



Side-channel Analysis of Cryptographic Implementations

Evaluation & Counter-Measures

Loïc Masure (loic.masure@lirmm.fr)

Forum InCyber, 1 Avril 2025



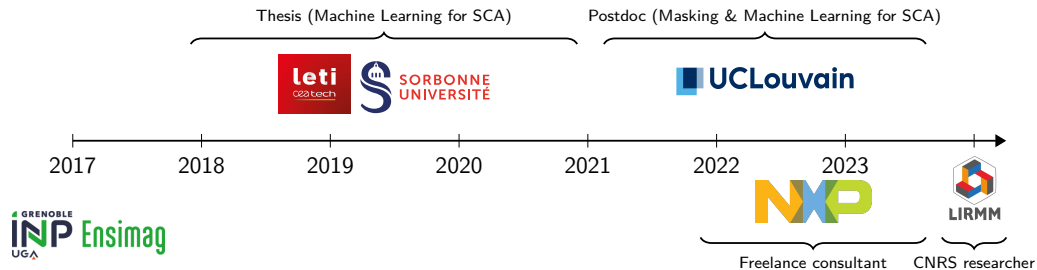
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Side-channel Analysis of Cryptographic Implementations



Who am I?



Agenda

Introduction: SCA

Device Certification

What is a Security Proof?

The Masking Countermeasure

Security Proof of Masking

Content

Introduction: SCA

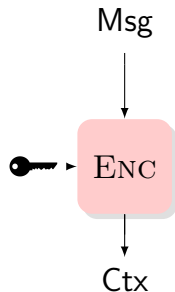
Device Certification

What is a Security Proof?

The Masking Countermeasure

Security Proof of Masking

Context : Side-Channel Analysis (SCA)

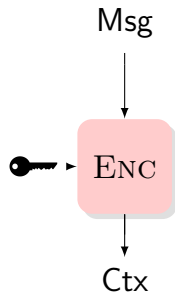


🔑: N bits ($\frac{N}{n}$ chunks of $n \ll N$ bits)

Black-box cryptanalysis: 2^N

Context : Side-Channel Analysis (SCA)

“Cryptographic algorithms don’t run on paper,

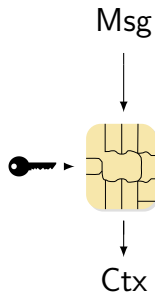


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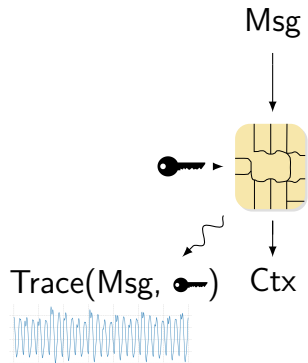


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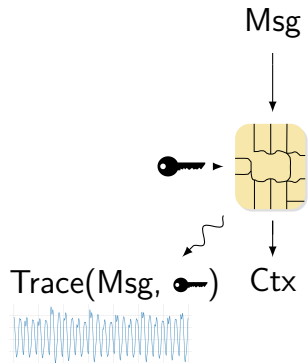


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🔑: N bits ($\frac{N}{n}$ chunks of $n \ll N$ bits)

Black-box cryptanalysis: 2^N

Divide-and-conquer: $2^n \cdot \frac{N}{n}$

\approx “quantum” break

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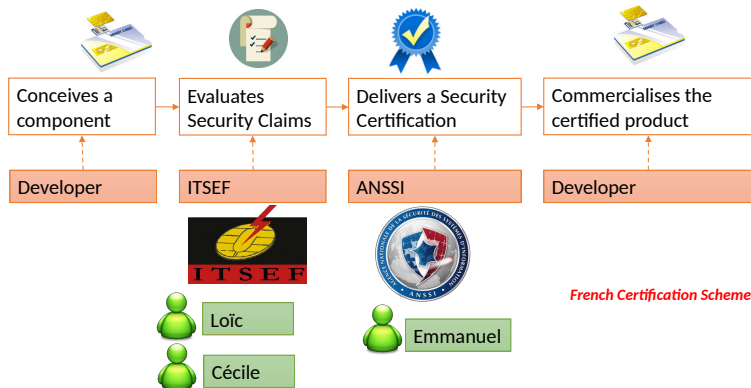
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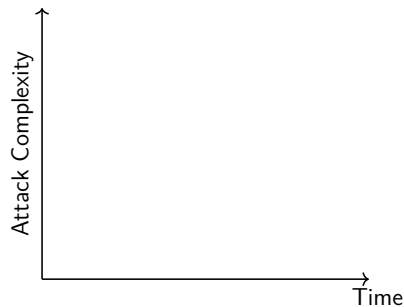
Security Proof of Masking

Certification against SCA



Security graded w.r.t. attack complexity in terms of human, material, and financial means

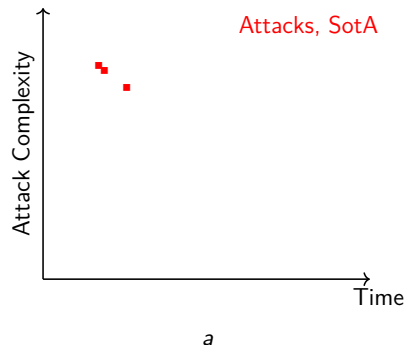
Evaluate Security against Side-Channel Attacks



Attack approach (industry):

^aShamelessly stolen to O. Bronchain

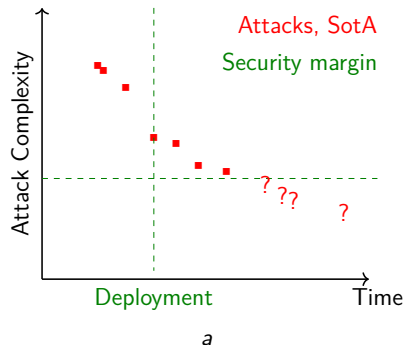
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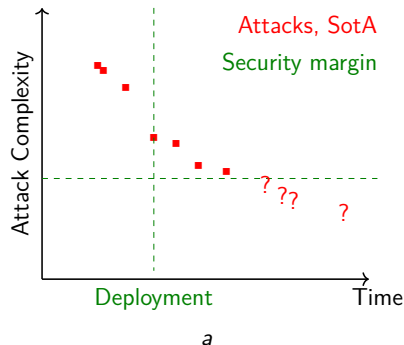
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Certification & deployment



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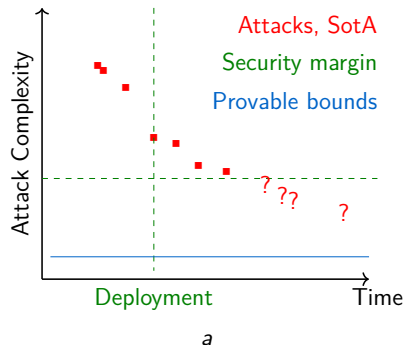
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Future improvement → reevaluation ✗

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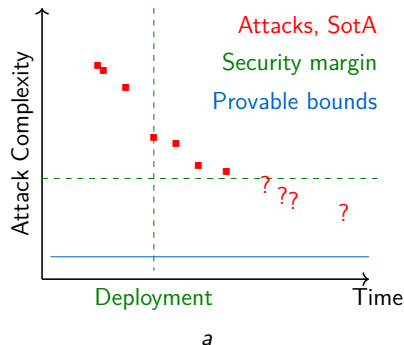
Approach by *proofs* (academia):

Rigorous approach ✓

Potentially conservative ✗

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Today's agenda: evaluation by proofs

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How to Evaluate Efficiently?

A good *evaluator* $\mathcal{E} \neq$ A good *adversary* \mathcal{A}

Security level:

Design-dependent ✓

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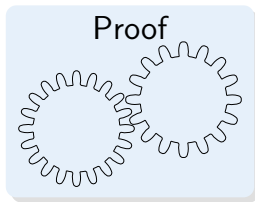
Design-dependent ✓

Device-dependent ✓

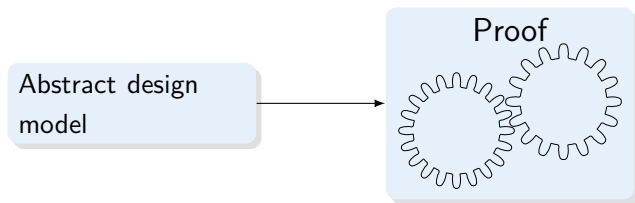
Adversary-dependent ✗

How to deal with this problem space?

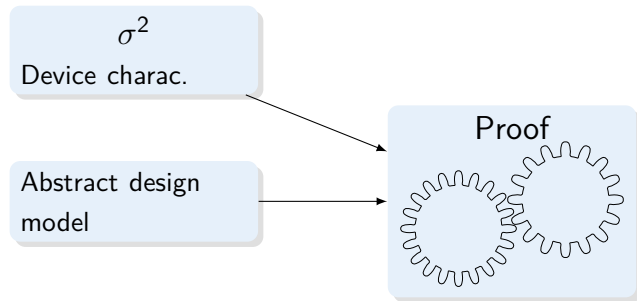
Security Proofs



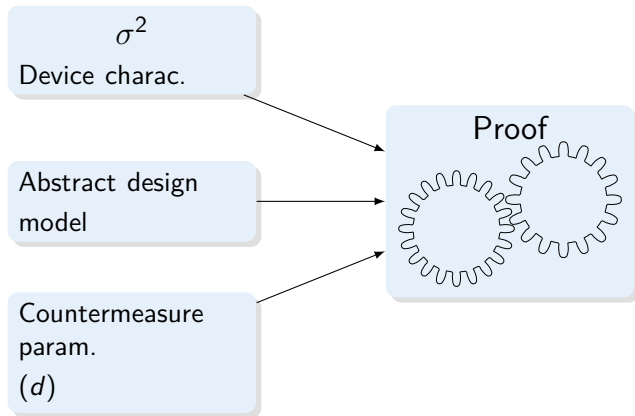
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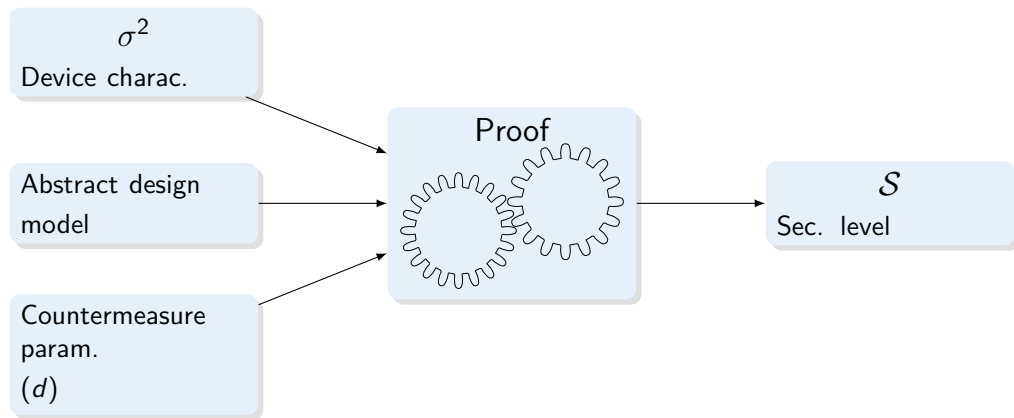
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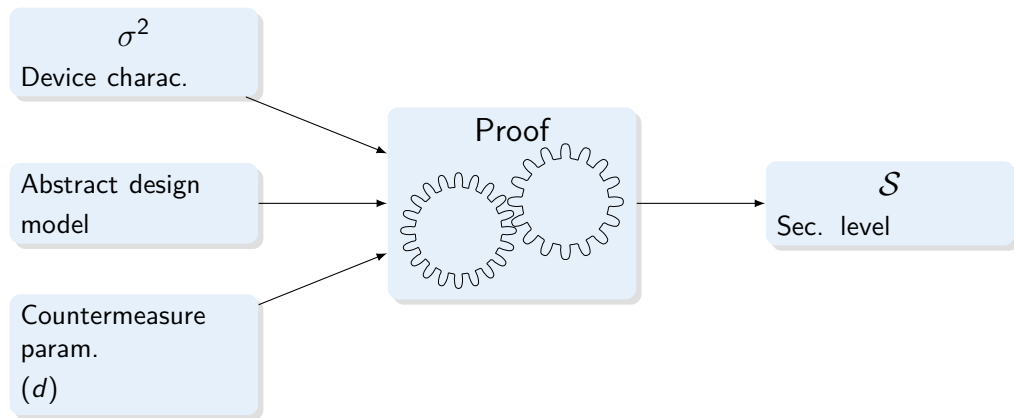
Security Proofs



Security Proofs



Security Proofs



“Any SCA attack requires at least \mathcal{S} queries”

Main Ingredient: Security Reductions

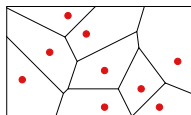


Figure: The set of all possible attacks : (adversary, leakages).

Reduction: “any attack from a given class is less powerful than the red-dot attack of the region”.

Main Ingredient: Security Reductions

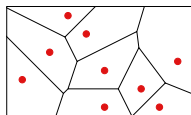


Figure: The set of all possible attacks : (adversary, leakages).

Reduction: “any attack from a given class is less powerful than the red-dot attack of the region”.

Data-processing inequality & Simulatability: If two attacks \mathcal{A}, \mathcal{B} are such that $\mathcal{A} = \mathcal{S}(\mathcal{B})$, then $\text{Success}(\mathcal{A}) \leq \text{Success}(\mathcal{B})$. Hence,

$$\max_{\mathcal{A} \in \mathcal{A}} \text{Success}(\mathcal{A}) \leq \max_{\mathcal{B} \in \mathcal{S}(\mathcal{A})} \text{Success}(\mathcal{B}).$$

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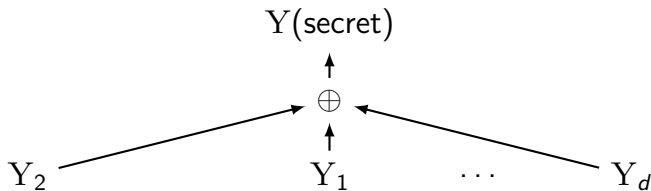
Security Proof of Masking

Masking: what is that ?

Masking, a.k.a. *MPC on silicon*:¹² secret sharing over a finite field $(\mathbb{F}, \oplus, \otimes)$
 $Y(\text{secret})$

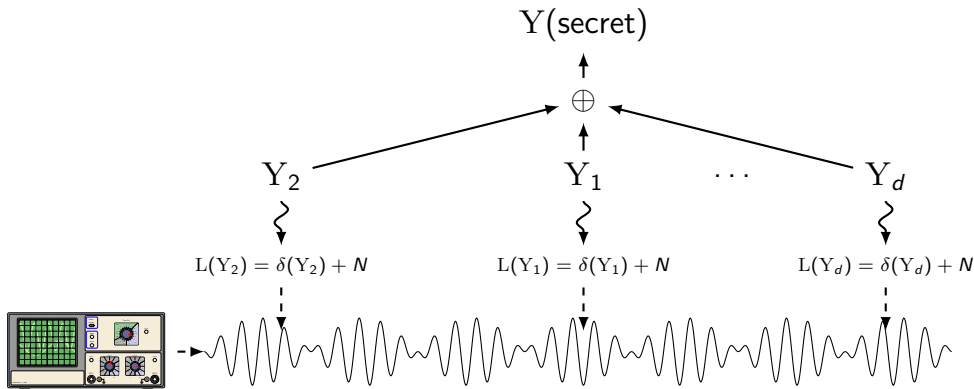
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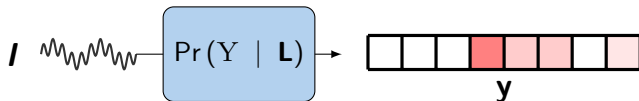


¹Chari et al., "Towards Sound Approaches to Counteract Power-Analysis Attacks".

²Goubin and Patarin, "DES and Differential Power Analysis (The "Duplication" Method)".

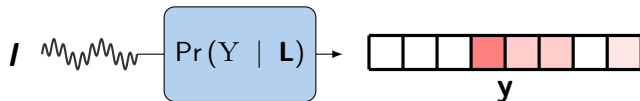
The Noisy Leakage Model

In this model, for each intermediate computation, the adversary gets a probability distribution about its operands:

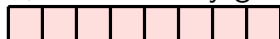


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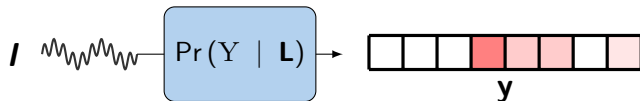


If, the adversary gets:

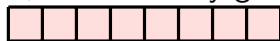


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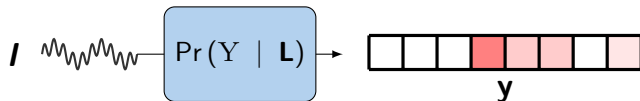


Very noisy

Sensitive computation unpredictable

The Noisy Leakage Model

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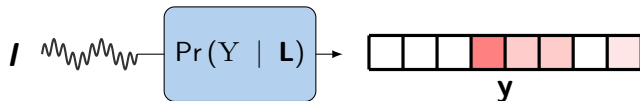


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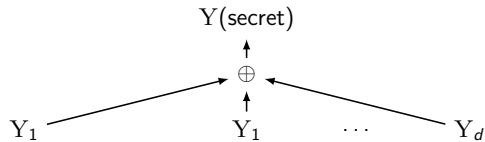
Low-noise

Exact prediction of the sensitive computation

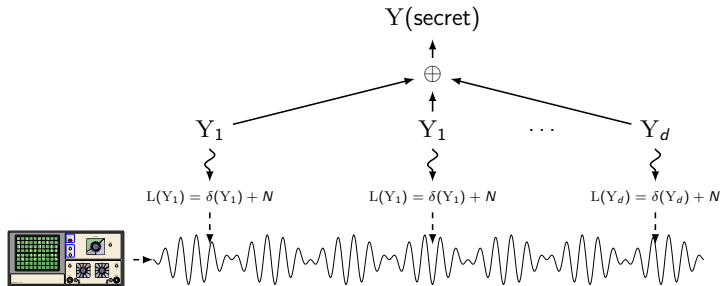
The Effect of Masking

Y(secret)

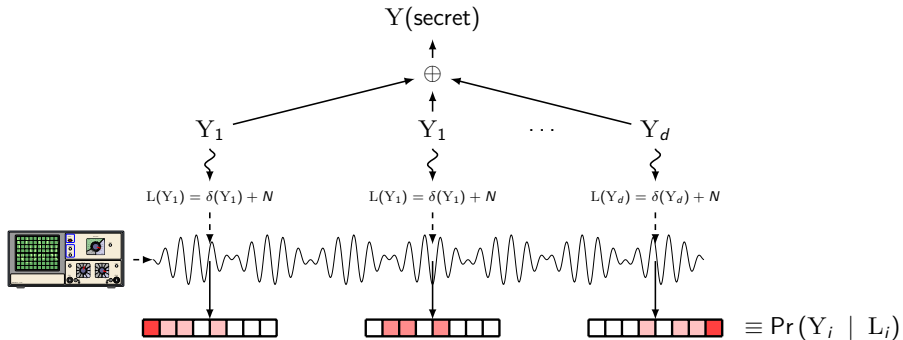
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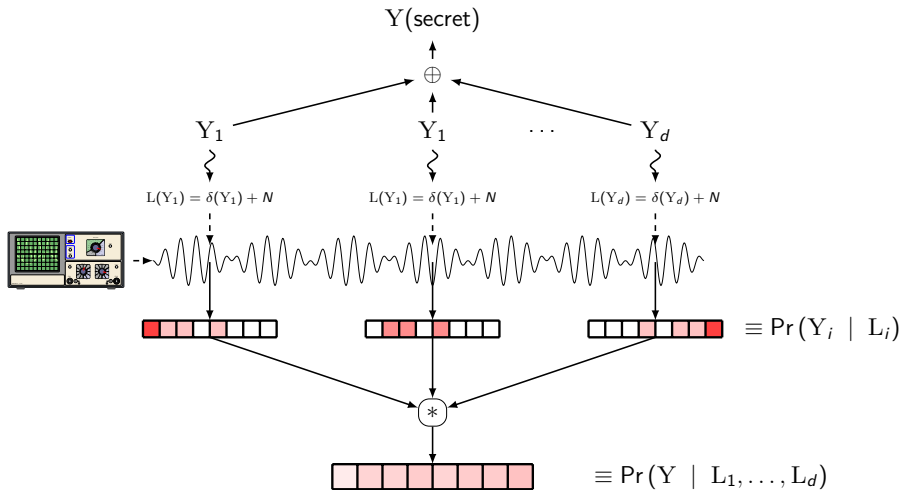
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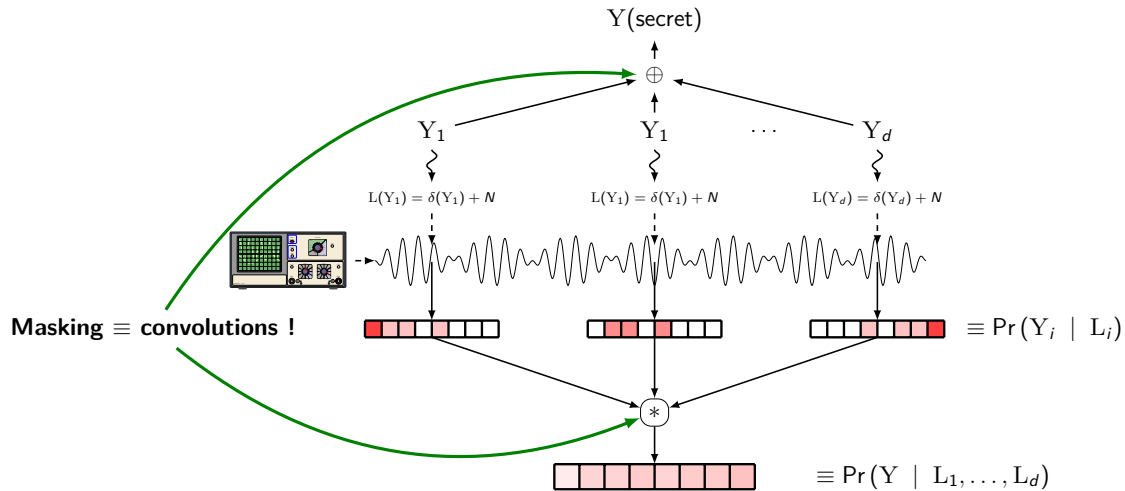
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Convolution = Noise Amplification

Simulation, for \mathbb{F}_{2^n} : $L(Y_i) = hw(Y_i) + \mathcal{N}(0; \sigma^2)$, hw = Hamming weight

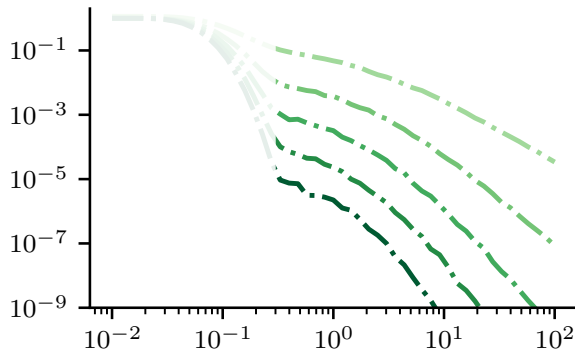


Figure: $MI(Y; \mathbf{L})$ vs. σ^2 , $2 \leq d \leq 6$

Content

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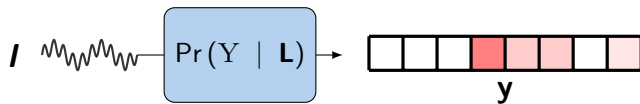
Device Certification

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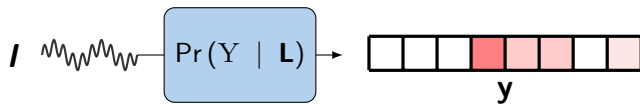
The Masking Countermeasure

Security Proof of Masking

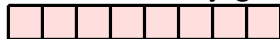
Recall on Noisy Leakage Model



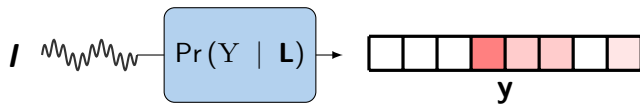
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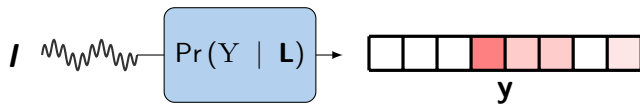
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Very noisy leakage

Y indistinguishable from blind guess

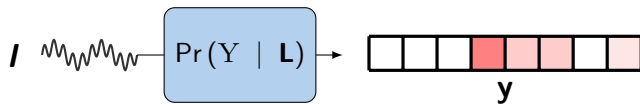
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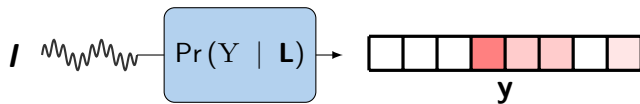
If, the adversary gets:



Low-noise leakage

Exact prediction for Y

Recall on Noisy Leakage Model



δ -NOISY ADVERSARY

Any intermediate computation Y leaks $L(Y)$ such that:

$$SD(Y; L) = \mathbb{E}_L \left[TV \left(\underbrace{\begin{array}{|c|c|c|c|c|c|c|c|} \hline \square & \square & \square & \text{red} & \text{light red} & \text{light red} & \square & \text{light red} \\ \hline \end{array}}_{\Pr(Y | L)}, \underbrace{\begin{array}{|c|c|c|c|c|c|c|c|} \hline \text{light red} & \text{light red} & \text{light red} & \text{light red} & \text{light red} & \text{light red} & \text{light red} & \text{light red} \\ \hline \end{array}}_{\Pr(Y)} \right) \right] \leq \delta$$

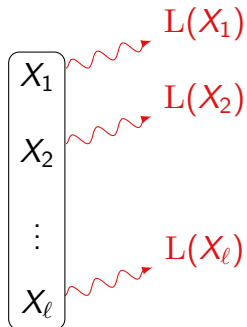
Security Proof for a Circuit

Consider a circuit with ℓ intermediate computations:

$$\begin{pmatrix} X_1 \\ X_2 \\ \vdots \\ X_\ell \end{pmatrix}$$

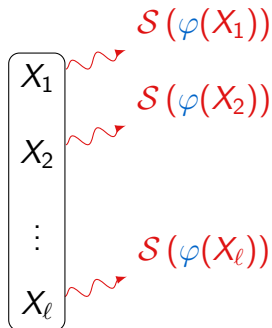
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Consider a circuit with ℓ δ -noisy intermediate computations:



Security Proof for a Circuit

Consider a circuit with ℓ δ -noisy intermediate computations:



LEMMA (SIMULATABILITY)

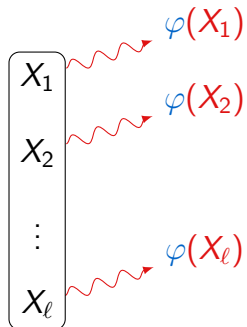
The leakage function L can be simulated from a *random probing adversary*: $\varphi(x)$ exactly reveals x with probability

$$\epsilon = 1 - \sum_l \min_x \Pr(L(x) = l) \leq \delta \cdot |\mathbb{F}|.^a$$

^aDuc, Dziembowski, and Faust, “Unifying Leakage Models: From Probing Attacks to Noisy Leakage”.

Security Proof for a Circuit

Consider a circuit with ℓ δ -noisy intermediate computations:



We may reduce to an adversary observing $\varphi(X)$ instead of $\mathcal{S}(\varphi(X))$ (Data Processing Inequality)

Security against a Random Probing Adversary

To succeed, at least d out of ℓ wires must be revealed to the adversary:

$$\Pr(\text{Adv. learns sth}) \leq \Pr(\text{At least } d \text{ wires revealed})$$

³Boucheron, Lugosi, and Massart, *Concentration Inequalities: A Nonasymptotic Theory of Independence*, P.24, and Ex. 2.11.

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THEOREM (CHERNOFF CONCENTRATION INEQUALITY³)

If ℓ wires, each independently revealed with proba. ϵ :

$$\Pr(\text{At least } d \text{ wires revealed}) \leq \left(\frac{e \cdot \ell \cdot \epsilon}{d} \right)^d$$

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Putting all Together

In our context, $\ell \leq \mathcal{O}(d^2)$, and $\epsilon \leq \delta \cdot |\mathbb{F}|$:

THEOREM (SECURITY BOUND)

For a single computation with $\ell \leq \mathcal{O}(d^2)$ gates:

$$\text{SD}(k; \mathbf{L}) \leq (\mathcal{O}(d) \cdot \delta \cdot |\mathbb{F}|)^d$$

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For the whole circuit \mathbb{C} , (work in progress),

$$\text{SD}(k; \mathbf{L}) \leq (\mathcal{O}(|\mathbb{C}|d) \cdot \delta \cdot |\mathbb{F}|)^d$$

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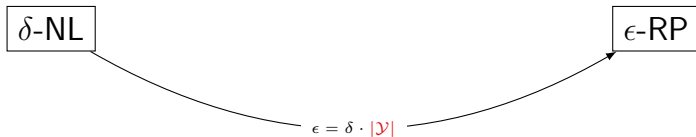
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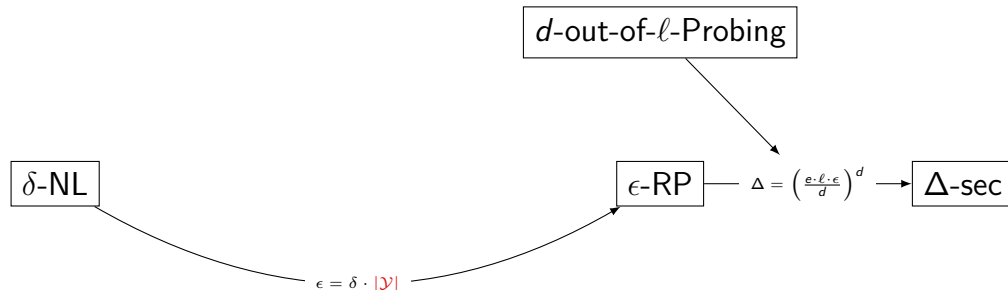
Wrap-Up of the Proof

δ -NL

Wrap-Up of the Proof



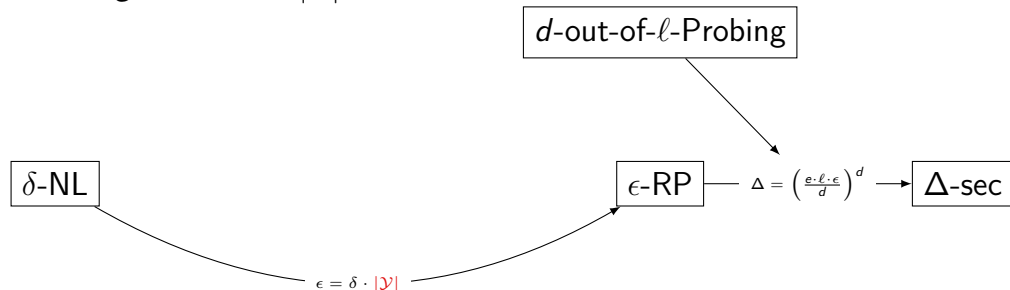
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⁴Brian, Dziembowski, and Faust, “From Random Probing to Noisy Leakages Without Field-Size Dependence”.

Wrap-Up of the Proof

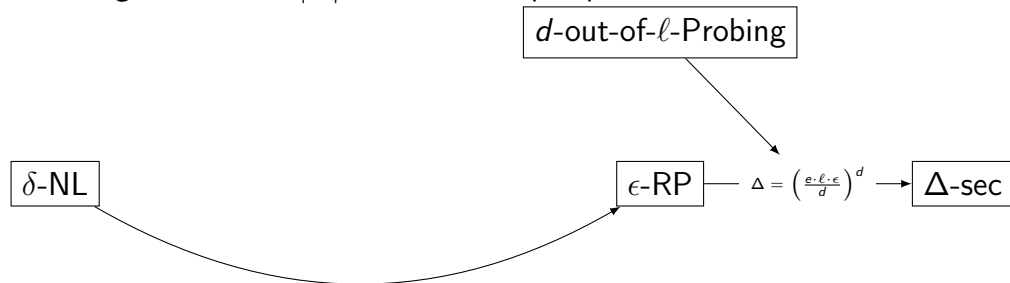
Bad *leakage rate* $\approx d \cdot |\mathbb{F}|$ **X**...



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Wrap-Up of the Proof

Bad *leakage rate* $\approx d \cdot |\mathbb{F}|$ ✗... but new perspectives⁴ ✓



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Perspectives

- Improving the reduction from Noisy Leakages to Random Probing

⁵Belaïd, Rivain, and Taleb, “On the Power of Expansion: More Efficient Constructions in the Random Probing Model”.

Perspectives

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⇒ wider gap between d -probing and ϵ -RP ✗



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Perspectives

- Improving the reduction from Noisy Leakages to Random Probing
- New constructions with leakage rates indep. of d^5
- Masking PQC, e.g., Kyber:
 - ★ Unefficient masking through decomposition into circuit ✗
 - ★ Needs bigger gadgets with other paradigm: pre-computation tables ✓
⇒ wider gap between d -probing and ϵ -RP ✗
 - ★ Masking-friendly schemes, e.g., Raccoon ? ✓

⁵Belaïd, Rivain, and Taleb, “On the Power of Expansion: More Efficient Constructions in the Random Probing Model”.



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