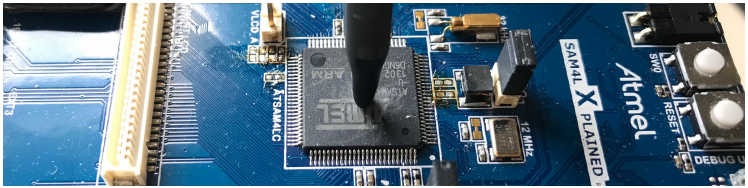


Computing over masked data with provable security

Loïc Masure (loic.masure@lirmm.fr)

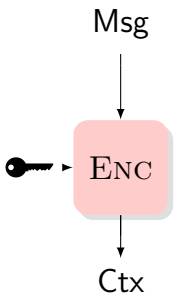
Séminaire ECO, Montpellier, 3 Septembre 2024

cnrs LIRMM UNIVERSITÉ DE MONTPELLIER

Agenda

Content

Context : Side-Channel Analysis (SCA)

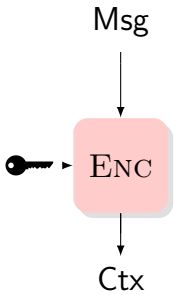


: N bits

Black-box cryptanalysis: 2^N

Context : Side-Channel Analysis (SCA)

"Cryptographic algorithms don't run on paper,



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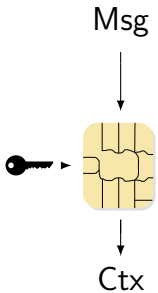

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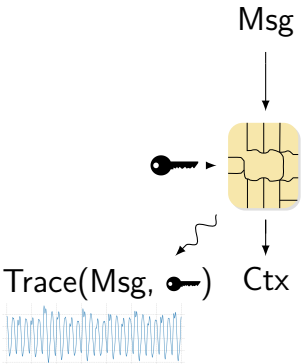

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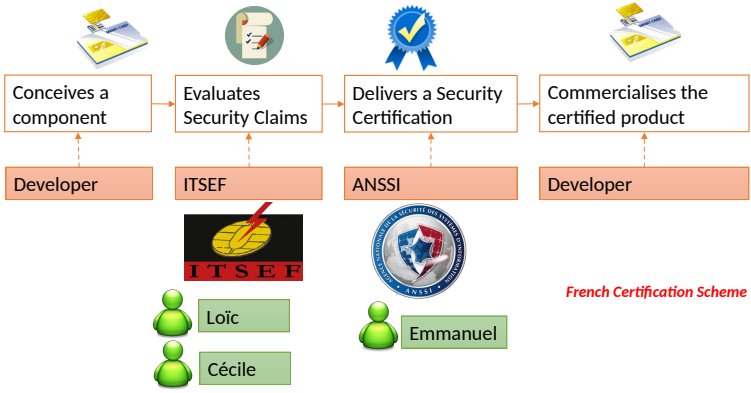
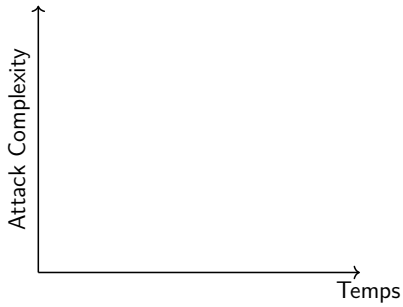
Content

Certification against SCA



Security graded w.r.t. attack complexity in terms of human, material, and financial means

Evaluate Security against Side-Channel Attacks



a

Attack approach (industry):

[a] Shamelessly stolen to O. Bronchain

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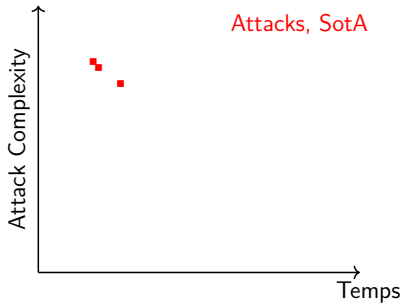

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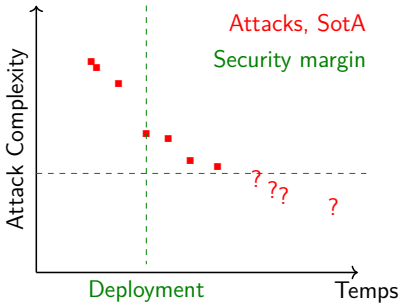

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Future improvement → reevaluation ✗

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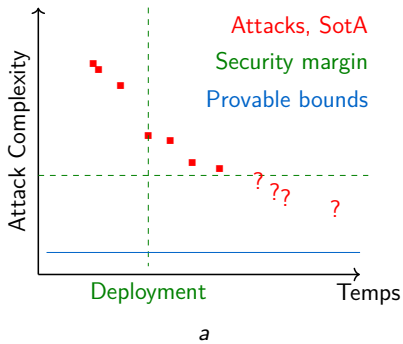

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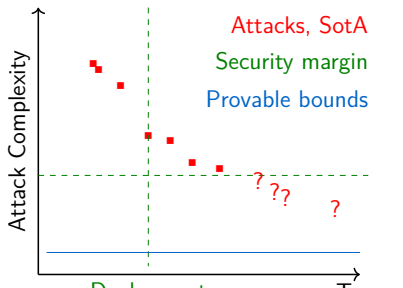

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Today's agenda: evaluation by proofs

Content

Masking: what is that ?

Masking, a.k.a. *MPC on silicon*:[12] secret sharing over a finite field $(\mathbb{F}, \oplus, \otimes)$

$$\mathrm{Y}(\text{secret})$$

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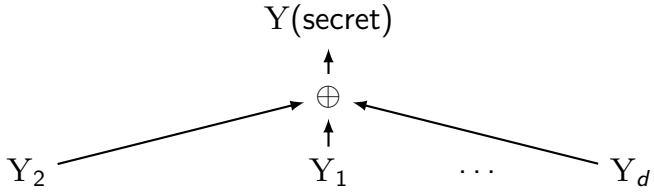

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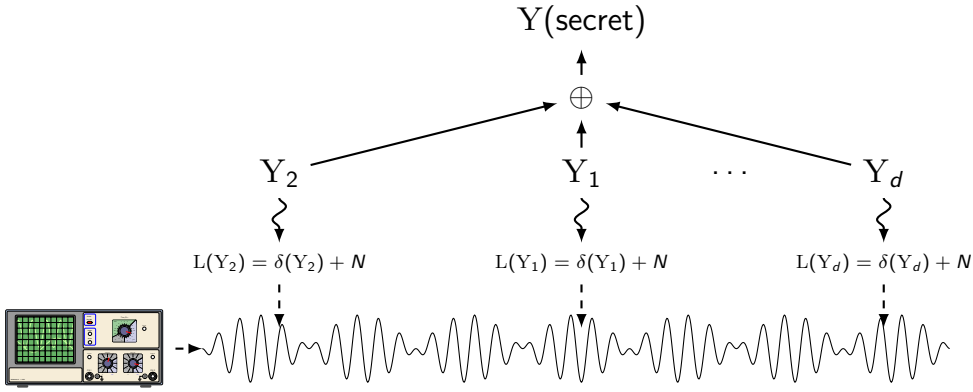


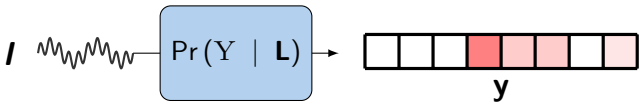
[1] Chari et al., "Towards Sound Approaches to Counteract Power-Analysis Attacks".

[2] Goubin and Patarin, "DES and Differential Power Analysis (The "Duplication" Method)".

Content

The Noisy Leakage Model

In this model, for each intermediate computation, the adversary gets a probability distribution about its operands:



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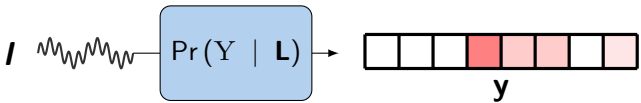

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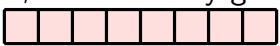

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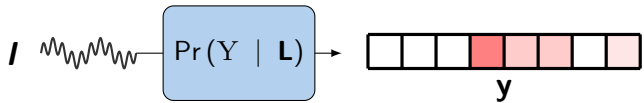
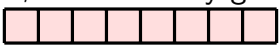


If, the adversary gets:



Very noisy
Sensitive computation unpredictable

The Noisy Leakage Model

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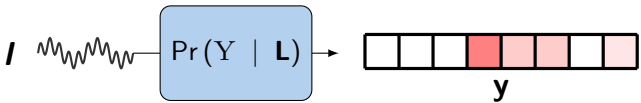

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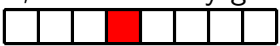

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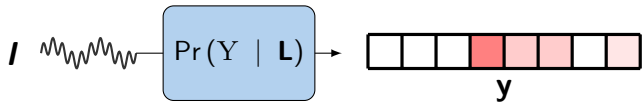


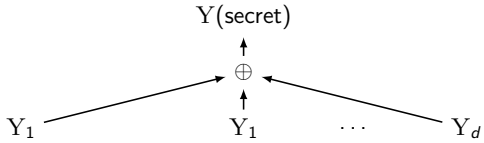
If, the adversary gets:

Low-noise
Exact prediction of the sensitive computation

The Effect of Masking

$Y(\text{secret})$

The Effect of Masking



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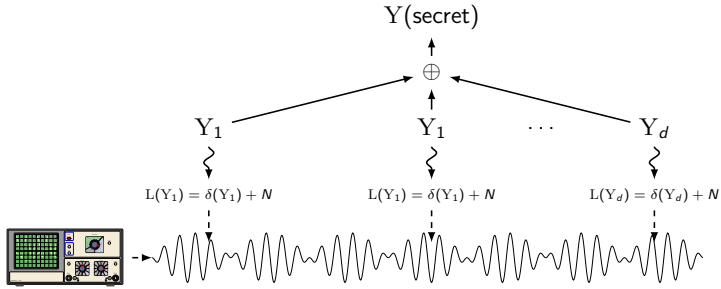

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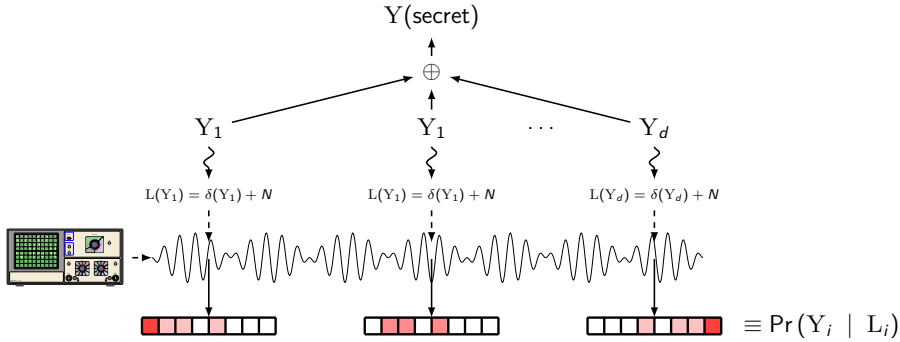

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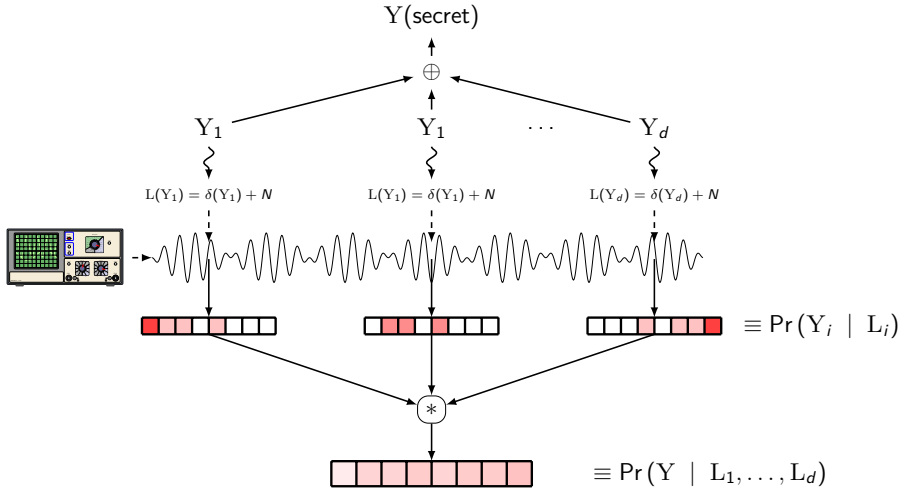

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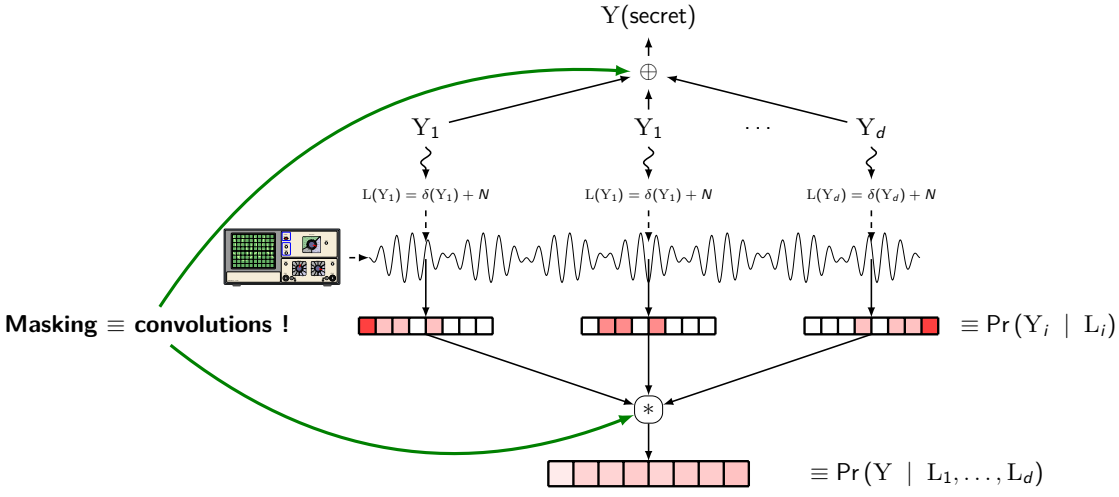

Security on a Single Encoding

Theorem (Mrs. Gerber's Lemma[3])

Given $\mathrm{Y} = \mathrm{Y}_1 \oplus \ldots \oplus \mathrm{Y}_d$*, and each* Y_i *with (indep.) side information* $\mathrm{L}_1, \ldots, \mathrm{L}_d$*, then for* $\eta^{-1} = 2\log(2)$*:*

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$\rightarrow$ Independent of the adversary ✓

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Convolution = Noise Amplification

Simulation, for $\mathbb{F}_{2^n}$: $L(Y_i) = hw(Y_i) + \mathcal{N}(0; \sigma^2)$, hw = Hamming weight

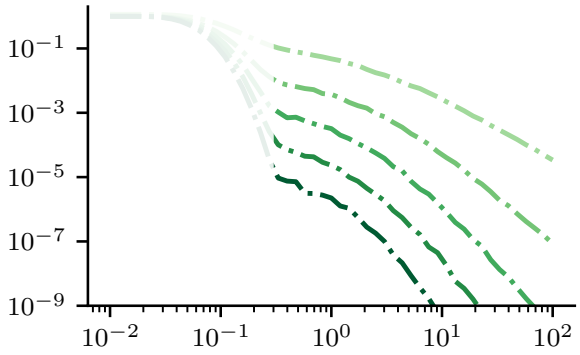

Figure: $\mathrm{MI}(\mathrm{Y}; \mathbf{L})$ vs. σ^2, $2 \leq d \leq 6$

Content

The Composition Paradigm

Idea to make a masked circuit

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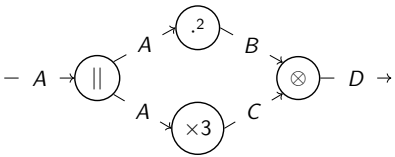

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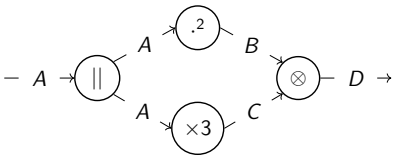

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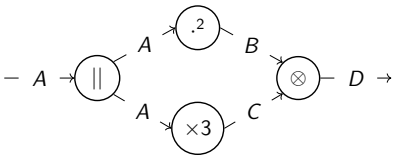

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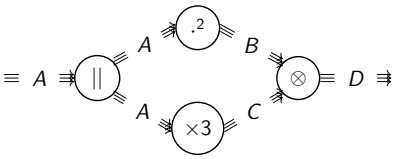

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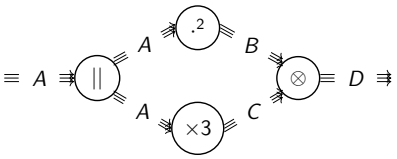

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Desired Properties

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Any tuple of t intermediate values $\perp$ secrets

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A set of probes $\mathcal{P}$ in a circuit $\mathbb{C}$ can be simulated with the input shares $\mathcal{I}$ if there exists an algorithm $\mathcal{S}$ (the *simulator*) such that

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The Composability Issue

✗NI is not always composable[7]

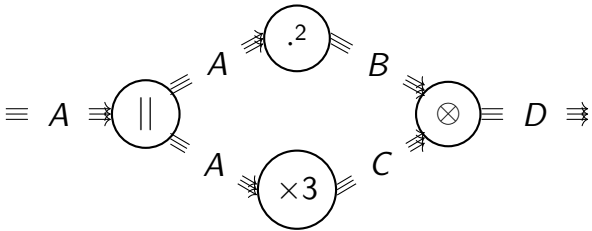

Figure: Two probes on D may depend on three probes of A !

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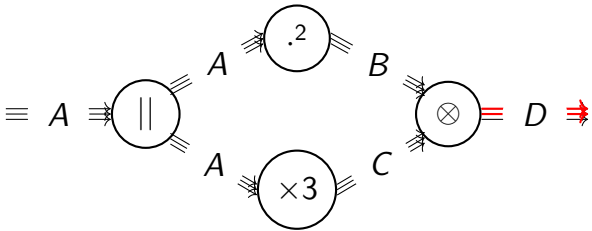

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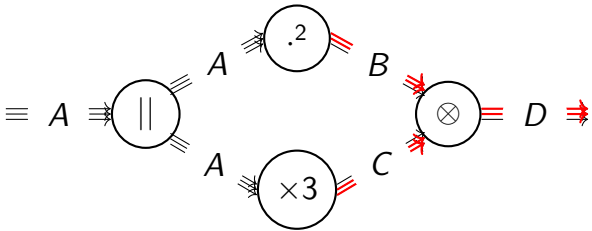

Figure: Two probes on D may depend on three probes of A !

[7] Coron et al., "Higher-Order Side Channel Security and Mask Refreshing".

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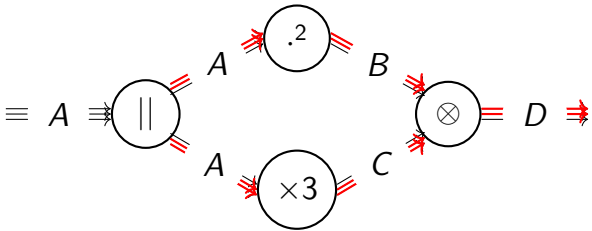

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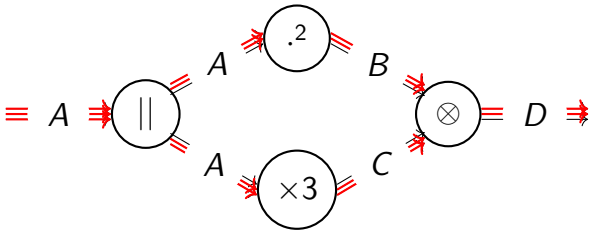

Figure: Two probes on D may depend on three probes of A !

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Strong Non-Interference[9]

Definition (t-Strong Non-Interference)

A gadget is t-SNI if any set of t_1 internal probes and t_2 output probes can be simulated with t_1 shares of each input sharing, and

$$t = t_1 + t_2$$

$\rightarrow$ SNI is composable ✓

$\rightarrow$ SNI $\implies$ NI $\implies$ privacy

Other composable notions: SNIu, PINI[8], robust probing, glitch-extended, ...

[8] Cassiers and Standaert, "Trivially and Efficiently Composing Masked Gadgets With Probe Isolating Non-Interference".

[9] Barthe et al., "Strong Non-Interference and Type-Directed Higher-Order Masking".

Masked addition gadget

Inputs:

$$\begin{aligned}[\![A]\!] &= (A_1, \ldots, A_d) \\ [\![B]\!] &= (B_1, \ldots, B_d)\end{aligned}$$

Output:

$$[\![C]\!] = (C_1, \ldots, C_d)$$

such that

$$\sum_i C_i = \left(\sum_i A_i\right) \oplus \left(\sum_i B_i\right)$$

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SecAdd algorithm:

$$\begin{aligned} C_1 &= A_1 \oplus B_1 \\ &\vdots \\ C_d &= A_d \oplus B_d \end{aligned}$$

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- t-NI + t-SNI refresh $\implies$ t-SNI ✓

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- NI, but not SNI ✗
- t-NI + t-SNI refresh $\implies$ t-SNI ✓
- Generalization: share-wise application of any affine map

Masked multiplication gadget

Inputs:

$$\begin{aligned}[\![A]\!] &= (A_1, \ldots, A_d) \\ [\![B]\!] &= (B_1, \ldots, B_d)\end{aligned}$$

Output:

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such that

$$\sum_i C_i = \left(\sum_i A_i\right) \otimes \left(\sum_i B_i\right)$$

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BadMult algorithm:

$$\begin{aligned}C_1 = &(A_1 \otimes B_1) \oplus (A_1 \otimes B_2) \oplus (A_1 \otimes B_3)\\ C_2 = &(A_2 \otimes B_1) \oplus (A_2 \otimes B_2) \oplus (A_2 \otimes B_3)\\ C_3 = &(A_3 \otimes B_1) \oplus (A_3 \otimes B_2) \oplus (A_3 \otimes B_3)\end{aligned}$$

Correct, but not 2-NI. Why ?

Masked multiplication gadget

Inputs:

$$[\![A]\!] = (A_1, \ldots, A_d)$$
$$[\![B]\!] = (B_1, \ldots, B_d)$$

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· SecMult is $(d-1)$-SNI ✓

Masked multiplication gadget

Inputs:

$$[\![A]\!] = (A_1, \ldots, A_d)$$
$$[\![B]\!] = (B_1, \ldots, B_d)$$

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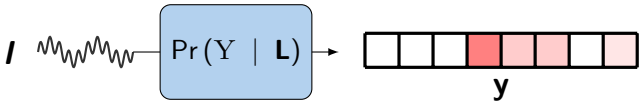
$$C_3 = (A_3 \otimes B_1 \ominus R_2) \oplus (A_3 \otimes B_2 \ominus R_3) \oplus (A_3 \otimes B_3)$$

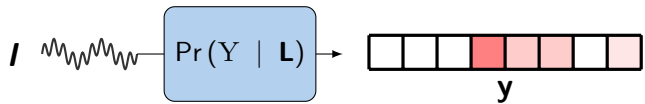
- SecMult is $(d-1)$-SNI ✓
- If $[\![B]\!] = (1, 0, \ldots, 0)$, then $\text{SecMult}([\![A]\!], [\![B]\!]) = \text{Refresh}([\![A]\!])$ ✓

Content

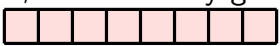
Recall on Noisy Leakage Model



Recall on Noisy Leakage Model



If, the adversary gets:



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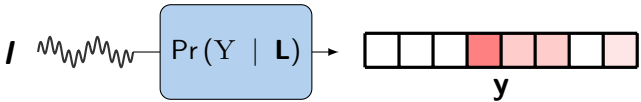

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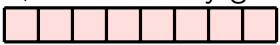
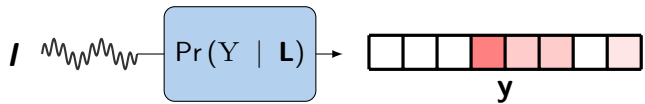


Very noisy leakage
Y indistinguishable from blind guess

Recall on Noisy Leakage Model



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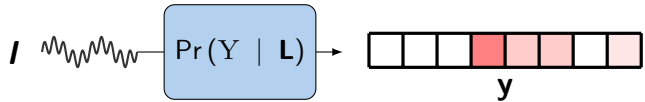
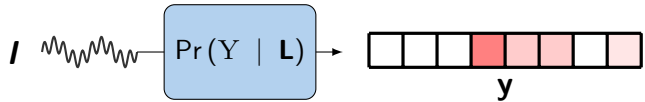


If, the adversary gets:

Low-noise leakage
Exact prediction for Y

Recall on Noisy Leakage Model



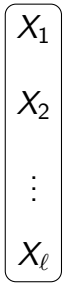
δ-NOISY ADVERSARY

Any intermediate computation Y leaks $\mathrm{L}(\mathrm{Y})$ such that:

$$\mathsf{SD}(\mathrm{Y};\mathrm{L}) = \mathop{\mathbb{E}}_{\mathrm{L}}\left[TV\left(\underbrace{}_{\Pr(\mathrm{Y}\mid\mathrm{L})}, \underbrace{}_{\Pr(\mathrm{Y})} \right)\right] \leq \delta$$

Security Proof for a Gadget

Consider a gadget with ℓ intermediate computations:



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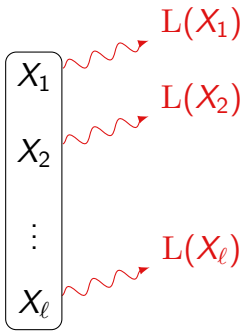

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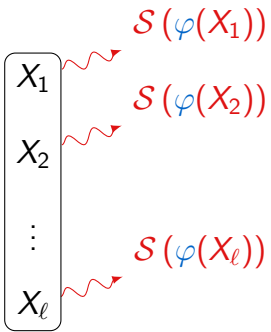

Lemma (Simulatability by RP)
The leakage function L *can be simulated from a random probing adversary:* $\varphi(x)$ *exactly reveals* x *with probability* $\epsilon = 1 - \sum_l \min_x \Pr(\mathrm{L}(x) = l) \leq \delta \cdot |\mathbb{F}|$.[a]

[a] Duc, Dziembowski, and Faust, "Unifying Leakage Models: From Probing Attacks to Noisy Leakage".

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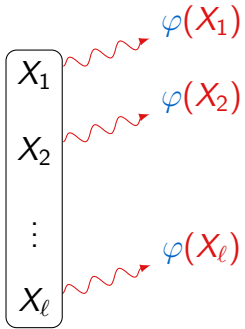

We may reduce to an adversary observing $\varphi(X)$ instead of $\mathcal{S}(\varphi(X))$ (Data Processing Inequality)

Proof of the Core Lemma (I)

Assume there exists such a simulator $\mathcal{S}$,

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Assume there exists such a simulator $\mathcal{S}$, we need to construct it for all inputs:

$$\begin{aligned} \Pr\left(\mathcal{S}\left(x\right) = l\right) &= \ldots, \text{ for all } x \\ \Pr\left(\mathcal{S}\left(\bot\right) = l\right) &= \ldots \end{aligned}$$

Constraints:

Proof of the Core Lemma (I)

Assume there exists such a simulator $\mathcal{S}$, we need to construct it for all inputs:

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$\rightarrow$ For all input x, $\Pr\left(\mathcal{S}\left(x\right)\right)$ should be a p.m.f. $(2 \cdot |\mathbb{F}|$ (in)equations)

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Constraints:

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$\rightarrow$ For the input $\bot$, $\Pr\left(\mathcal{S}\left(\bot\right)\right)$ should be a p.m.f. (2 (in)equations)

$\rightarrow$ For any x, l, $\Pr\left(\mathcal{S}\left(\varphi(x)\right) = l\right) = \Pr\left(\mathrm{L}(x) = l\right)$ $(|\mathbb{F}| \times |\mathcal{L}|$ equations)

Proof of the Core Lemma (II)

Let us start from the last constraint. For any x and any l:

$$\Pr\left(\mathrm{L}(x) = l\right) \;=\; \Pr\left(\mathcal{S}\left(\varphi(x)\right) = l\right)$$

Proof of the Core Lemma (II)

Let us start from the last constraint. For any x and any l:

$$\begin{aligned}\Pr\left(\mathrm{L}(x) = l\right) &= \Pr\left(\mathcal{S}\left(\varphi(x)\right) = l\right) \\ &= \Pr\left(\varphi(x) = x\right) \cdot \Pr\left(\mathcal{S}\left(x\right) = l\right) + \Pr\left(\varphi(x) = \bot\right) \cdot \Pr\left(\mathcal{S}\left(\bot\right) = l\right)\end{aligned}$$

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&= \epsilon \cdot \Pr\left(\mathcal{S}\left(x\right) = l\right) + (1 - \epsilon) \cdot \Pr\left(\mathcal{S}\left(\bot\right) = l\right)
\end{aligned}$$

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Hence,

$$0 \leq \Pr\left(\mathcal{S}\left(\bot\right)=l\right) = \frac{\overbrace{\Pr\left(\mathrm{L}(x)=l\right)-\epsilon\cdot\Pr\left(\mathcal{S}\left(x\right)=l\right)}^{\text{Should not depend on } X}}{1-\epsilon}$$

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$$0 \leq \Pr\left(\mathcal{S}\left(x\right)=l\right) = \frac{\Pr\left(\mathrm{L}(x)=l\right)-\pi(l)}{\epsilon} \quad (2)$$

Proof of the Core Lemma (II)

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Hence,

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Is there any ϵ such that $\geq$ and $\geq$ are valid?

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In other words,

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Furthermore, summing (1) over l, by definition of probability distributions,

$$\sum_l \pi(l) = \underbrace{\sum_l \Pr\left(\mathrm{L}(x) = l\right)}_{=1} - \epsilon \cdot \underbrace{\sum_l \Pr\left(\mathcal{S}(x) = l\right)}_{=1}$$

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$$0 \leq \pi(l) \leq \Pr\left(\mathrm{L}(x) = l\right) \text{ for any } x$$

In other words,

$$0 \leq \pi(l) \leq \min_x \Pr\left(\mathrm{L}(x) = l\right)$$

Furthermore, summing (1) over l, by definition of probability distributions,

$$\sum_l \pi(l) = \underbrace{\sum_l \Pr\left(\mathrm{L}(x) = l\right)}_{=1} - \epsilon \cdot \underbrace{\sum_l \Pr\left(\mathcal{S}(x) = l\right)}_{=1} = 1 - \epsilon$$

Hence,

$$\epsilon = 1 - \sum_l \pi(l) \geq 1 - \sum_l \min_x \Pr\left(\mathrm{L}(x) = l\right)$$

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To succeed, at least d out of ℓ wires must be revealed to the adversary:

$$\Pr(\text{Adv. learns sth}) \leq \Pr(\text{At least } d \text{ wires revealed})$$

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Q11: Prove the inequality from a particular case of Chernoff inequality[10]

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Putting all Together

In our context, $\ell \leq \mathcal{O}\left(d^2\right)$ (for $\otimes$ gadget), and $\epsilon \leq \delta \cdot |\mathbb{F}|$:

Theorem (Security Bound)

For a single gadget with $\ell \leq \mathcal{O}\left(d^2\right)$ intermediate computations:

$$\mathsf{SD}\left(k; \mathbf{L}\right) \leq \mathcal{O}\left(\left(7e \cdot d \cdot \delta \cdot |\mathbb{F}|\right)^d\right)$$

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For the whole circuit $\mathbb{C}$, $d/2$-region probing[11] security implies

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Wrap-Up of the Proof

δ-NL

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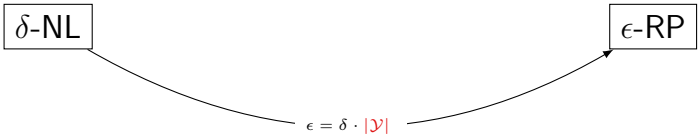

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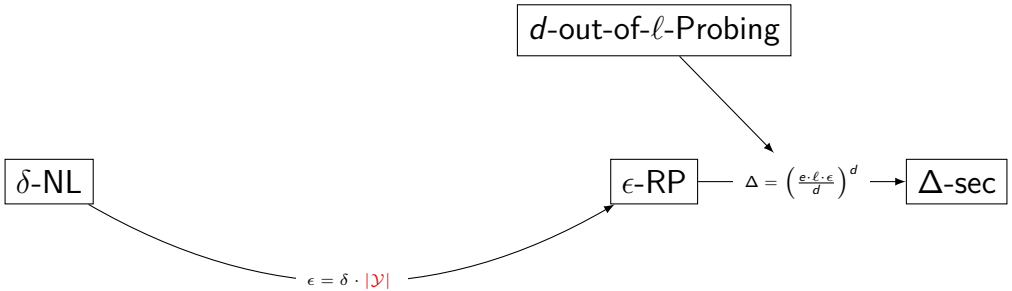

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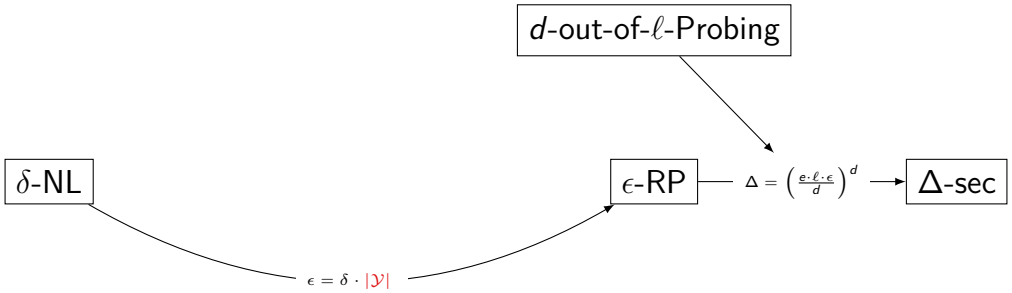

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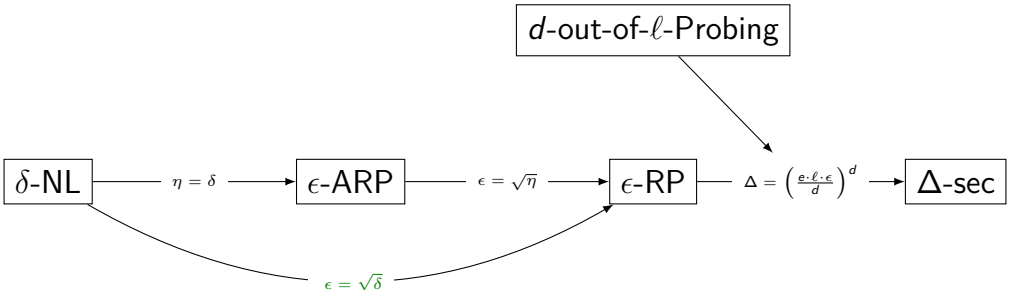

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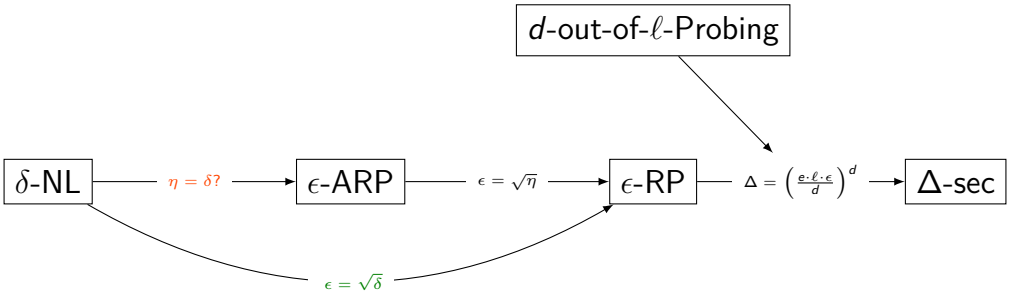
Problem: Gap with the definition of ARP[13] ✗



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Perspectives

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    $\Longrightarrow$ wider gap between d-probing and ϵ-RP ✗
  - ⋆ Masking-friendly schemes, *e.g.*, Raccoon ? ✓

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Pointers

Interested ?

- Coron's keynote at CARDIS 23 on masking lattice-based cryptography
- Cassiers' keynote at COSADE 23 on masking composability
- Nicolas Bordes' thesis with nice examples of probing notions.

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