



Au Bal des Implémentations Masquées

Quand et comment le masquage peut protéger contre les attaques par canaux auxiliaires

Loïc Masure

Séminaire LJK, Grenoble, 23 Mai 2024



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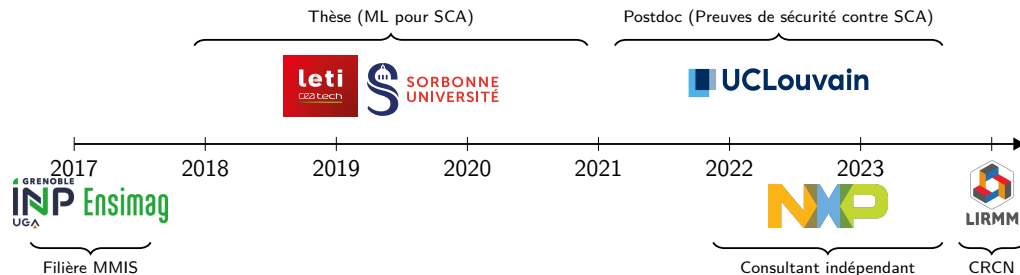


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UNIVERSITÉ DE
MONTPELLIER

Qui suis-je ?



Content

Introduction: SCA

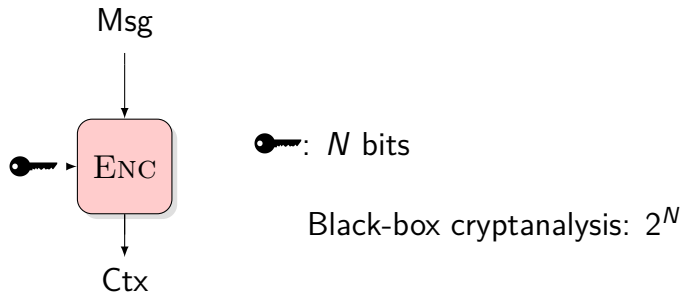
Masking an Implementation

The Effect of Masking

Masking in Prime Fields

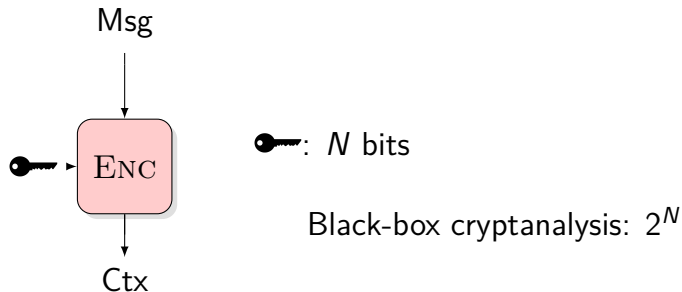
Conclusion

Context : Side-Channel Analysis (SCA)



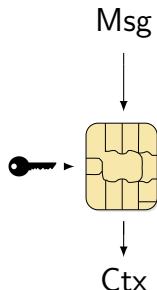
Context : Side-Channel Analysis (SCA)

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“Cryptographic algorithms don’t run on paper, they run on physical devices”

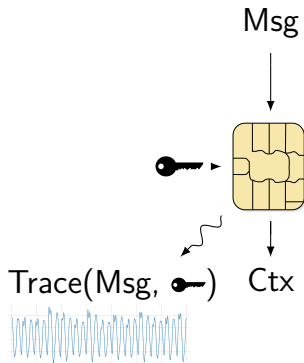


key: N bits

Black-box cryptanalysis: 2^N

Context : Side-Channel Analysis (SCA)

“Cryptographic algorithms don’t run on paper, they run on physical devices”



Black-box cryptanalysis: 2^N

Side Channel = Unintended Communication Channel

Example: the Washington Pizza Index¹

NEWS

CRUSTY D.C. VETERAN SAYS WAR IS NEAR

By Cox News Service
Chicago Tribune • Published: Jan 16, 1991 at 12:00 am



WASHINGTON — The pizza index indicates military action is imminent in the Persian Gulf, a Domino's delivery official said Tuesday.

Record numbers of late-night pizzas have been delivered this week to the White House, Pentagon and State Department, said Frank Meeks, owner of several Washington-area Domino's outlets.

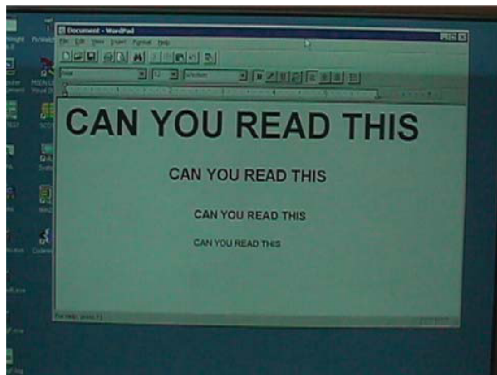
Similar order patterns came immediately before the invasions of Panama and Grenada, Meeks said.

The increase in pizza orders at key government buildings after 10 p.m. is "very unusual," Meeks said. "I don't think they're sitting around watching Redskins reruns."

Figure: Chicago Tribune, **Jan. 16 1991**, the day before *Desert Storm* operation began.

¹Reality questioned: <http://home.xnet.com/~warinner/pizzacites.html>

What is a Side Channel? A First Example



(a) A good old monitor



(b) Reconstruction from EM field

Figure: An example from Koç, *Cryptographic Engineering*.

An Exemplary SCA on Crypto

RSA: Modular exponentiation over large ($\approx$ 2000-bit wide) integers

An Exemplary SCA on Crypto

RSA: Modular exponentiation over large (≈ 2000 -bit wide) integers

Square-and-Multiply:

$$M^k = M^{\sum_i k_i \cdot 2^i} = \prod_i (M^{k_i})^{2^i}$$

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Step 0: M^{k_N} then **square**

Step 1: $\times M^{k_{N-1}}$ then **square**

...

Step i: $\times M^{k_{N-i}}$ then **square**

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Op.	Guess
$\times M$	
square	
square	
square	
$\times M$	
square	
$\times M$	
square	

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Op.	Guess
$\times M$ square	$k_N = 1$
square square square	$k_{N-1} = 0$
$\times M$ square	
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$\times M$ square	$k_{N-4} = 1$

Can you guess the key from the Oscilloscope?

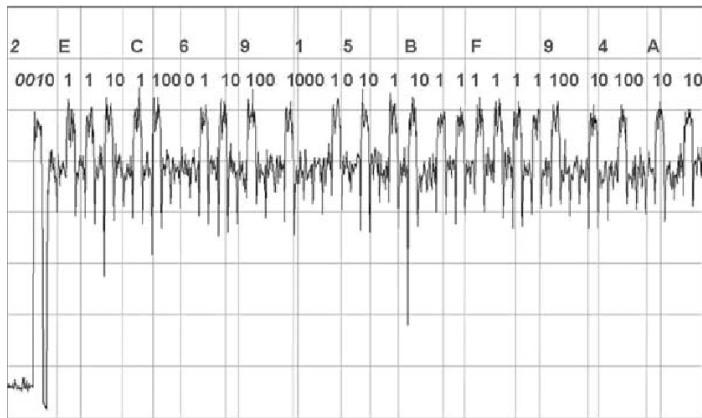


Figure: Power consumption. Illustration from Koç, *Cryptographic Engineering*

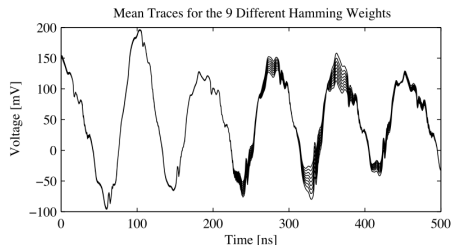
Power Analysis on Symmetric Key

Power consumption: each bit x_i of a data chunk X stored in a register²

$x_i = 0 \implies$ register voltage = 0

$x_i = 1 \implies$ register voltage $\neq 0$

Overall consumption of X is $\propto \sum_i x_i$

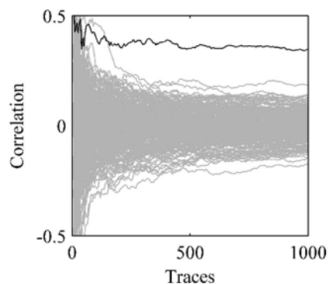
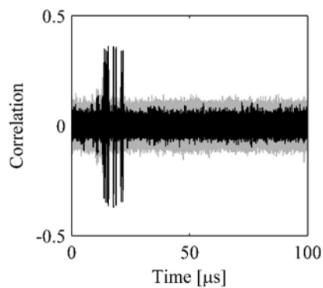


²Mangard, Oswald, and Popp, *Power analysis attacks - revealing the secrets of smart cards*.

Power Analysis on Symmetric Key

Power consumption: each bit x_i of a data chunk X stored in a register²

Key guessed by a statistical test leveraging the correlation between the Hamming weight of data and the power consumption



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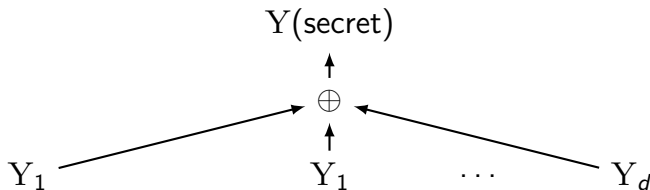
Masking: what is that ?

Masking, aka *MPC on silicon*: linear secret sharing over a finite field $(\mathbb{F}, \star, \cdot)$
 $Y(\text{secret})$

Introduced by Chari *et al.*, Goubin & Patarin (Crypto, Ches 99)

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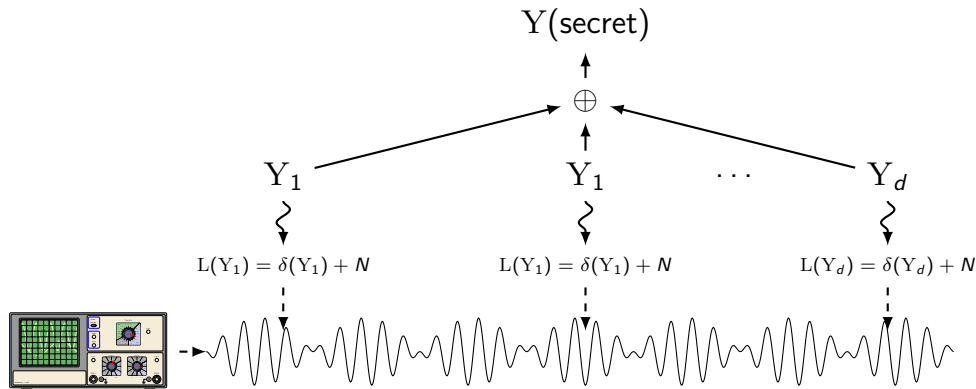
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How to Calculate over Masked Data? Outline

Over finite field $\mathbb{F}$, write each operation as a polynomial, *i.e.*, a sequence of

³If deemed useful, I have some appendix for the proof in the general case :-)

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Over finite field $\mathbb{F}$, write each operation as a polynomial, *i.e.*, a sequence of

- $\mathbb{F}$ -affine functions (*e.g.*, $\oplus$): $f(\sum_i Y_i) = \sum_i f(Y_i)$;

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 - Spans d^2 shares
 - Needs to *securely* compress into d shares.
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In this talk we only focus on the leakage of *one* d -sharing only³

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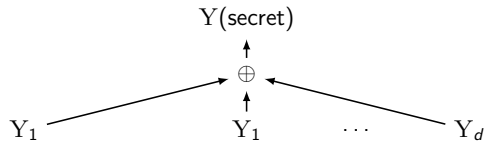
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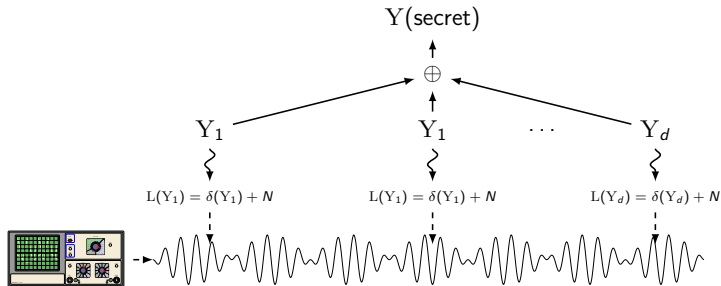
The Effect of Masking

Y(secret)

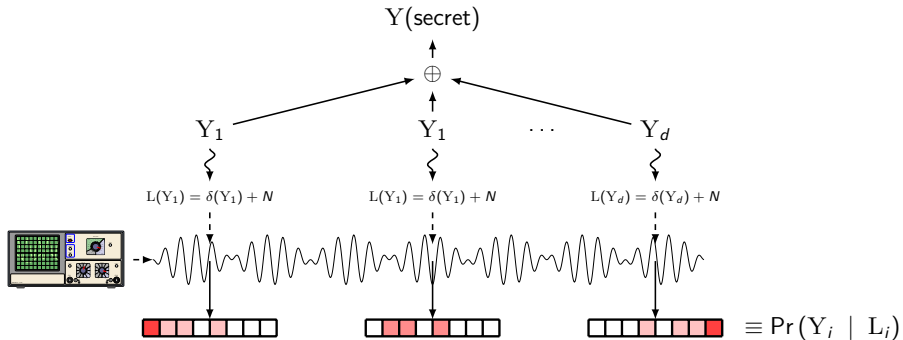
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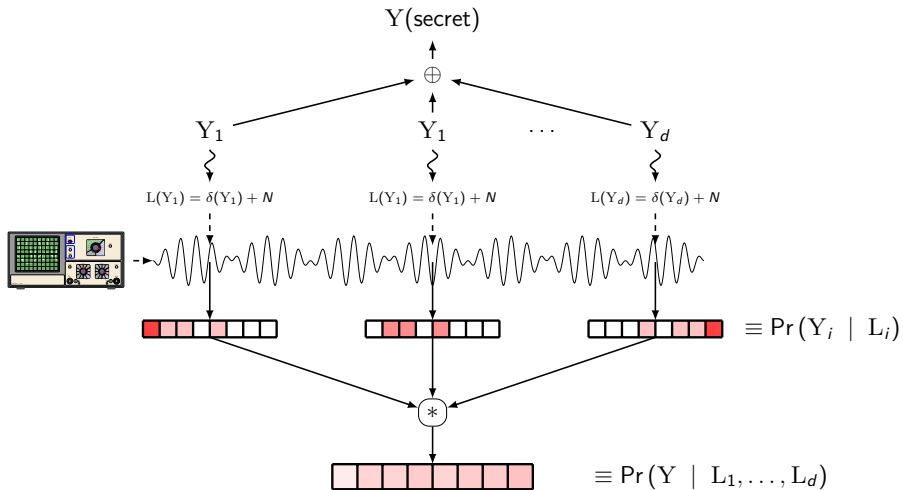
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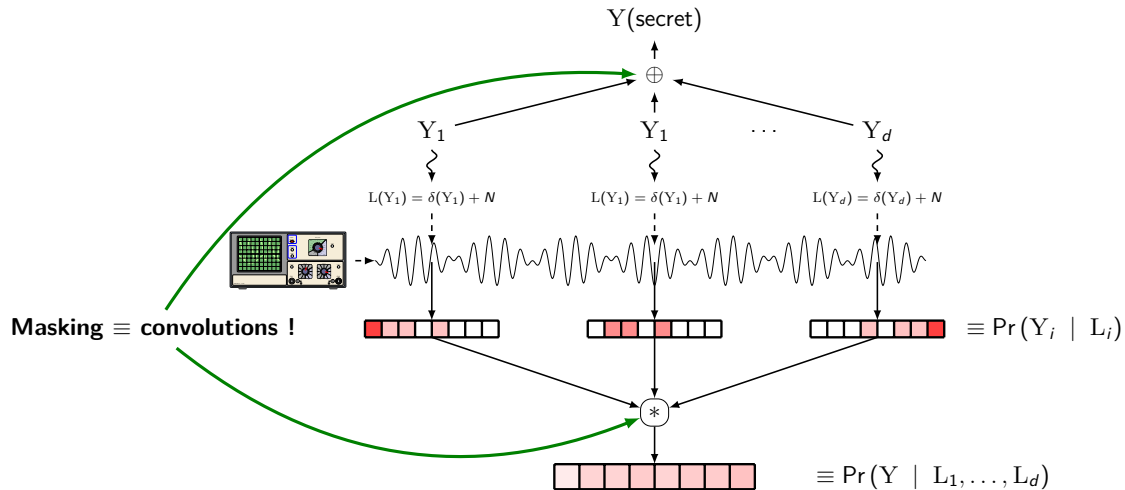
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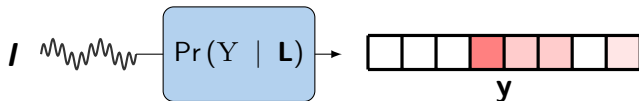


The Effect of Masking



The Noisy Leakage Model

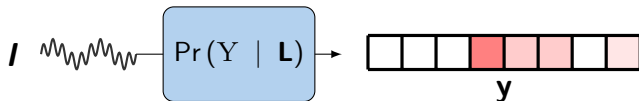
In this model, for each intermediate computation, the adversary gets a probability distribution about its operands:



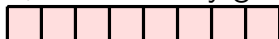
⁴ D : Kullback-Leibler (KL) divergence, total variation, Euclidean norm, ...

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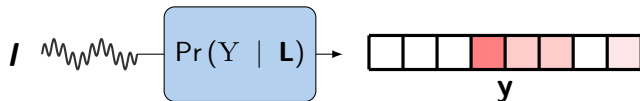
If, the adversary gets:



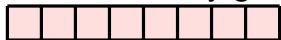
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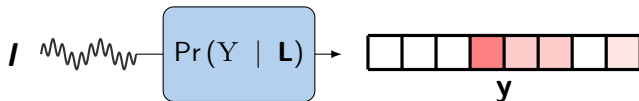
Very noisy

Sensitive computation unpredictable

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The Noisy Leakage Model

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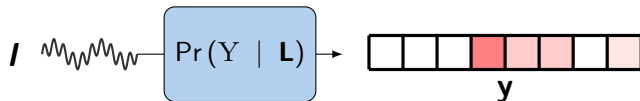
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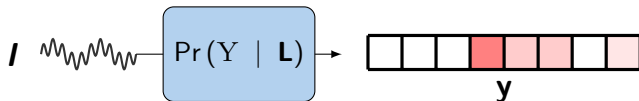
Low-noise

Exact prediction of the sensitive computation

⁴ D : KL divergence, total variation, Euclidean norm, ...

The Noisy Leakage Model

In this model, for each intermediate computation, the adversary gets a probability distribution about its operands:



δ -NOISY ADVERSARY

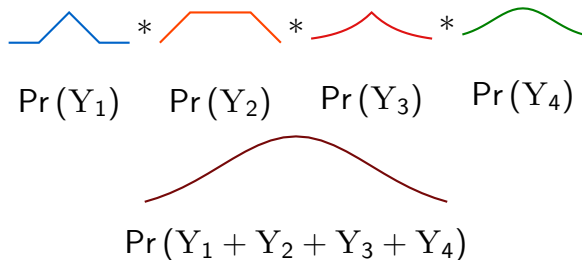
All the p.m.f.s accessed by the adversary are δ -close⁴ to the uniform:

$$D \left(\begin{array}{|c|c|c|c|c|c|c|c|} \hline \square & \square & \square & \text{red} & \text{light red} & \text{light red} & \square & \text{light red} \\ \hline \end{array}, \begin{array}{|c|c|c|c|c|c|c|c|} \hline \text{light red} & \text{light red} & \text{light red} & \text{light red} & \text{light red} & \text{light red} & \text{light red} & \text{light red} \\ \hline \end{array} \right) \leq \delta$$

⁴ D : KL divergence, total variation, Euclidean norm, ...

The Secret Power of Convolutions

Central Limit Theorem: Assume real-valued random variables Y_i



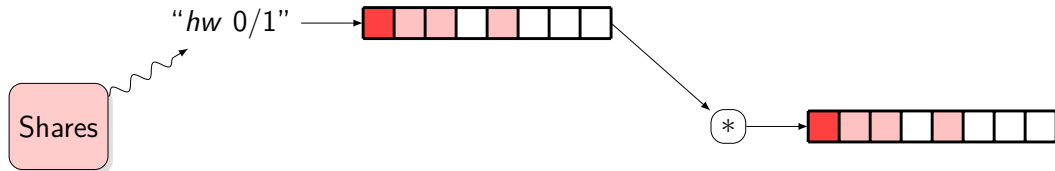
Then the sum is (approximately) distributed like a Gaussian⁵

Interesting property of Gaussian: maximizes the entropy (*i.e.*, uncertainty)⁶

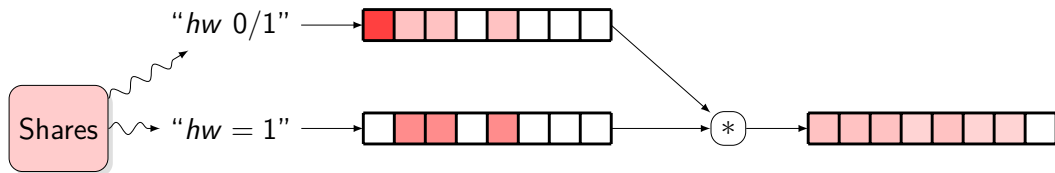
⁵With mild assumptions, but we'll get back to that ...

⁶Out of all Probability Density Functions (p.d.f.s) of same mean and variance

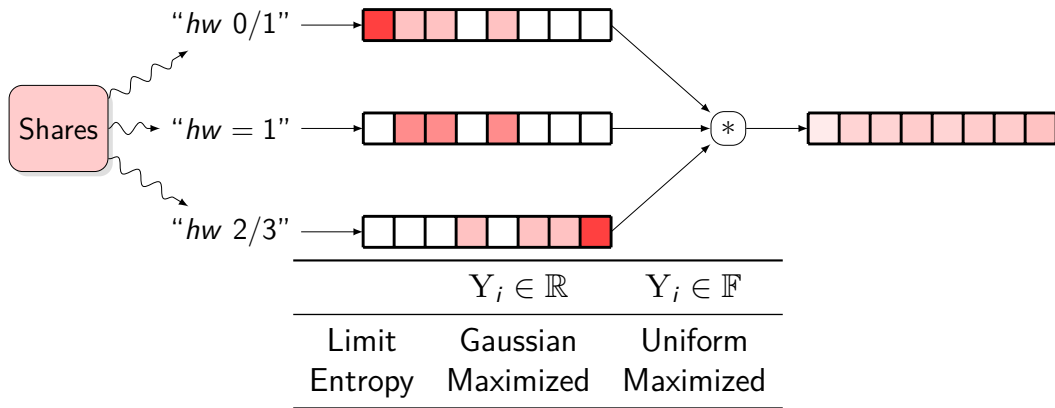
CLT also Works in Finite Groups/Fields !



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Fast Fourier Transform also apply over finite fields !

Quantitative version of CLT

THEOREM (MRS. GERBER'S LEMMA⁷)

Given $Y = Y_1 \oplus \dots \oplus Y_d$, and each Y_i with (indep.) side information $L_1, \dots, L_d$, then for $\eta^{-1} = 2 \log(2)$:

⁷Béguinot et al., “Removing the Field Size Loss from Duc et al.’s Conjectured Bound for Masked Encodings”.

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$$\text{MI}(Y; \mathbf{L}) \leq \prod_{i=1}^d \frac{\text{MI}(Y_i; L_i)}{\eta} + \mathcal{O} \left(\prod_{i=1}^d \text{MI}(Y_i; L_i)^2 \right) \text{ in } \mathbb{F}_{2^n}$$

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→ Security $\propto \frac{1}{\text{MI}(Y; \mathbf{L})} \implies$ increases **exponentially fast** with d ✓

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→ Independent of the adversary ✓

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Convolution = Noise Amplification

Simulation, for $\mathbb{F}_{2^n}$: $L(Y_i) = hw(Y_i) + \mathcal{N}(0; \sigma^2)$, hw = Hamming weight

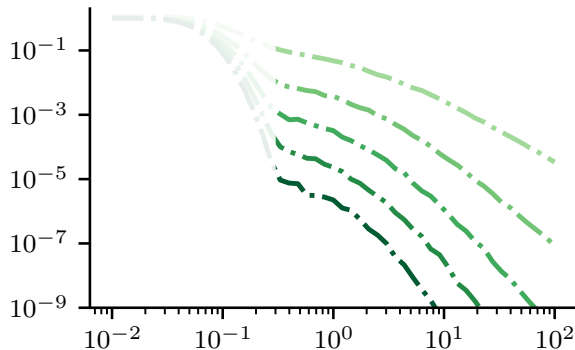
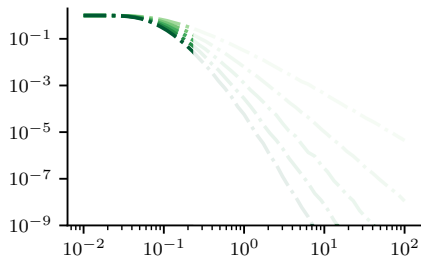


Figure: $MI(Y; L)$ vs. σ^2 , $2 \leq d \leq 6$

Masking in a Low-Noise Setting

Does masking always work in a low-noise setting ?



Observation:

Secret always leaks > 1 bit, regardless of d

Explanation:

$$\text{lsb}(Y_1 \oplus \dots \oplus Y_d) = \text{lsb}(Y_1) \oplus \dots \oplus \text{lsb}(Y_d)$$

Figure: $MI(Y; \text{Trace})$ vs. σ^2 , $2 \leq d \leq 6$

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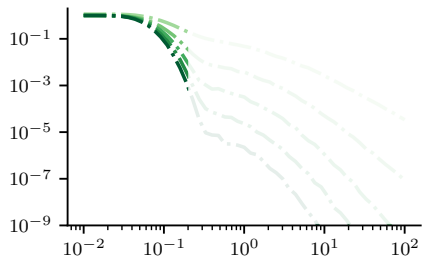


Figure: $MI(Y; \text{Trace})$ vs. σ^2 , $2 \leq d \leq 6$

Observation:

Secret always leaks > 1 bit, regardless of d

Explanation:

$$\text{hw}(Y_1 \oplus \dots \oplus Y_d) = \sum_i \text{hw}(Y_i) - 2 \cdot (\dots)$$

Parity of $\text{hw}(Y)$: **cosets of $\mathbb{F}_{2^n}$**

Corollary: parallelism is no cure either

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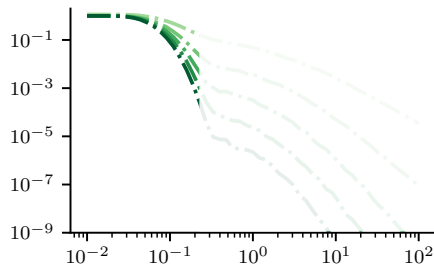
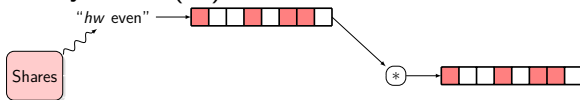


Figure: $MI(Y; \text{Trace})$ vs. σ^2 , $2 \leq d \leq 6$

Explanation:

Parity of $\text{hw}(Y)$: **cosets of $\mathbb{F}_{2^n}$**



Masking in a Low-Noise Setting

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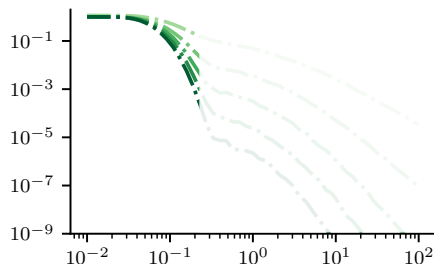
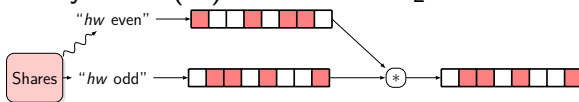


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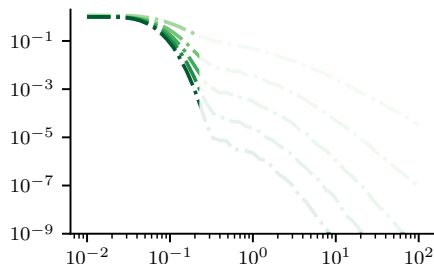
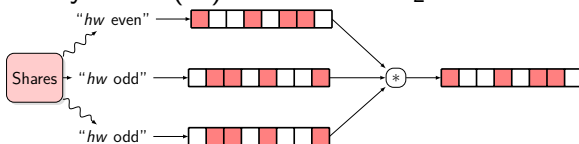


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Conditions for Sound Masking

What conditions the distributions  of each share must fit?

⁸Stromberg, “Probabilities on a Compact Group”.

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“CENTRAL LIMIT THEOREM” (QUALITATIVE)⁸

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In $\mathbb{R}$: mild assumption

→ Only $\mathbb{Z}$ and $\mathbb{Q}$ (and their respective subgroups)

→ Negligible measure over $\mathbb{R}$

In finite $\mathbb{F}$: no longer mild in finite fields ...

⁸Stromberg, “Probabilities on a Compact Group”.

Two Solutions

Two Solutions

Solution 1: Make sure to leak < 1 bit per share:

- Support of PMF always larger than any coset
- Work with any $\mathbb{F}$ (usually chosen to fit the cipher) ✓
- **Leakage-dependent: not always verified** ✗

Two Solutions

Solution 2: Choose $\mathbb{F}$ without any non-trivial subgroup, *i.e.*, $\mathbb{F}_p$, p prime:

- No assumption on the leakage ✓
- Major change of paradigm:
 - Fix $\mathbb{F}$ masking-friendly first,
 - Then build crypto upon it ✓

Comparing Binary and Prime Fields: a Simulation

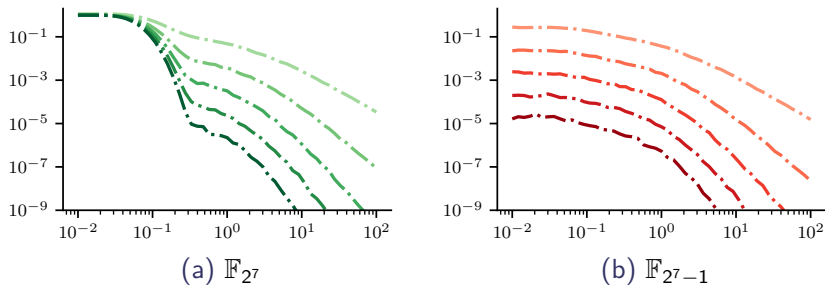


Figure: Comparing binary and prime fields.

Content

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How to leverage?

Q: How can we make use of masking in $\mathbb{F}_p$ to effectively and efficiently protect crypto implementations?

A: Ideally, we need algorithms that work in implementation-friendly prime fields, such as **small-Mersenne-prime fields** ($\mathbb{F}_{2^n-1}$), and use only simple field arithmetic ($+$, $-$, $\cdot$)

Complex in Software? Not really!

Field Addition in $\mathbb{F}_{2^n-1}$ in C/C++ and ARM Assembly ($c = a + b \bmod p$)

<code>c = a+b;</code>	<code>ADD r0,r0,r1</code>
	<code>UBFX r1,r0,#0,#n</code>
<code>c = (c & p) + (c >> n);</code>	<code>ADD r0,r1,r0,ASR #n</code>

Field Multiplication in $\mathbb{F}_{2^n-1}$ in C/C++ and ARM Assembly ($c = a \cdot b \bmod p$)

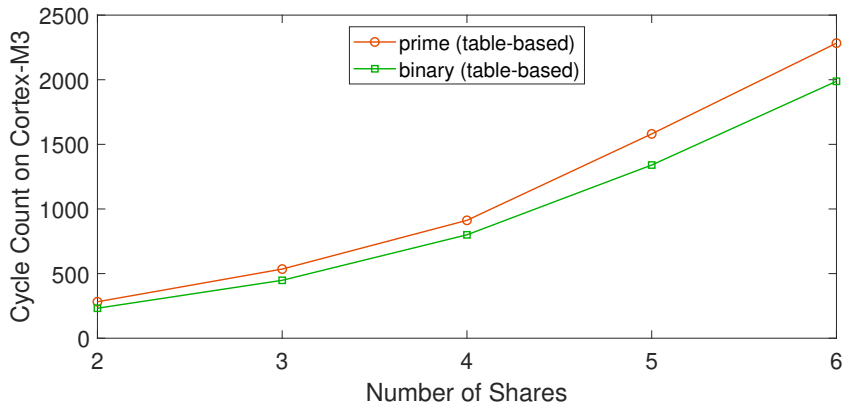
<code>c = a*b;</code>	<code>MUL r0,r1,r0</code>
	<code>UBFX r1,r0,#0,#n</code>
<code>c = (c & p) + (c >> n);</code>	<code>ADD r0,r1,r0,ASR #n</code>
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→ Only works for sufficiently small integers (< 16 bit for multiplication operands on ARM Cortex-M3)

→ If $c < p$ is strictly needed for the addition result, then $c \stackrel{?}{=} p$ needs to be checked after reduction

Software Case Study: Masked S-box

Naive implementation of masked $x^5 + 2$ using 3 consecutive ISW multiplications:

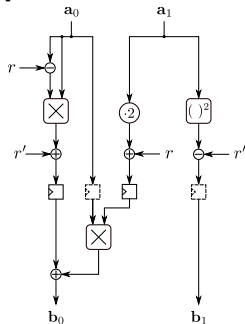


Dealing with Non-Linearity

In $\mathbb{F}_p$, every $\mathbb{F}_2$ -linear mapping, e.g. $\cdot^2$, becomes non-linear ✗

Ches 23: new masked squaring more efficient than naive multiplication⁹

Eurocrypt 24: serves as a building block for a Generalized Feistel cipher¹⁰



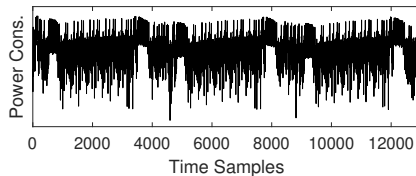
In $\mathbb{F}_{2^n-1}$, $2 \cdot x$: cyclic shift of the bits

- Almost free in hardware
- Interesting property for later ...

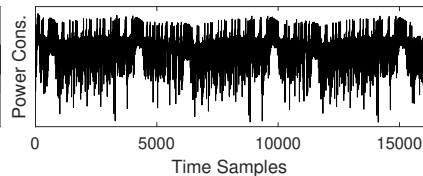
⁹Cassiers et al., “Prime-Field Masking in Hardware and its Soundness against Low-Noise SCA Attacks”.

¹⁰Grassi et al., “Generalized Feistel Ciphers for Efficient Prime Field Masking”.

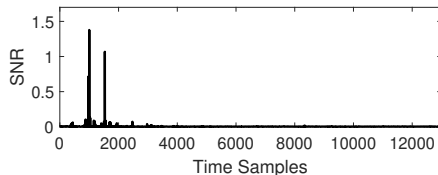
Masked $x^5 + 2$ (naive) in Software, Log/Alog tables



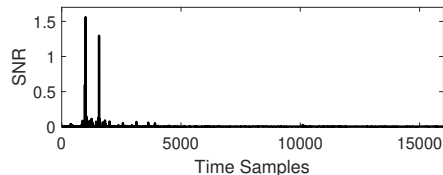
(a) Cortex-M3 sample trace, $\mathbb{F}_{2^7}$.



(b) Cortex-M3 sample trace, $\mathbb{F}_{2^7-1}$.

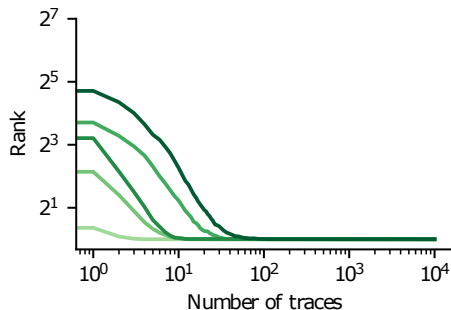
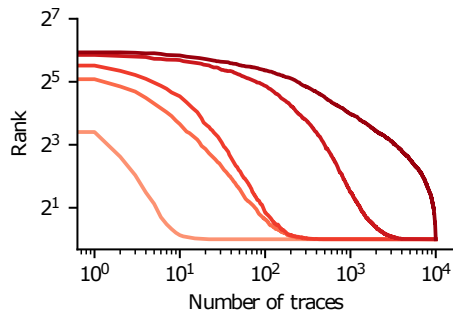


(c) SNR of input share 0, $\mathbb{F}_{2^7}$.

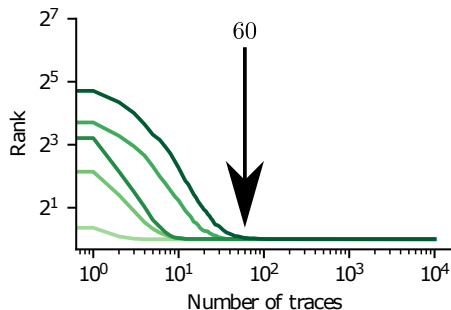
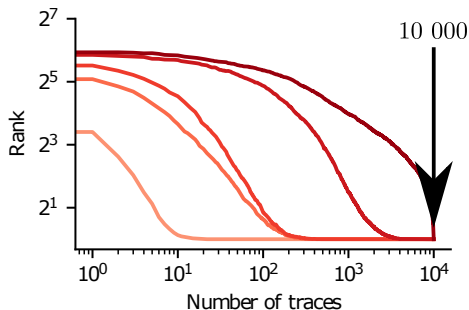


(d) SNR of input share 0, $\mathbb{F}_{2^7-1}$.

Software, Horizontal SASCA Attack for 2-6 Shares

(a) $\mathbb{F}_{2^7}$ (b) $\mathbb{F}_{2^7-1}$

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 - Largest encoding within n bits
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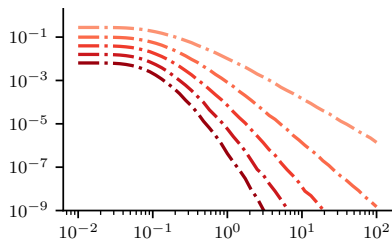
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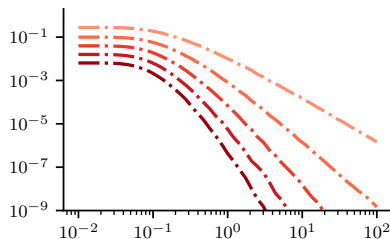
- Prime characteristic, for leakage resilience
- Size of a Mersenne number $2^n - 1$ for implementation efficiency
 - Largest encoding within n bits
 - Nice implementation for modulo reductions, for $\times 2, \dots$
- **What about the size of Mersenne prime p ?**

What is the Effect of Field Size ?

LSB = Least Significant Bit. One bit leaked on every share.



(a) LSB, $n = 7$.



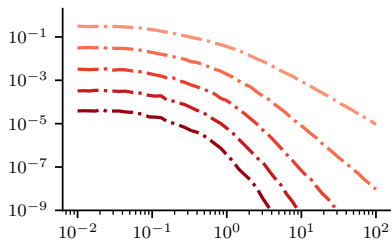
(b) LSB, $n = 13$.

Figure: MI vs. σ^2 , for LSB.

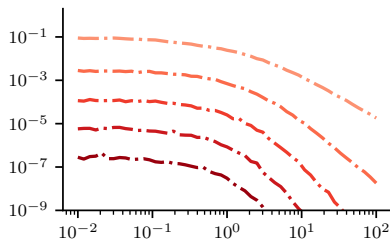
Observation: no effect of the field size $\times$

What is the Effect of Field Size ?

HW = Hamming Weight. $\approx \log(n)$ bits leaked on every share.



(a) HW, $n = 7$.



(b) HW, $n = 13$.

Figure: MI vs. σ^2 , for HW.

Observation: increasing the field size helps resilience ✓

Wait a Minute ...

“CENTRAL LIMIT THEOREM” (QUANTITATIVE)¹¹

If each share is δ -leaky, for $\delta < 1$, then the secret is $\mathcal{O}(\delta^d)$ -leaky.

First Intuition: “*the leakier the shares, the leakier the masked secret*”

¹¹Béguinot et al., “Removing the Field Size Loss from Duc et al.’s Conjectured Bound for Masked Encodings”; Dziembowski, Faust, and Skórski, “Optimal Amplification of Noisy Leakages”.

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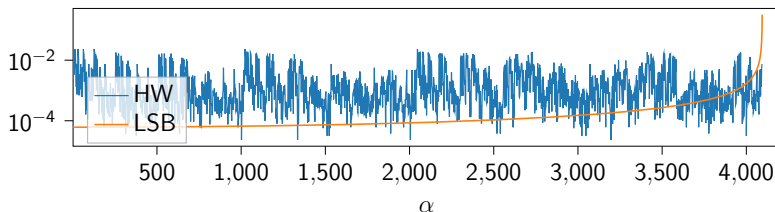
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Masking $\equiv$ Convolution

Masking $\equiv$ Convolution $\equiv$ Fourier Analysis

Poisson summation formula:

“The leakage-resilience can be read in the maximum amplitude of the Fourier spectrum”



Fourier Analysis for LSB

Related works¹² and ours show secret to be $\Theta\left(\left(\frac{2}{\pi}\right)^d\right)$ -leaky

Independent of p !

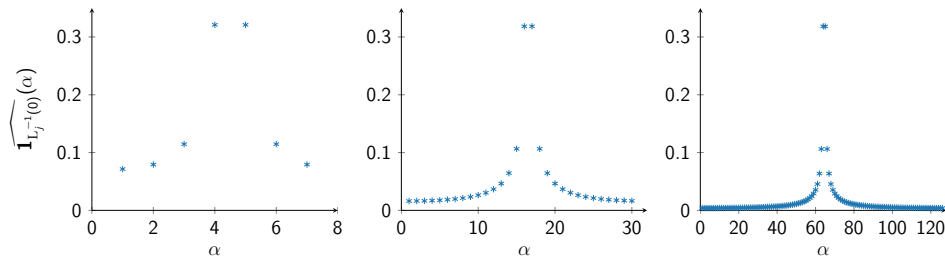


Figure: α vs $\widehat{\mathbf{1}_{L_j^{-1}(0)}}(\alpha)$ for $\alpha \in \mathbb{F}^*$ in the LSB leakage model, for $p = 7, 31, 127$

¹²Benhamouda et al., “On the Local Leakage Resilience of Linear Secret Sharing Schemes”.

Fourier Analysis for HW

At first glance, messier spectrum than for LSB — *i.e.* harder to analyze ...

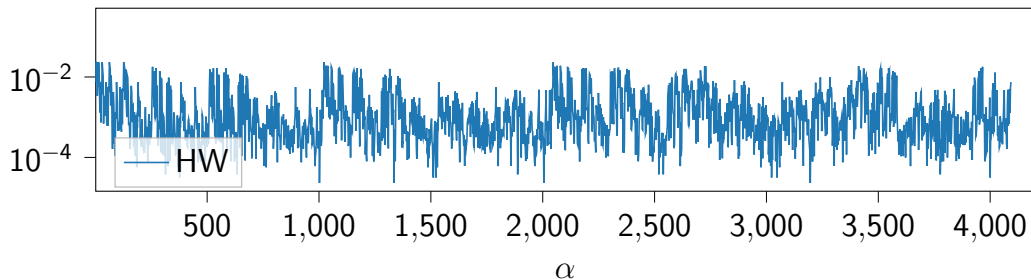


Figure: Fourier spectrum (1st half) of $\mathbf{1}_{\text{hw}^{-1}(n/2)}$ and for $n = 13, p = 2^n - 1$.

Fourier Analysis for HW

More regular patterns in log scale

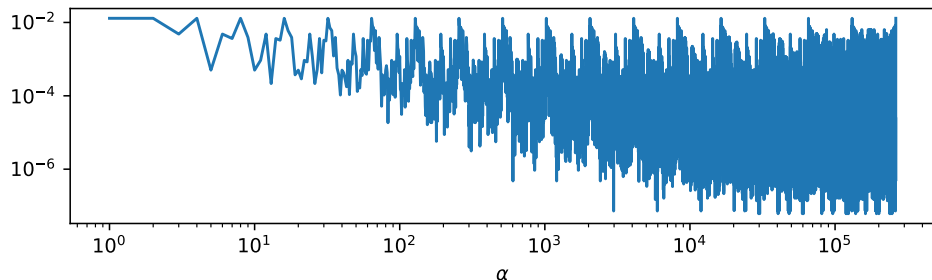


Figure: Fourier spectrum (1st half) of $\mathbf{1}_{\text{hw}^{-1}(n/2)}$ and for $n = 17, p = 2^n - 1$.

Explanation

Remember that in $\mathbb{F}_{2^n-1}$, $\cdot 2$ is a cyclic shift of the bits

¹³Faust et al., “Connecting Leakage-Resilient Secret Sharing to Practice: Scaling Trends and Physical Dependencies of Prime Field Masking”, Eurocrypt 24.

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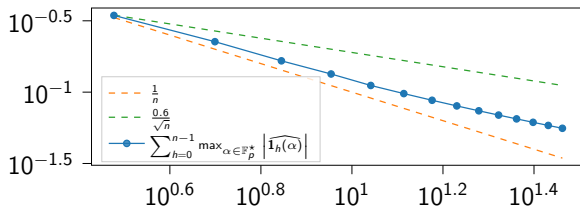
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Working over binary fields: prone to attacks in low-noise

Working over prime fields: more leakage resilient

Opens perspectives for *Post-Quantum* crypto (works over prime fields) and new primitives for symmetric crypto

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-  Béguinot, J. et al. “Removing the Field Size Loss from Duc et al.’s Conjectured Bound for Masked Encodings”. In: *Constructive Side-Channel Analysis and Secure Design - 14th International Workshop, COSADE 2023, Munich, Germany, April 3-4, 2023, Proceedings*. Ed. by E. B. Kavun and M. Pehl. Vol. 13979. Lecture Notes in Computer Science. Springer, 2023, pp. 86–104. DOI: [10.1007/978-3-031-29497-6_5](https://doi.org/10.1007/978-3-031-29497-6_5). URL: https://doi.org/10.1007/978-3-031-29497-6_5.

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References IV

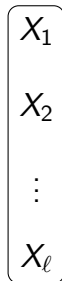
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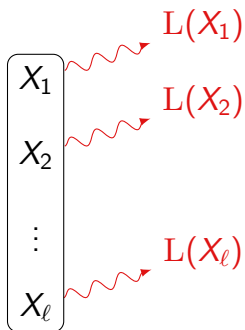
Provable Masking over the whole Circuit

Consider a **leaky** gadget (e.g., emulating a $\otimes$) with ℓ wires:



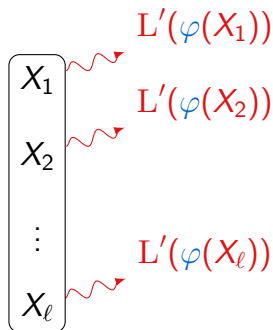
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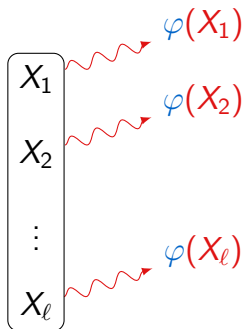


If $L(\cdot)$ noisy enough, can be *simulated* by a **random probing adversary**: $\varphi(x)$ exactly reveals x with some probability ϵ .¹⁴

¹⁴Duc, Dziembowski, and Faust, “Unifying Leakage Models: From Probing Attacks to Noisy Leakage”.
 Loïc Masure Au Bal des Implémentations Masquées

Provable Masking over the whole Circuit

Consider a **leaky** gadget (e.g., emulating a $\otimes$) with ℓ wires:



Upper bound by removing $\mathbf{L}'$ (Data Processing Inequality)

Security against a Random Probing Adversary

Assuming the gadget to be “properly” implemented, at least d out of ℓ wires must be revealed to leak the secret for a successful attack:

THEOREM (CHERNOFF)

If ℓ wires, each independently revealed with proba. ϵ :

$$\Pr(\text{At least } d \text{ wires revealed}) \leq \left(\frac{2e \cdot \ell \cdot \epsilon}{d} \right)^d$$

Still exponential security ✓

Strong requirements on the noise level to get non-trivial values of ϵ ✗