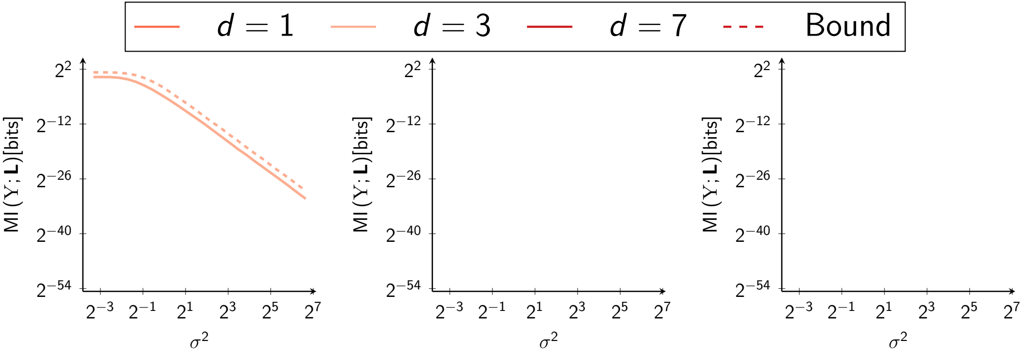

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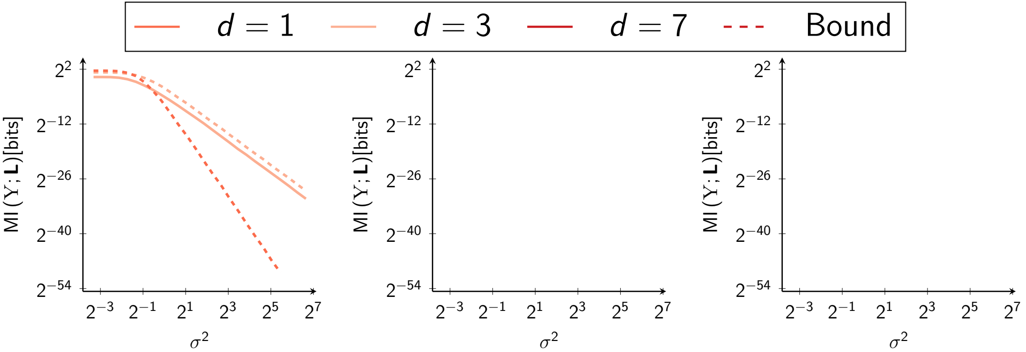

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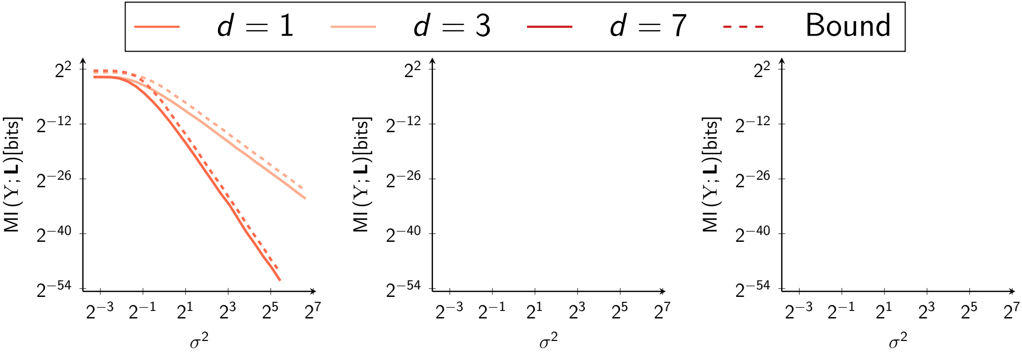

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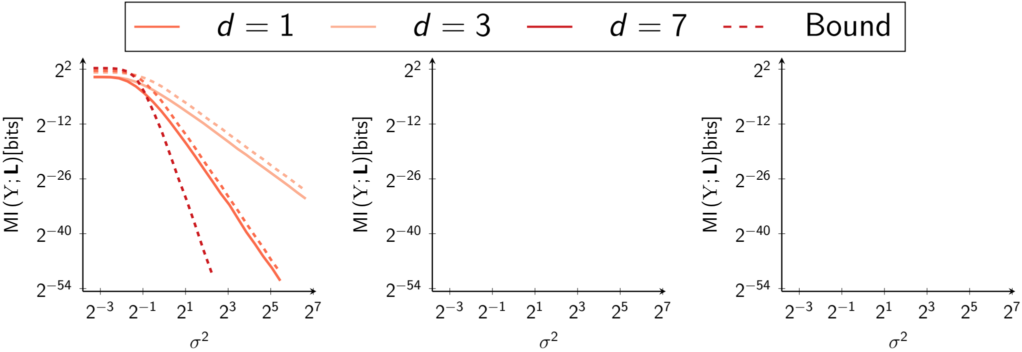

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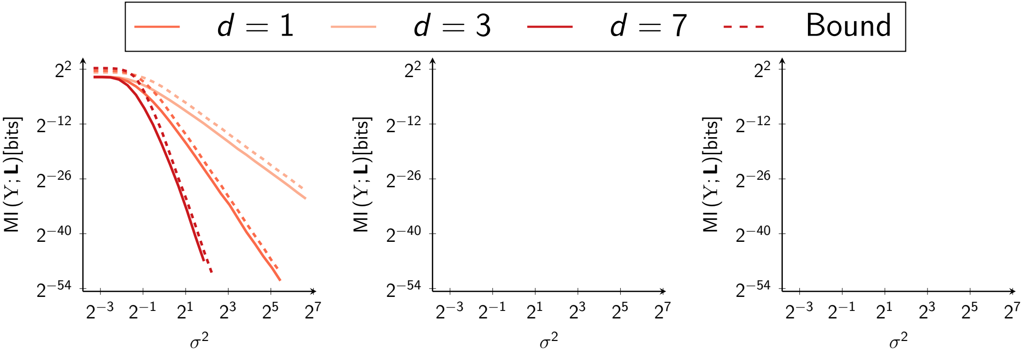

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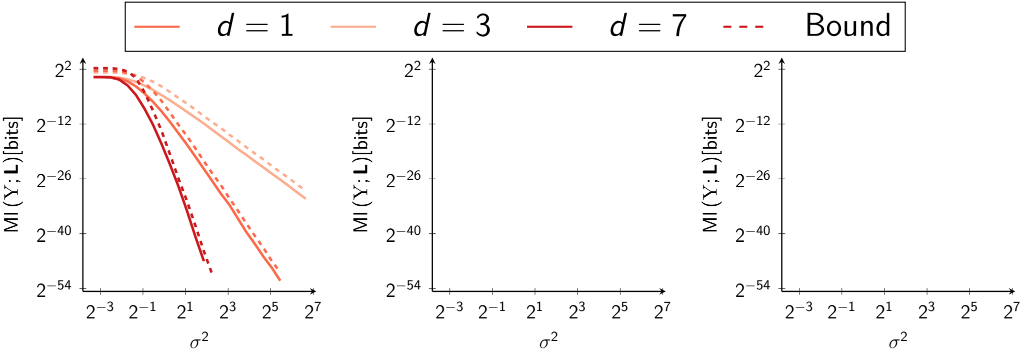

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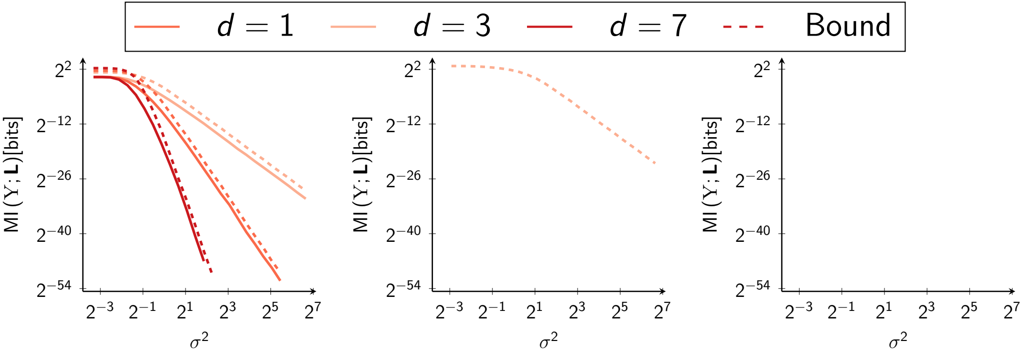

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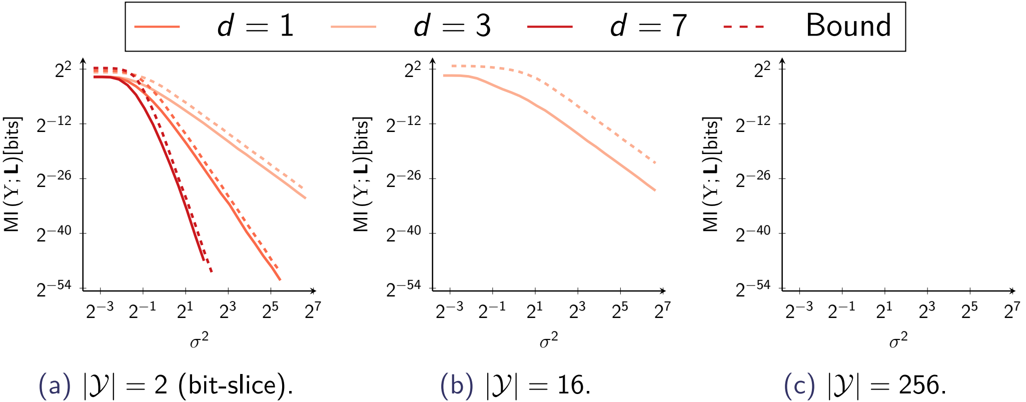

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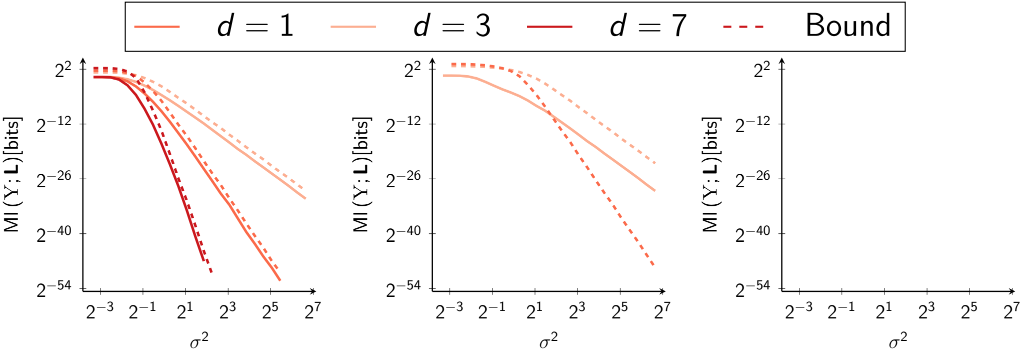

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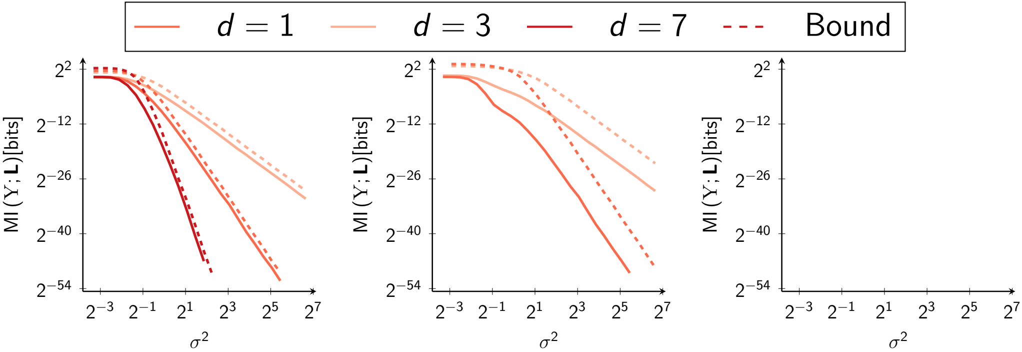

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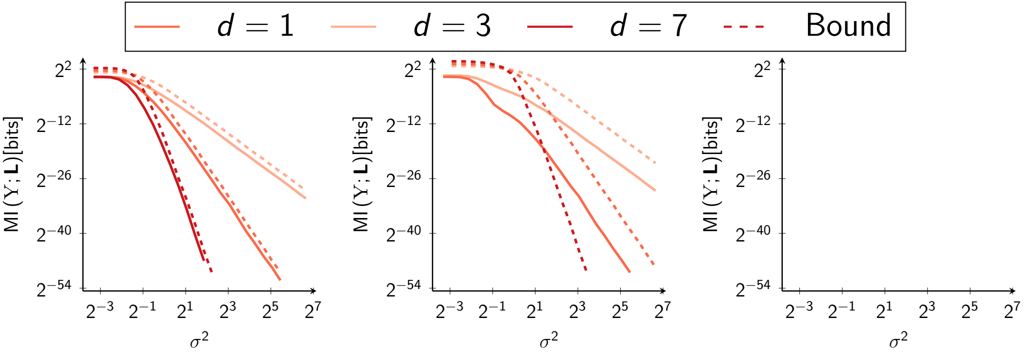

(a) $|\mathcal{Y}| = 2$ (bit-slice).   (b) $|\mathcal{Y}| = 16$.   (c) $|\mathcal{Y}| = 256$.

Figure: MI (plain) and new MI upper bound (dashed) for different field sizes.

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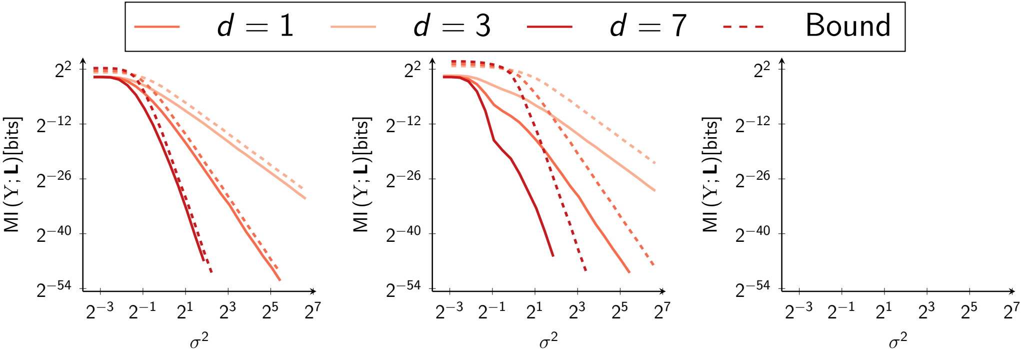

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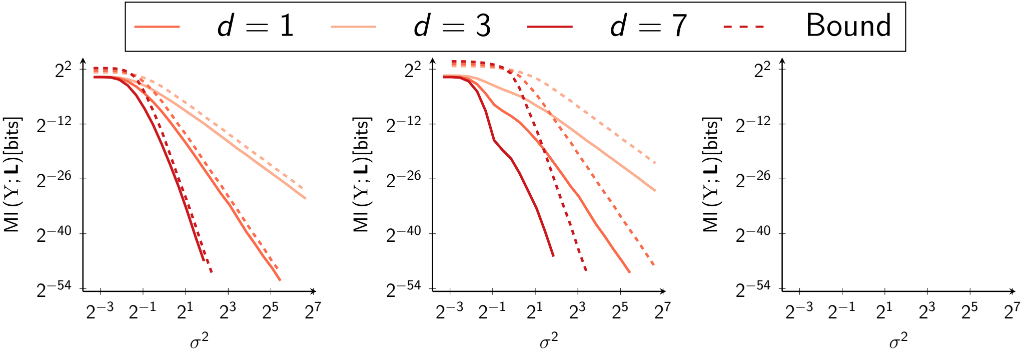

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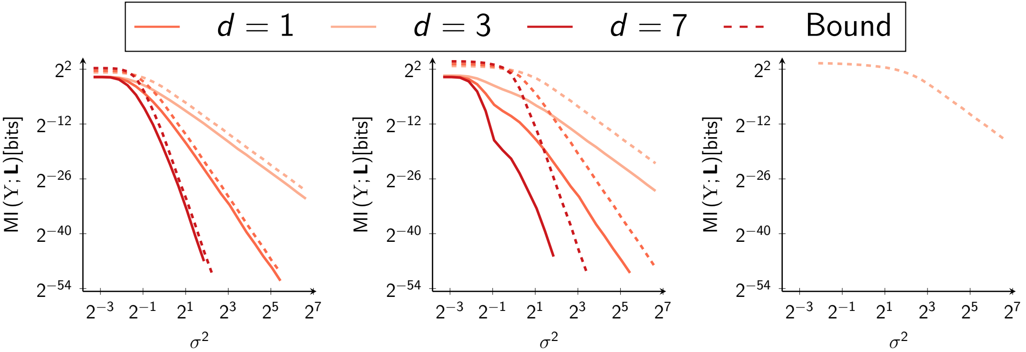

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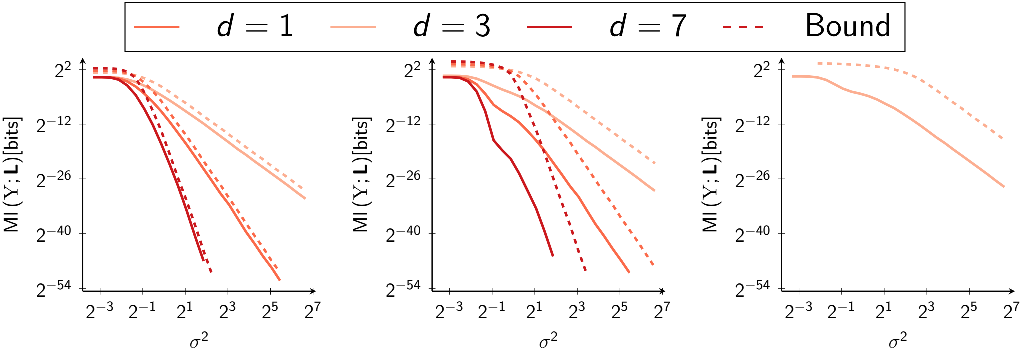

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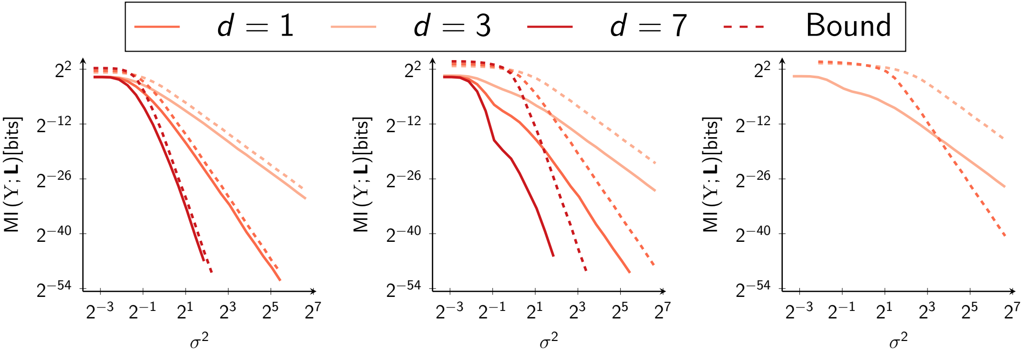

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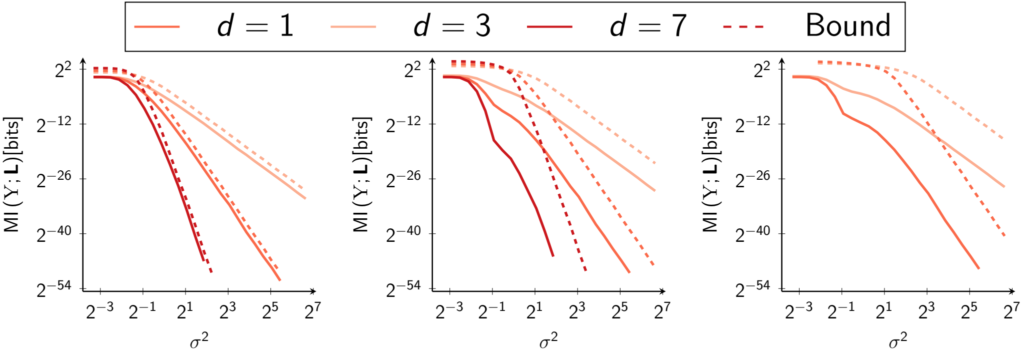

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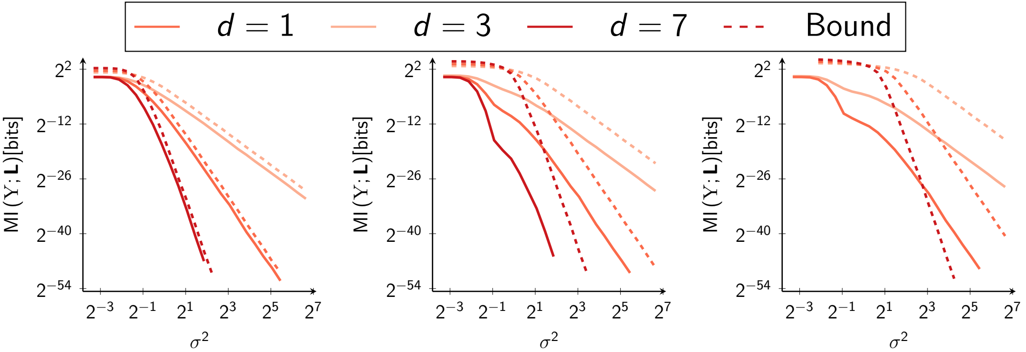

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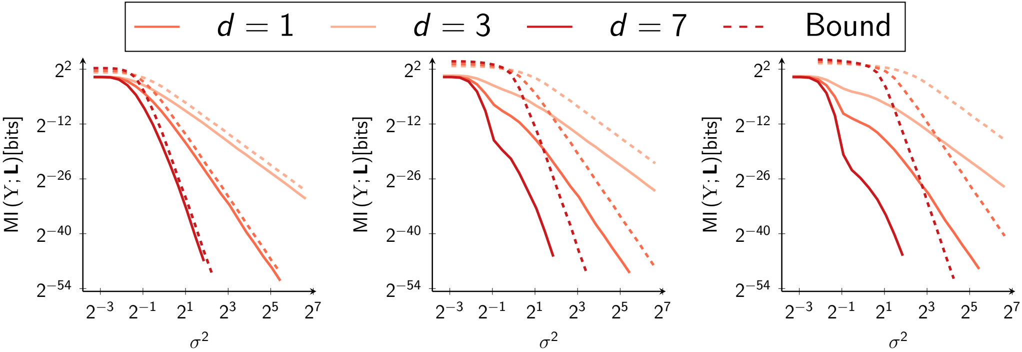

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Content

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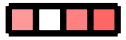
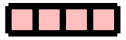
How to measure the discrepancy between $\mathrm{p}_{\mathrm{Y}_i \mid \boldsymbol{l}_i} \approx$  and $\mathrm{p}_{\mathrm{Y}_i} =$  ?

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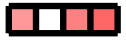
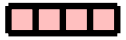
How to measure the discrepancy between $\mathsf{p}_{\mathrm{Y}_i \mid \boldsymbol{l}_i} \approx$  and $\mathsf{p}_{\mathrm{Y}_i} =$  ?

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MI relates well to SR ✓
Not convenient with convolutions ✗

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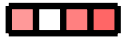
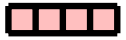
How to measure the discrepancy between $\mathrm{p}_{\mathrm{Y}_i \mid \boldsymbol{l}_i} \approx$  and $\mathrm{p}_{\mathrm{Y}_i} =$  ?

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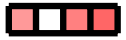
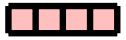
How to measure the discrepancy between $\mathsf{p}_{\mathrm{Y}_i \mid \boldsymbol{l}_i} \approx$  and $\mathsf{p}_{\mathrm{Y}_i} =$  ?

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Can we leverage both advantages ?

Back and Forth Between Metrics

Theorem (Pinsker's Inequalities)

Allows to convert TV to KL divergence, and inversely:

$$2\log(2)\,\mathrm{TV}\,(\mathrm{p};\mathrm{m})^2 \underset{\textit{Pinsker}}{\leq} \mathrm{D}(\mathrm{p} \parallel \mathrm{m})$$

Back and Forth Between Metrics

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Allows to convert TV to KL divergence, and inversely:

$$2\log(2)\,\mathsf{TV}\left(\mathsf{p};\mathsf{m}\right)^2 \underset{\text{Pinsker}}{\leq} \mathsf{D}(\mathsf{p}\parallel\mathsf{m}) \underset{\text{Reversed Pinsker}}{\leq} \log_2\left(1+2\left|\mathcal{Y}\right|\mathsf{TV}\left(\mathsf{p};\mathsf{m}\right)^2\right) \leq 2\log(2)\left|\mathcal{Y}\right|\mathsf{TV}\left(\mathsf{p};\mathsf{m}\right)^2$$

The Core Ingredient: the *Xor* Lemma, I

Theorem (Xor Lemma[4])

If $\mathrm{Y} = \mathrm{Y}_0 \star \ldots \star \mathrm{Y}_d$, *then*

$$\mathsf{TV}\left(\mathsf{p}_{\mathrm{Y} \mid \boldsymbol{l}}; \mathsf{p}_{\mathrm{Y}}\right) \leq 2^d \prod_{i=0}^{d} \mathsf{TV}\left(\mathsf{p}_{\mathrm{Y}_i \mid \boldsymbol{l}_i}; \mathsf{p}_{\mathrm{Y}_i}\right) \qquad \textit{(Local)}$$

$$\mathsf{SD}\left(\mathrm{Y}; \mathbf{L}\right) \leq 2^d \prod_{i=0}^{d} \mathsf{SD}\left(\mathrm{Y}_i; \mathbf{L}_i\right) \qquad \textit{(Average)}$$

This is where the noise **amplification** comes from

[4] Dziembowski, Faust, and Skórski, "Optimal Amplification of Noisy Leakages".

The Core Ingredient: the *Xor* Lemma, II

Theorem (Xor Lemma, KL-version)

$$\mathsf{D}(\mathsf{p}_{\mathrm{Y} \mid \boldsymbol{I}} \parallel \mathsf{p}_{\mathrm{Y}}) \leq \log_2 \left(1 + |\mathcal{Y}| \prod_{i=0}^{d} \left(2 \log(2) \mathsf{D}(\mathsf{p}_{\mathrm{Y}_i \mid \boldsymbol{I}_i} \parallel \mathsf{p}_{\mathrm{Y}_i}) \right) \right) \qquad \textit{(Local)}$$

Last Ingredient: Local vs. Average Metrics

By assumption, the leakages $\mathbf{L}_i$ on each share Y_i are independent
"The expectation of the product = the product of expectations"

$$\mathsf{MI}\left(\mathrm{Y}; \mathbf{L}\right) = \underset{\mathbf{L}}{\mathbb{E}}\left[\mathsf{D}(\mathsf{p}_{\mathrm{Y} \mid \mathbf{L}} \parallel \mathsf{p}_{\mathrm{Y}})\right]$$

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$$\mathsf{MI}\left(\mathrm{Y};\mathbf{L}\right) \underset{\text{Xor Lemma}}{\leq} \underset{\mathbf{L}}{\mathbb{E}}\left[\log_2\left(1+|\mathcal{Y}|\cdot\prod_{i=0}^{d}\left(C\cdot \mathsf{D}(\mathsf{p}_{\mathrm{Y}_i\mid\mathbf{L}_i}\parallel \mathsf{p}_{\mathrm{Y}_i})\right)\right)\right]$$

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$$\mathsf{MI}\left(\mathrm{Y};\mathbf{L}\right) \underset{\text{Jensen}}{\leq} \log_2\left(1 + |\mathcal{Y}| \cdot \mathop{\mathbb{E}}_{\mathbf{L}}\left[\prod_{i=0}^{d}\left(C \cdot \mathsf{D}(\mathsf{p}_{\mathrm{Y}_i \mid \mathbf{L}_i} \parallel \mathsf{p}_{\mathrm{Y}_i})\right)\right]\right)$$

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By assumption, the leakages $\mathbf{L}_i$ on each share Y_i are independent
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$$\mathrm{MI}\left(\mathrm{Y};\mathbf{L}\right) \underset{\text{Indep.}}{\leq} \log_2\left(1 + |\mathcal{Y}| \cdot \prod_{i=0}^{d}\left(C \cdot \mathop{\mathbb{E}}_{\mathbf{L}}\left[\mathrm{D}(\mathrm{p}_{\mathrm{Y}_i \mid \mathbf{L}_i} \parallel \mathrm{p}_{\mathrm{Y}_i})\right]\right)\right)$$

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Theorem (Xor-Lemma, MI-version)

$$\mathsf{MI}\left(\mathrm{Y};\mathbf{L}\right) \leq 2\log(2)\,|\mathcal{Y}| \prod_{i=0}^{d}\left(\mathsf{MI}\left(\mathrm{Y}_i;\mathbf{L}_i\right)/\tau\right) , \qquad (1)$$

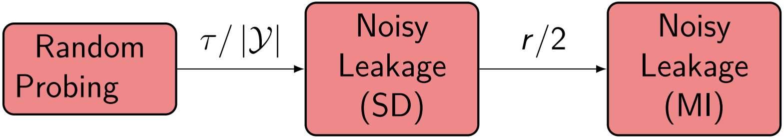
$\tau = \frac{1}{2\log(2)} \approx 0.72$

Why Former Papers are not Tight?

Warning ! Pinsker allows also to convert SD to MI, but not inversely:

$$2\log(2)\,\mathrm{SD}\left(\mathrm{Y};\mathbf{L}\right)^2 \underset{\text{Pinsker + Jensen}}{\leq} \mathrm{MI}\left(\mathrm{Y};\mathbf{L}\right) \not\lesssim 2\log(2)\,\mathrm{SD}\left(\mathrm{Y};\mathbf{L}\right)^2$$

Duc *et al.*'s result relies on the following reduction



Figure

References I

Chérisey, E. de et al. "Best Information is Most Successful: Mutual Information and Success Rate in Side-Channel Analysis". In: *IACR Transactions on Cryptographic Hardware and Embedded Systems* 2019.2 (2019), pp. 49–79. DOI: 10.13154/tches.v2019.i2.49-79. URL: https://tches.iacr.org/index.php/TCHES/article/view/7385.

Duc, A., S. Faust, and F. Standaert. "Making Masking Security Proofs Concrete (Or How to Evaluate the Security of Any Leaking Device), Extended Version". In: *J. Cryptology* 32.4 (2019), pp. 1263–1297. DOI: 10.1007/s00145-018-9277-0. URL: https://doi.org/10.1007/s00145-018-9277-0.

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