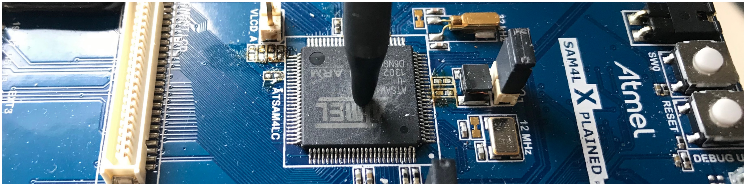


Side-channel Analysis of Cryptographic Implementations

Evaluation & Counter-Measures

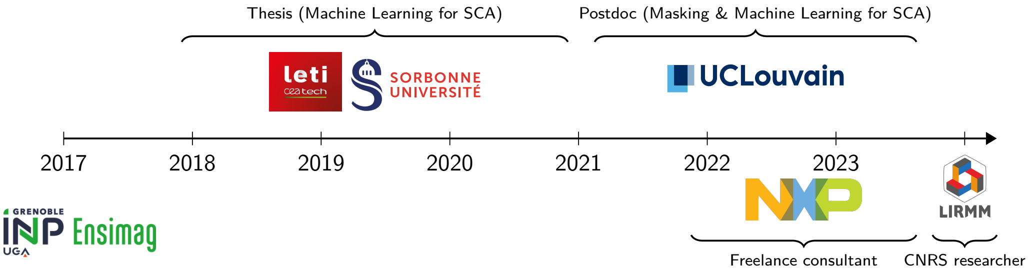
Loïc Masure (loic.masure@lirmm.fr)

PQ-TLS Summer School, Anglet, 20 – 21 June 2024

Acknowledgements

This work received funding from the France 2030 program, managed by the French National Research Agency under grant agreement No. ANR-22-PETQ-0008 PQ-TLS

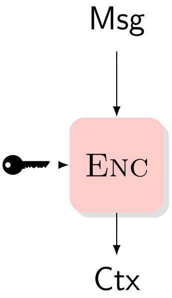
Who am I?


Thesis (Machine Learning for SCA)
Postdoc (Masking & Machine Learning for SCA)
2017
2018
2019
2020
2021
2022
2023
Freelance consultant
CNRS researcher

Agenda

Content

Context : Side-Channel Analysis (SCA)



: N bits

Black-box cryptanalysis: 2^N

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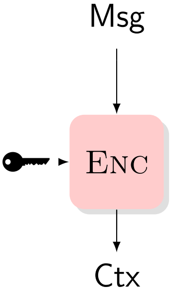
"Cryptographic algorithms don't run on paper,



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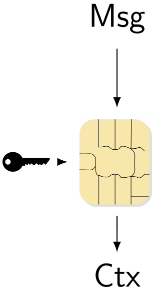

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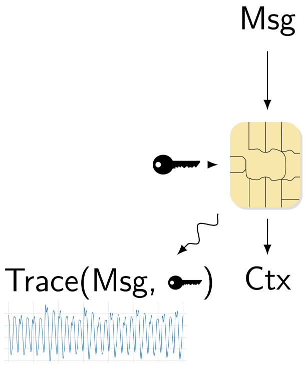

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Black-box cryptanalysis: 2^N

Side Channel = Unintended Communication Channel

Example: the Washington Pizza Index[1]

NEWS

CRUSTY D.C. VETERAN SAYS WAR IS NEAR

By Cox News Service
Chicago Tribune • Published: Jan 16, 1991 at 12:00 am

WASHINGTON — The pizza index indicates military action is imminent in the Persian Gulf, a Domino`s delivery official said Tuesday.

Record numbers of late-night pizzas have been delivered this week to the White House, Pentagon and State Department, said Frank Meeks, owner of several Washington-area Domino`s outlets.

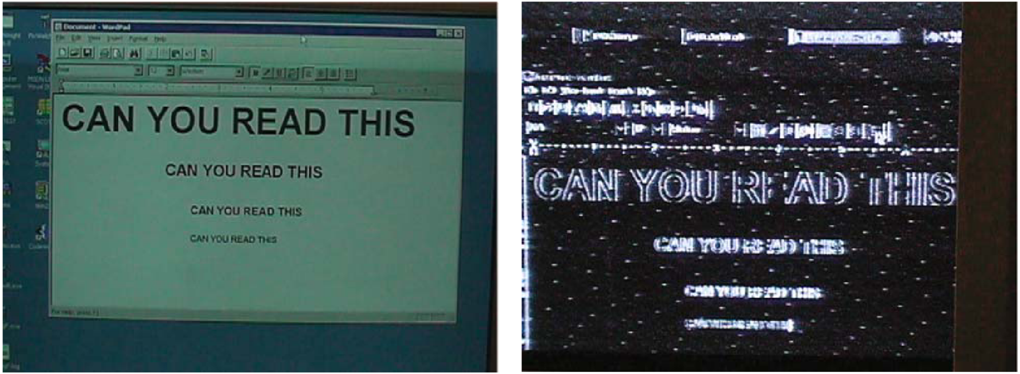
Similar order patterns came immediately before the invasions of Panama and Grenada, Meeks said.

The increase in pizza orders at key government buildings after 10 p.m. is ''very unusual,'' Meeks said. ''I don`t think they`re sitting around watching Redskins reruns.''

Figure: Chicago Tribune, **Jan. 16 1991**, the day before *Desert Storm* operation began.

[1] Reality questioned: `http://home.xnet.com/~warinner/pizzacites.html`

What is a Side Channel? A First Example



(a) A good old monitor (b) Reconstruction from EM field

Figure: An example from Koç, *Cryptographic Engineering*.

An Exemplary SCA on Crypto

RSA: Modular exponentiation over large ($\approx$ 2000-bit wide) integers

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$$M^k = M^{\sum_i k_i \cdot 2^i} = \prod_i (M^{k_i})^{2^i}$$

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Step 0: M^{k_N} then square
Step 1: $\times M^{k_{N-1}}$ then square
...
Step i: $\times M^{k_{N-i}}$ then square

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...

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Op.	Guess
$\times M$	
square	
square	
square	
$\times M$	
square	
$\times M$	
square	

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Op.	Guess
$\times M$	$k_N = 1$
square	
square	
square	
$\times M$	
square	
$\times M$	
square	

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...
Step i: if $k_{N-i} = 1$, $\times M$ then square

Op.	Guess
$\times M$	$k_N = 1$
square	
square	$k_{N-1} = 0$
square	
$\times M$	
square	
$\times M$	
square	

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Op.	Guess
$\times M$	$k_N = 1$
square	
square	$k_{N-1} = 0$
square	$k_{N-2} = 0$
$\times M$	
square	
$\times M$	
square	

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Op.	Guess
$\times M$	$k_N = 1$
square	
square	$k_{N-1} = 0$
square	$k_{N-2} = 0$
$\times M$	$k_{N-3} = 1$
square	
$\times M$	
square	

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square	$k_{N-2} = 0$
$\times M$ square	$k_{N-3} = 1$
$\times M$ square	$k_{N-4} = 1$

Can you guess the key from the Oscilloscope?

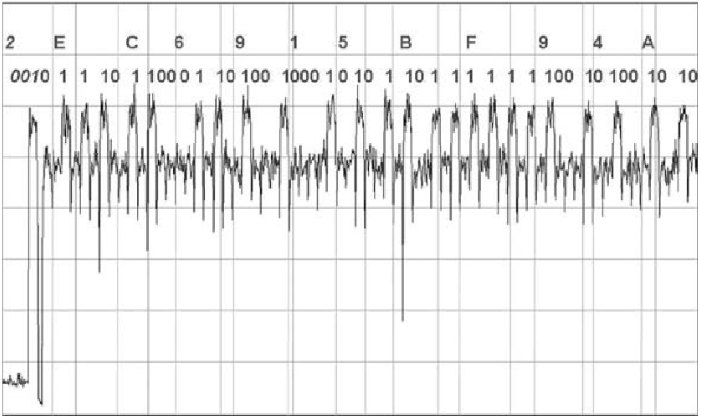
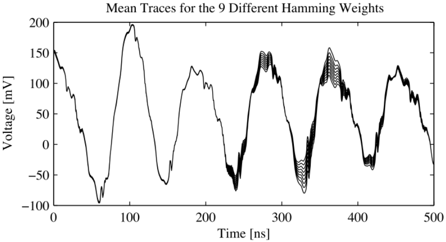


Figure: Power consumption. Illustration from Koç, *Cryptographic Engineering*

Power Analysis on Symmetric Key

Power consumption: each bit x_i of a data chunk X stored in a register[2]

$x_i = 0 \implies$ register voltage $= 0$
$x_i = 1 \implies$ register voltage $\neq 0$
Overall consumption of X is proportional to $\mathrm{hw}(X) = \sum_i x_i$
hw = Hamming Weight



[2] Mangard, Oswald, and Popp, *Power analysis attacks - revealing the secrets of smart cards*.

Practical Attack on AES, with Correlation

In practice: $\mathrm{L}_{\mathrm{P}} \propto \mathsf{hw}(k \oplus \mathrm{P}) + \mathcal{N}(0, \sigma^2)$

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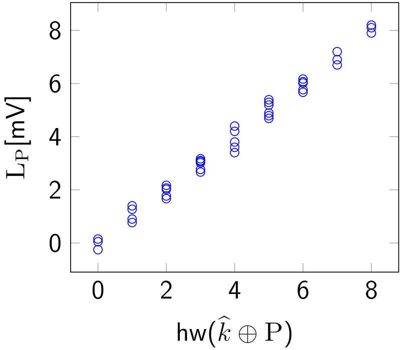
Distinguisher with a statistical test: for all key hypothesis $\widehat{k} = 0, 1, 2, \ldots$
If $\widehat{k} = k^\star$, then $\mathrm{L_P}$ should be highly correlated with $\mathsf{hw}(\widehat{k} \oplus \mathrm{P})$

$$\rho_{\widehat{k}} = \frac{\underset{\mathrm{P},\mathcal{N}}{\mathsf{Cov}}\left(\mathrm{L_P}, \mathsf{hw}(\widehat{k} \oplus \mathrm{P})\right)}{\sqrt{\underset{\mathrm{P},\mathcal{N}}{\mathsf{Var}}\left(\mathrm{L_P}\right)} \cdot \sqrt{\underset{\mathrm{P},\mathcal{N}}{\mathsf{Var}}\left(\mathsf{hw}(\widehat{k} \oplus \mathrm{P})\right)}} \approx \pm 1$$

$\widehat{k} = k^\star$



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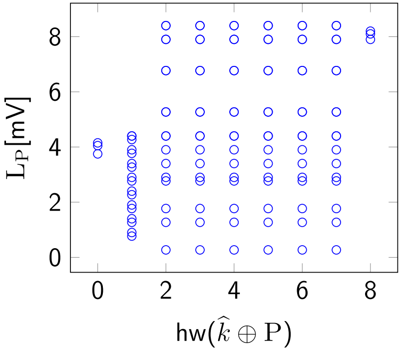
Distinguisher with a statistical test: for all key hypothesis $\widehat{k} = 0, 1, 2, \ldots$
If $\widehat{k} \neq k^\star$, then $\mathrm{L_P}$ should be poorly correlated with $\mathsf{hw}(\widehat{k} \oplus \mathrm{P})$

$\widehat{k} \neq k^\star$

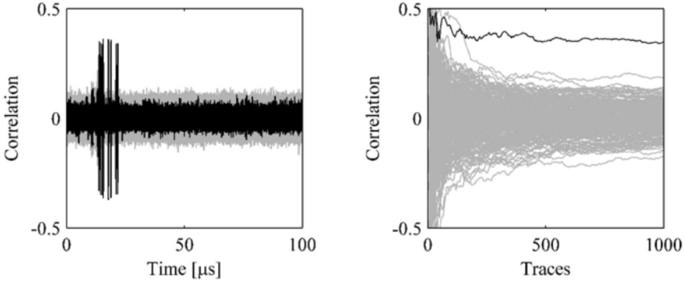
$$\rho_{\widehat{k}} = \frac{\underset{\mathrm{P},\mathcal{N}}{\mathsf{Cov}}\left(\mathrm{L_P}, \mathsf{hw}(\widehat{k} \oplus \mathrm{P})\right)}{\sqrt{\underset{\mathrm{P},\mathcal{N}}{\mathsf{Var}}\left(\mathrm{L_P}\right)} \cdot \sqrt{\underset{\mathrm{P},\mathcal{N}}{\mathsf{Var}}\left(\mathsf{hw}(\widehat{k} \oplus \mathrm{P})\right)}} \ll \pm 1$$



Power Analysis on Symmetric Key

Power consumption: each bit x_i of a data chunk X stored in a register[3]

Key guessed by a statistical test leveraging the correlation between the Hamming weight of data and the power consumption



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It's Demo Time

Application of the Correlation Attack on a `ChipWhisperer`

`CW` = Target device (8-bit MCU) + Oscilloscope

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It's Demo Time

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`CW` = Target device (8-bit MCU) + Oscilloscope

- **Q1**: What is the attack complexity ?

  One n-bit key chunk: $\mathcal{O}\left(2^n\right)$

  N-bit full key: divide-and-conquer $\implies \mathcal{O}\left(\frac{N}{n} \cdot 2^n\right) \approx$ "quantum" break

Counter-Measure

Q2: Can you find a simple counter-measure for this attack ?

[4] Rivain, Prouff, and Doget, "Higher-Order Masking and Shuffling for Software Implementations of Block Ciphers".

[5] Coron and Kizhvatov, "An Efficient Method for Random Delay Generation in Embedded Software".

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Data complexity: $N_a \propto \frac{1}{\rho^2}$

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→ What can the adversary do? *Integrated* attack: $\rho_{\text{shuffling,integrated}} = \frac{\rho}{\sqrt{t}}$

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Contradicting Goals

For correlation attacks, we usually target AddRoundKey or the SubBytes (bijective between each other)

- **Q3**: What's the "best" target ?

→ When targeting hw($k \oplus \mathrm{P}$), $k^\star$ and $\overline{k^\star}$ undistinguishable: *ghost* peaks

→ Problem solved when targeting hw(Sbox[$k \oplus \mathrm{P}$])

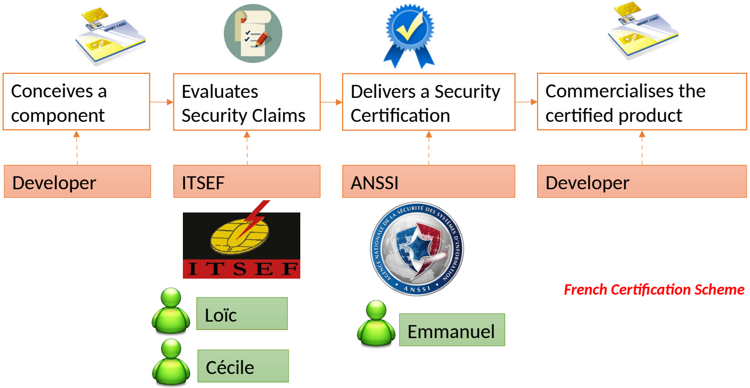
Contradicting goal: Sbox brings confusion to thwart cryptanalysis, but helps side-channel analysis[6]

[6]Prouff, "DPA Attacks and S-Boxes".

Content

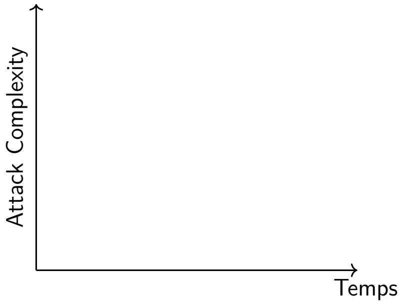
Content

Certification against SCA



Security graded w.r.t. attack complexity in terms of human, material, and financial means

Evaluate Security against Side-Channel Attacks



[a]

Attack approach (industry):

[a] Shamelessly stolen to O. Bronchain

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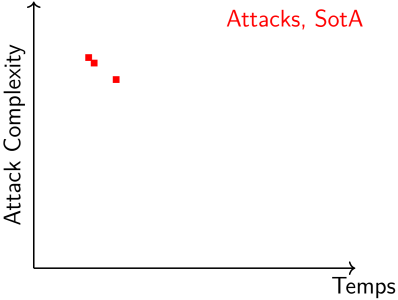

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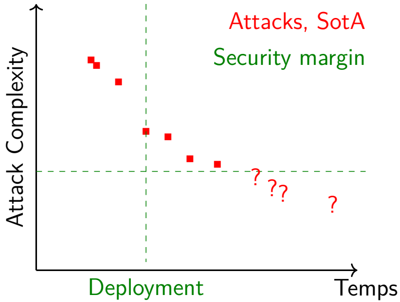

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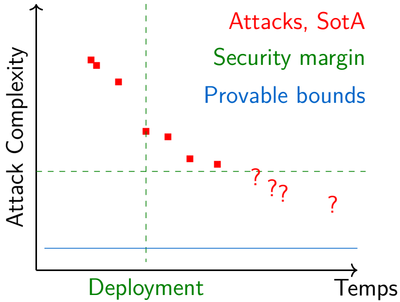

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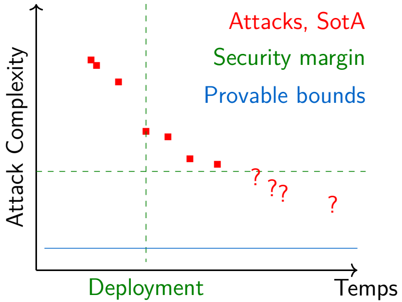

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Agenda: evaluation by attack (today), evaluation by proofs (tomorrow)

How to Evaluate Efficiently? Interlude

A good *evaluator* $\mathcal{E} \neq$ A good *adversary* $\mathcal{A}$

[7] Analogy with real estate

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Evaluating = guessing how much the best $\mathcal{A}$ must pay to succeed[7]

[7] Analogy with real estate

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→ Quantify the complexity of each step

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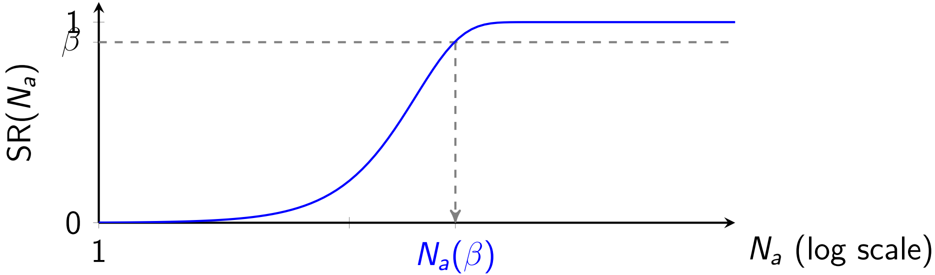
$\rightarrow$ Decompose each attack step

$\rightarrow$ Quantify the complexity of each step

$\Longrightarrow$ Finding evaluation shortcuts

[7] Analogy with real estate

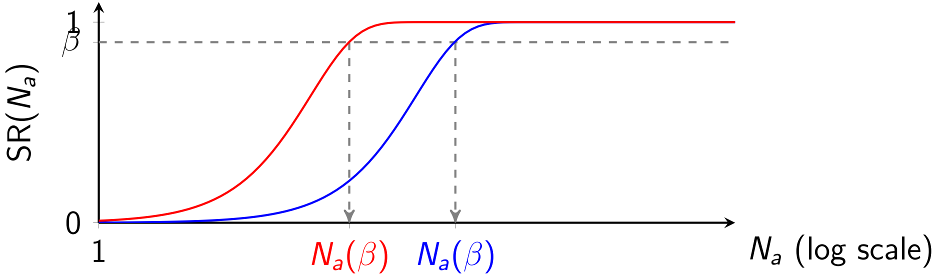
Assessing an Attack: the Success Rate[8]



- SR: probability to succeed the attack within N_a queries to the target

[8] In the following, we focus on *data* complexity only. All known attacks are computationally efficient.

Assessing an Attack: the Success Rate[8]



- SR: probability to succeed the attack within N_a queries to the target
- Allows to compare attacks: $\mathcal{A}_1 \underset{\beta}{<} \mathcal{A}_2$ iff for a fixed $N_a(\beta) > N_a(\beta)$

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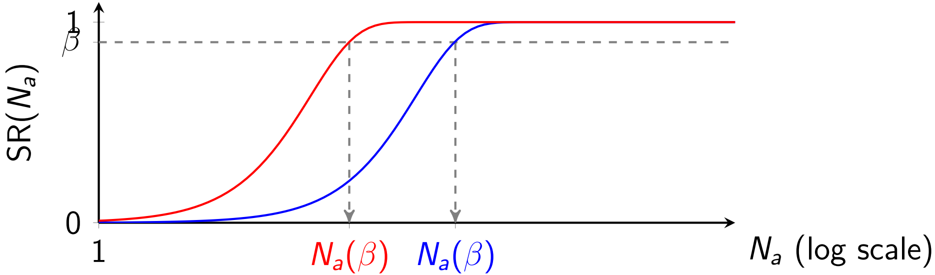
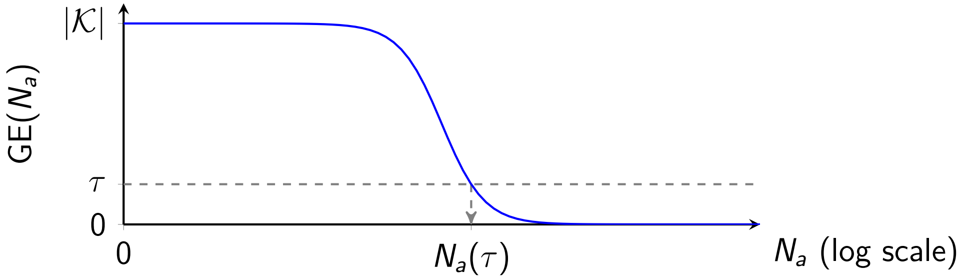


- SR: probability to succeed the attack within N_a queries to the target
- Allows to compare attacks: $\mathcal{A}_1 \underset{\beta}{<} \mathcal{A}_2$ iff for a fixed $N_a(\beta) > N_a(\beta)$
- **Q4 [full key to D&C]**: Prove that $N_a(\beta, \mathrm{full}) = N_a(\beta^{\frac{n}{|\mathcal{K}|}}, \mathrm{word})$

[8] In the following, we focus on *data* complexity only. All known attacks are computationally efficient.

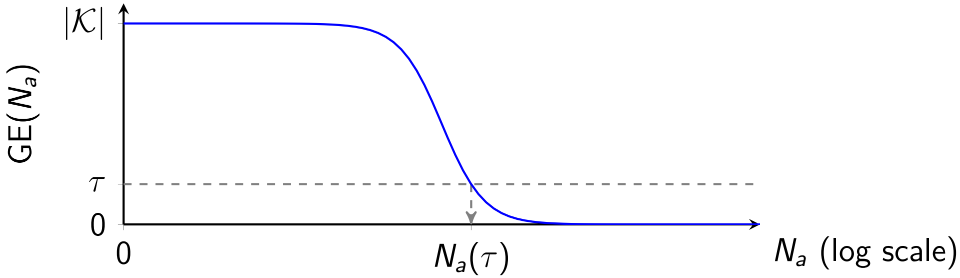
Assessing an Attack: the Guessing Entropy



- GE: average rank of the right key, among all key hypotheses

[9] David and Wool, *A Bounded-Space Near-Optimal Key Enumeration Algorithm for Multi-Dimensional Side-Channel Attacks*.

Assessing an Attack: the Guessing Entropy



- GE: average rank of the right key, among all key hypotheses
- Allows to quantify the key remaining *enumeration work*

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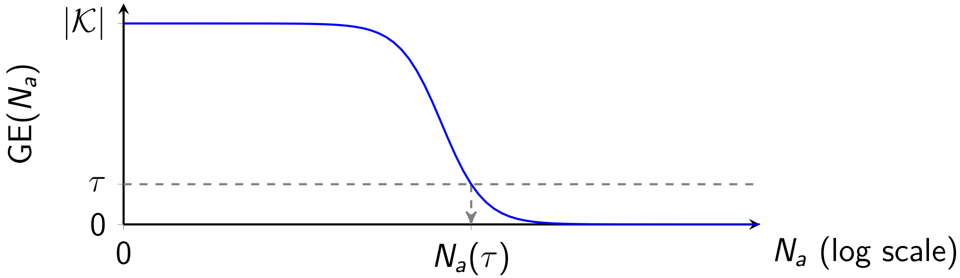
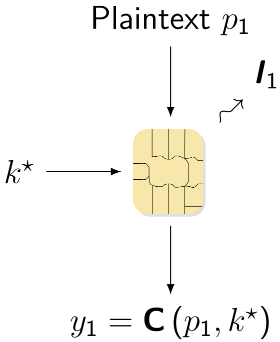
Assessing an Attack: the Guessing Entropy



- GE: average rank of the right key, among all key hypotheses
- Allows to quantify the key remaining *enumeration work*
- **Q5 [full key to D&C]**:[9] Prove that $N_a(\tau, \mathrm{full}) \geq N_a(\tau^{\frac{n}{|\mathcal{K}|}}, \mathrm{word})$

[9] David and Wool, *A Bounded-Space Near-Optimal Key Enumeration Algorithm for Multi-Dimensional Side-Channel Attacks*.

Optimal Attack: Maximum Likelihood[10]



[10] Heuser, Rioul, and Guilley, “Good Is Not Good Enough - Deriving Optimal Distinguishers from Communication Theory”.

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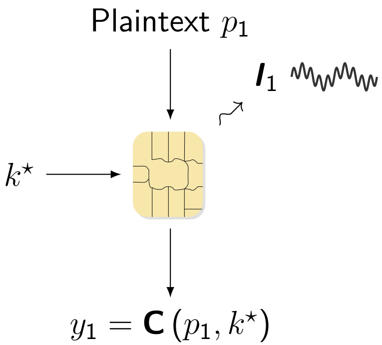

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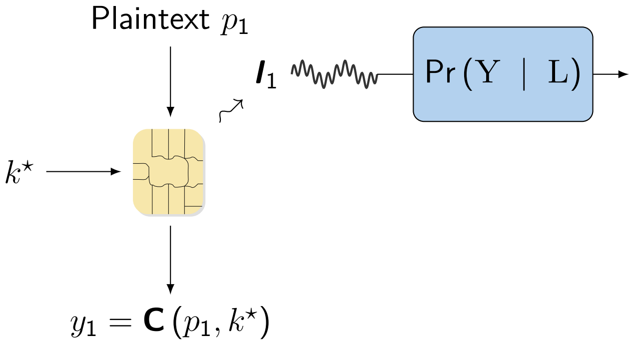

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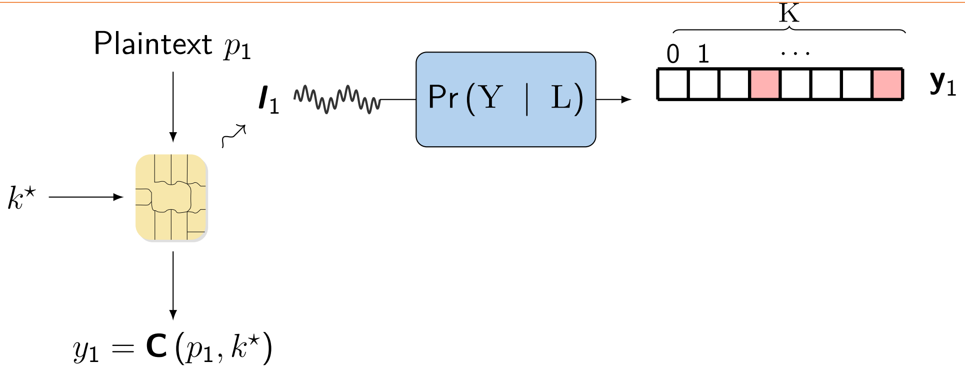

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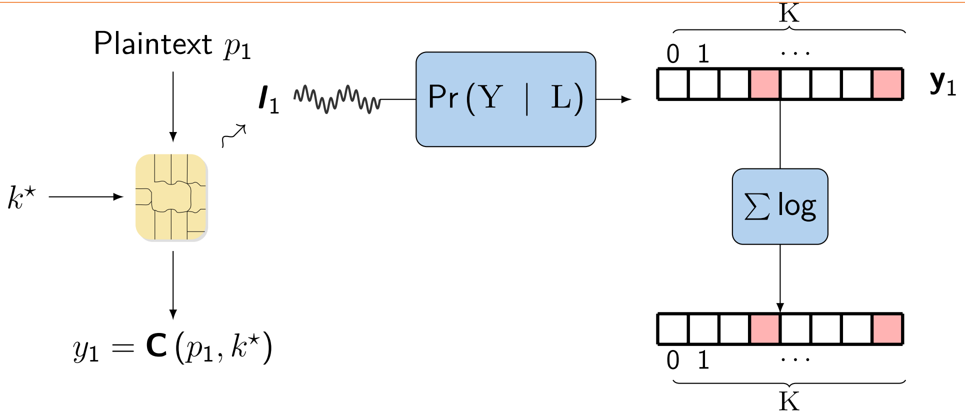

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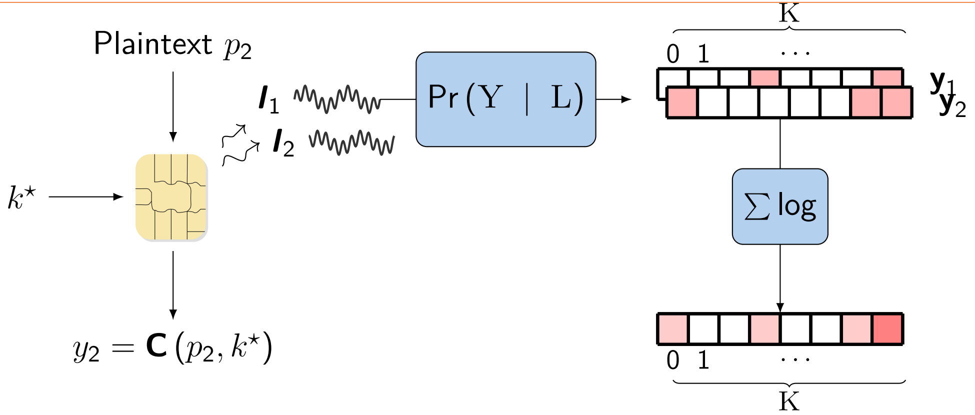

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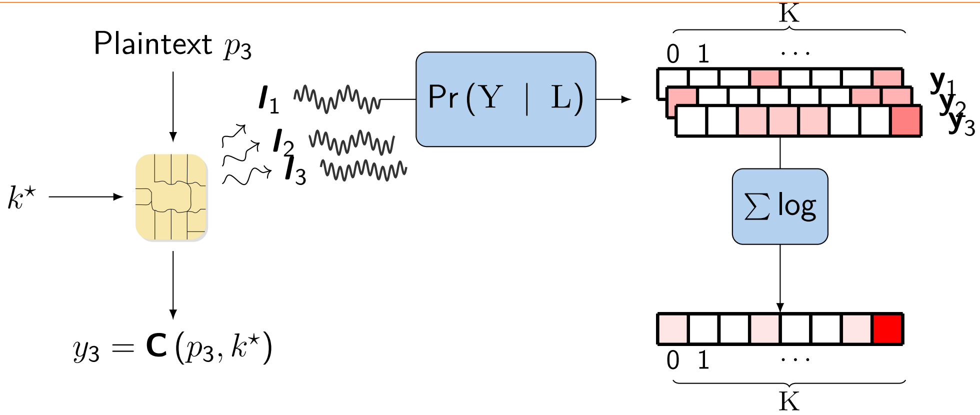

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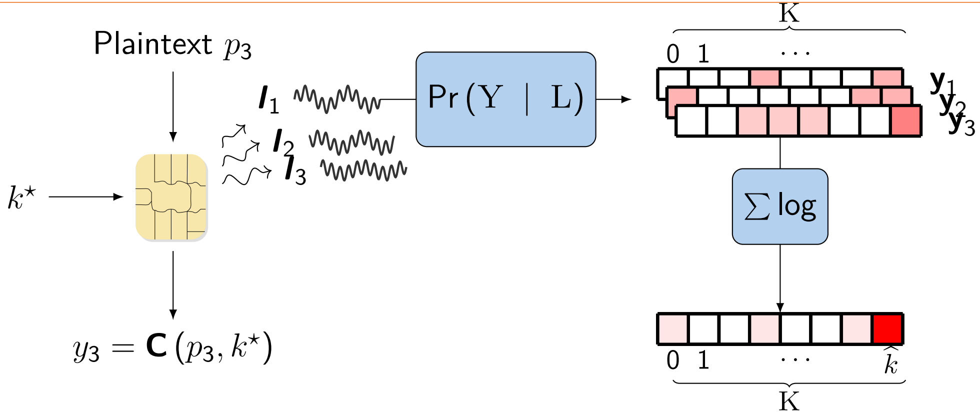

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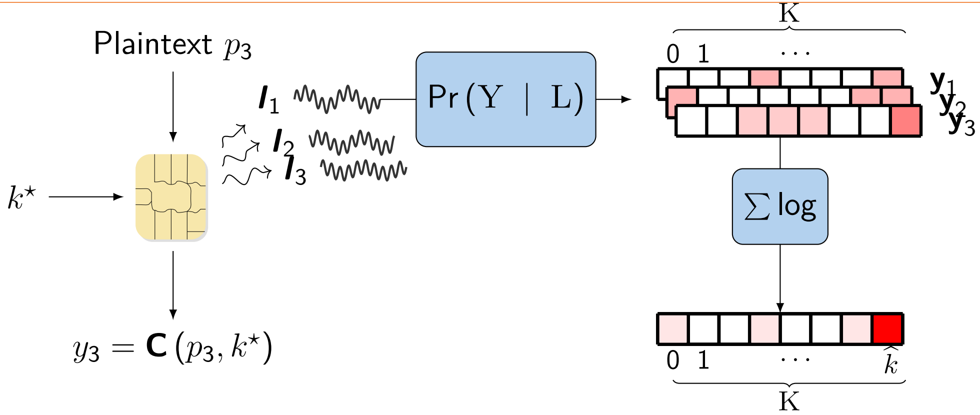
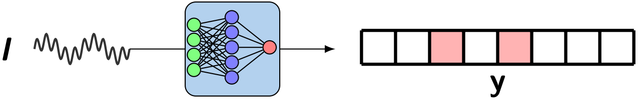


Problem: $\Pr(Y \mid \mathbf{L})$ unknown (device-dependent)

[10] Heuser, Rioul, and Guilley, "Good Is Not Good Enough - Deriving Optimal Distinguishers from Communication Theory".

Content

Deep Learning (DL) for SCA



Deep Neural Network (DNN)

General way to modelize, *i.e.*, to convert leakage into probabilities

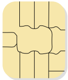
$$F : \left| \begin{array}{ccc} \mathcal{L} & \longrightarrow & \mathcal{P}(\mathcal{Y}) \\ \boldsymbol{l} & \longmapsto & \mathbf{y} = F(\boldsymbol{l}) \approx \Pr\left(\mathrm{Y} \mid \mathbf{L} = \boldsymbol{l}\right) \end{array} \right. \tag{1}$$

$F(\boldsymbol{l})$: output of a Directed Acyclic Graph (DAG) of computation:

- Each node: elementary function $f_i(\cdot, \theta_i)$
- θ_i: *parameters* fully describing f_i

Shape of the DAG, nature of the classes of functions: *architecture* of the DNN.

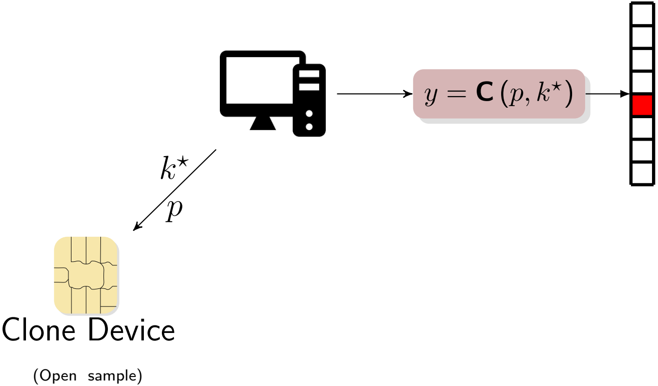
Profiled SCA = Supervised Learning Problem



Clone Device

(Open sample)

Profiled SCA = Supervised Learning Problem



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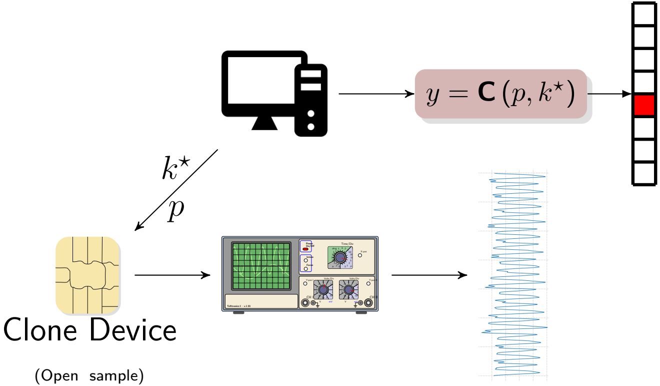

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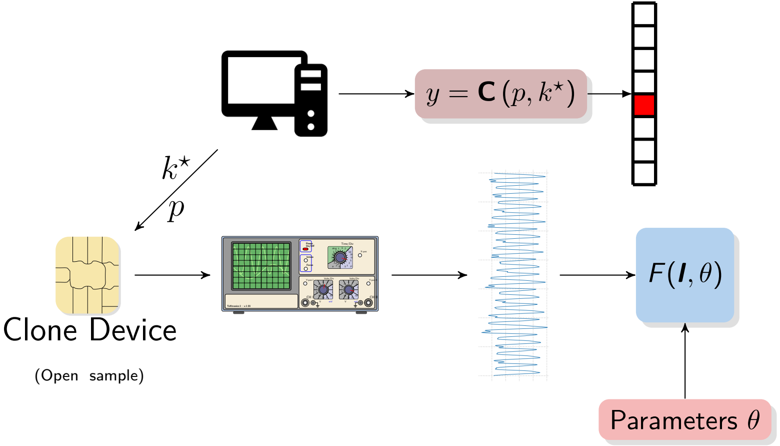

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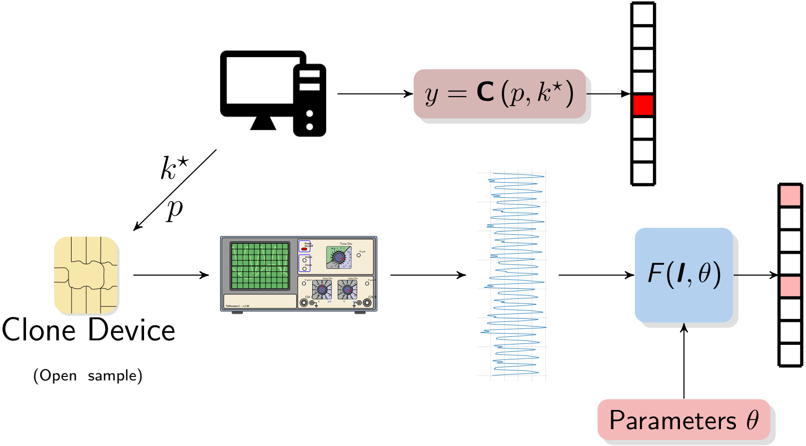

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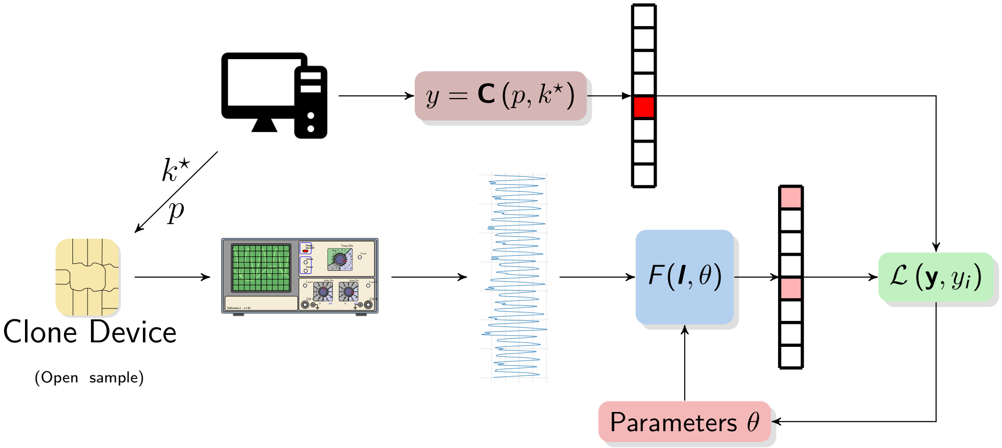
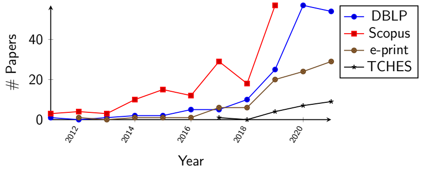


$\mathcal{L}()$: loss function to minimize, with *gradient descent*

The Deep Learning (DL) hype in SCA

- ➔ **Space 2016:** DL breaks *masking*[11]
- ➔ **Ches 2017:** CNNs efficiently tackles *misalignment*[12]
- ➔ **Ches 2019:** non-profiled attacks[13]
- ➔ *De facto* standard for evaluations
- ➔ Dedicated sessions in conferences



[11] Maghrebi, Portigliatti, and Prouff, "Breaking Cryptographic Implementations Using Deep Learning Techniques"

[12] Cagli, Dumas, and Prouff, "Convolutional Neural Networks with Data Augmentation Against Jitter-Based Countermeasures - Profiling Attacks Without Pre-processing"

[13] Timon, "Non-Profiled Deep Learning-based Side-Channel attacks with Sensitivity Analysis"

Content

Use Case: SCA against code polymorphism I

Implementation of AES from mbedTLS on ARM-Cortex M4 architecture

- T-table implementation with 32 bit variables

$100,000$ traces acquired for each target ($\leq$ a day)

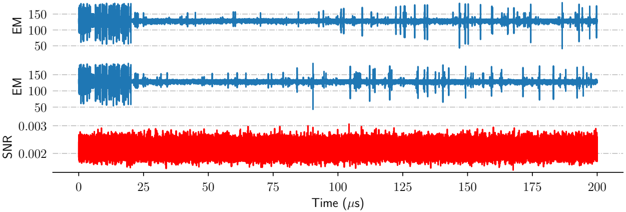


Figure: Two examples of traces (blue) and the Signal-to-Noise Ratio (SNR) (red)

No SNR peak $\implies$ a layman attacker fails, even with $N_a = 10^5$ traces

Use Case:SCA against code polymorphism II[14]

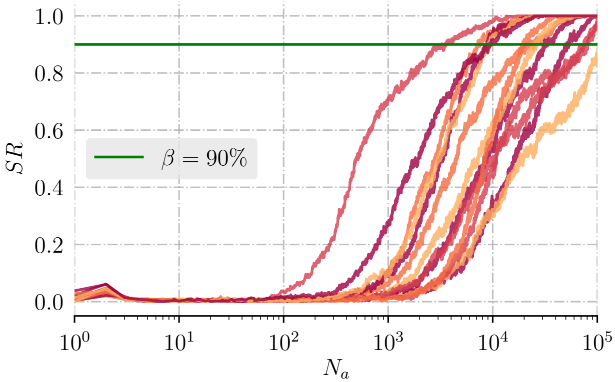


Figure: Layman Attacker with Re-alignment

[14] Masure et al., "Deep Learning Side-Channel Analysis on Large-Scale Traces - A Case Study on a Polymorphic AES".

Use Case:SCA against code polymorphism II[14]

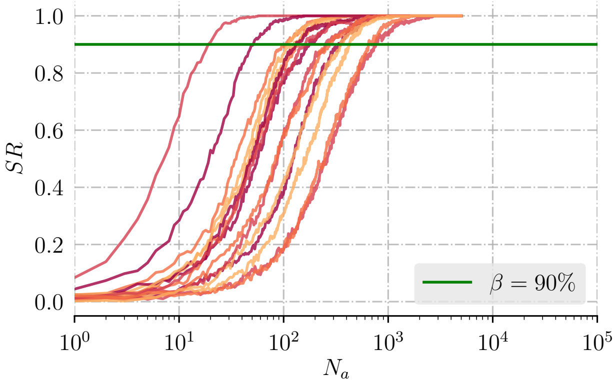


Figure: Attacker with Re-alignment and a Clone device

[14] Masure et al., "Deep Learning Side-Channel Analysis on Large-Scale Traces - A Case Study on a Polymorphic AES".

Use Case:SCA against code polymorphism II[14]

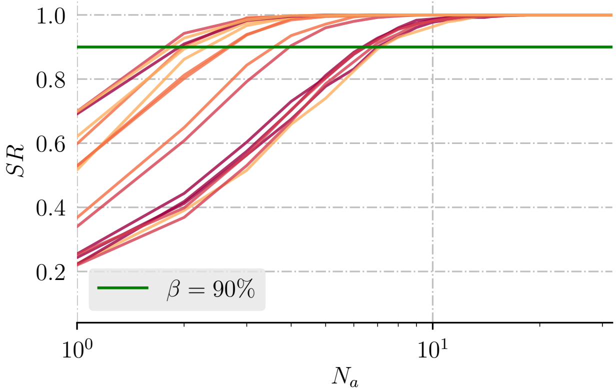


Figure: Attacker without Re-alignment but with a Clone device and deep learning

[14] Masure et al., "Deep Learning Side-Channel Analysis on Large-Scale Traces - A Case Study on a Polymorphic AES".

Post-Mortem Sensitivity Analysis

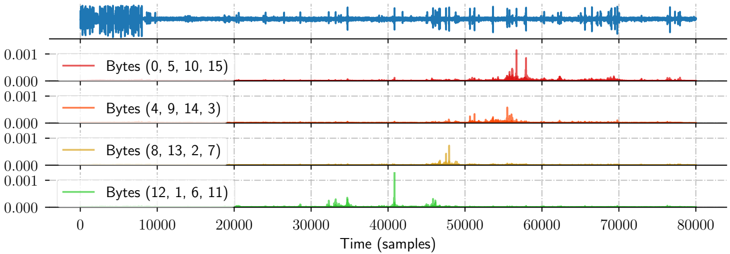


Figure: Gradient Visualization against code polymorphism

Forensics: *"Where does my leakage come from"*?

0	4	8	12
1	5	9	13
2	6	10	14
3	7	11	15

X0X1X2X3

(a) AES state after the first AddRoundKey

0	4	8	12
5	9	13	1
10	14	2	6
15	3	7	11

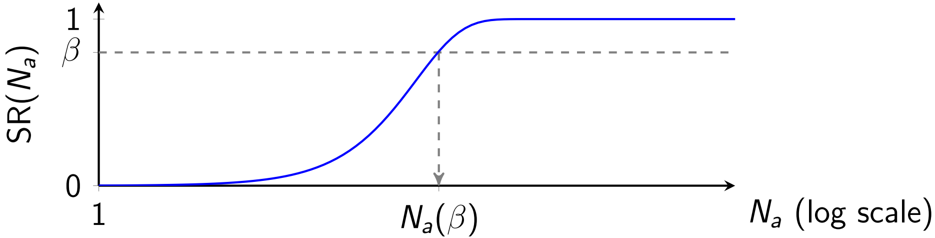
X0X1X2X3

(b) AES state at the end of the ShiftRows

Content

Expensive Metrics to Estimate

Needs to estimate the whole Success Rate (SR) curve to derive $N_a(\beta)$



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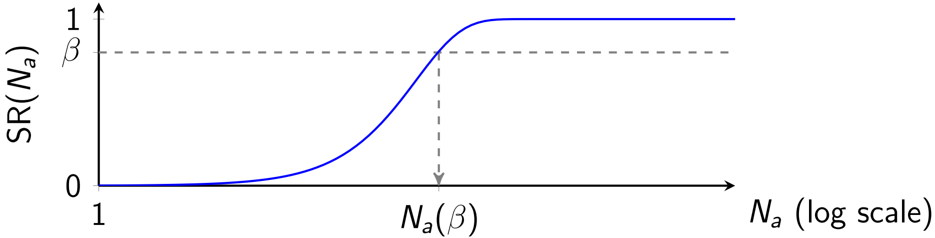

Naive est. of $N_a(\beta)$ expensive:

- Requires $N_v \geq N_a(\beta)$ ✓
- Complexity $\mathcal{O}\left(N_{est} \cdot N_v\right)$ ✗

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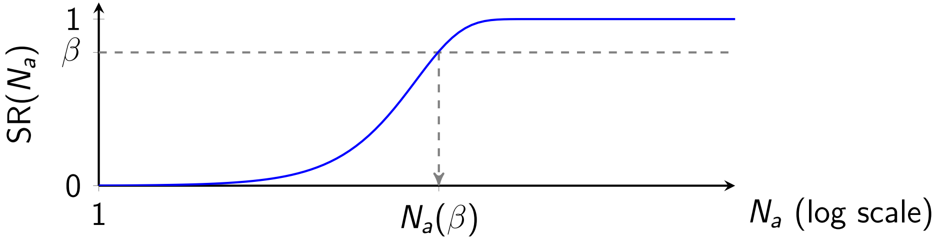

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Cheaper alternative: re-sampling

- Requires $N_v \gg N_a(\beta)$ ✓
- Biased method ✓

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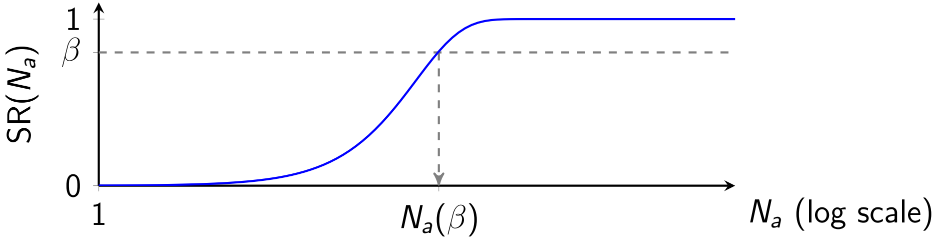

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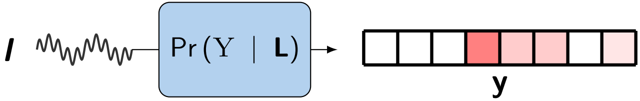
Wish: find a shortcut to estimate $N_a(\beta)$

Shortcut

Solution: characterize the predictions of the adversary's model



[15] D: Kullback - Leibler (KL) divergence, total variation, Euclidean norm, ...

Shortcut

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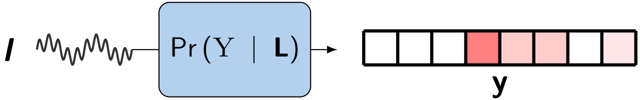
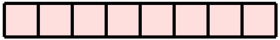


If, the adversary gets:



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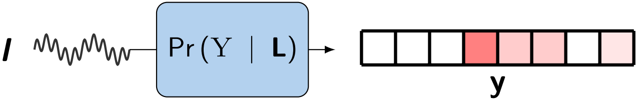

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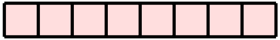

Very noisy leakage

k unpredictable: $N_a(\beta) \to \infty$, if $\beta > \frac{1}{|\mathcal{K}|}$

[15] D: KL divergence, total variation, Euclidean norm, ...

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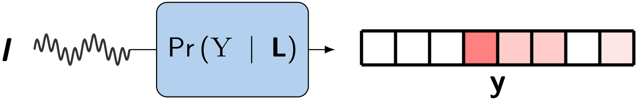

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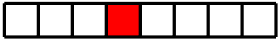

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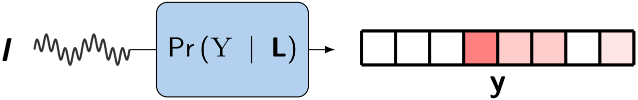

If, the adversary gets:

Low-noise leakage

Exact prediction: $N_a(\beta) = 1$

[15] D: KL divergence, total variation, Euclidean norm, ...

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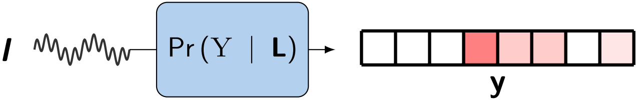
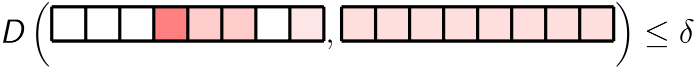


δ-noisy adversary

All the p.m.f.s accessed by the adversary are δ-close[15] to the uniform:



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A First Attempt: the Statistical Distance (SD)

Definition (Statistical Distance (SD))

Statistical Distance (SD) upper bounds the probability to distinguish two leakage distributions given two different keys (useful for cryptographers):

$$\mathsf{SD}\left(\mathrm{Y};\mathbf{L}\right) = \mathop{\mathbb{E}}_{\mathbf{L}}\left[\mathsf{TV}\left(\mathsf{p}_{\mathrm{Y}\mid\mathbf{L}};\mathsf{p}_{\mathrm{Y}}\right)\right], \text{ where } \mathsf{TV}\left(\mathsf{p};\mathsf{m}\right) = \frac{1}{2}\sum_{y\in\mathcal{Y}}\left|\mathsf{p}\left(y\right)-\mathsf{m}\left(y\right)\right|$$

[16] Lower bound: tensorization of SD. Upper bound: Chernoff inequality (+ Slud's for tightness).

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Lemma (Tensorization)

Denote $\mathsf{SD}(\mathrm{Y};\mathbf{L})$ *by* δ, *then the following bounds are tight (**Q6**: prove it):*[16]

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$$\Omega\left(\frac{\beta}{\delta}\right) \leq N_a(\beta)$$

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[16] Lower bound: tensorization of SD. Upper bound: Chernoff inequality (+ Slud's for tightness).

A Second Attempt: the Mutual Information (MI)

Definition (Mutual Information (MI))

$$\mathsf{MI}(\mathrm{Y}; \mathbf{L}) = \mathop{\mathbb{E}}_{\mathbf{L}}\left[\mathsf{D}(\mathsf{p}_{\mathrm{Y} \mid \mathbf{L}} \parallel \mathsf{p}_{\mathrm{Y}})\right], \text{ where } \mathsf{D}(\mathsf{p} \parallel \mathsf{m}) = \sum_{y \in \mathcal{Y}} \mathsf{p}(y) \log\left(\frac{\mathsf{p}(y)}{\mathsf{m}(y)}\right)$$

Lemma (Fano Inequality)

Denote $\mathsf{MI}(\mathrm{Y}; \mathbf{L})$ *by* δ*, then the following inequality is tight (Shannon's coding theorem):*[17]

$$\frac{f(\beta)}{\delta} \leq N_a(\beta)$$

[17] Cover and Thomas, *Elements of information theory (2. ed.)*

Use Cases with Univariate Leakage, Gaussian Noise

Leakage model of shape $\mathrm{L} = \delta(\mathcal{Y}) + N$

$\rightarrow$ Upper bound of MI from Signal-to-Noise Ratio (SNR)

$$N_a(\beta) \geq \frac{f(\beta)}{\mathsf{MI}(\mathrm{Y};\mathrm{L})}$$

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$\rightarrow$ Upper bound of MI from Signal-to-Noise Ratio (SNR)

$$N_a(\beta) \geq \frac{f(\beta)}{\frac{1}{2}\log\left(1 + \frac{\underset{\mathrm{Y}}{\mathsf{Var}}\left(\mathbb{E}[\mathrm{L} \mid \mathrm{Y}]\right)}{\underset{\mathrm{Y}}{\mathbb{E}}\left[\mathsf{Var}(\mathrm{L} \mid \mathrm{Y})\right]}\right)}$$

Use Cases with Univariate Leakage, Gaussian Noise

Leakage model of shape $\mathrm{L} = \delta(\mathcal{Y}) + N$

$\rightarrow$ Upper bound of MI from Signal-to-Noise Ratio (SNR)

$$N_a(\beta) \geq \frac{f(\beta)}{\frac{1}{2}\log\left(1 + \frac{\mathsf{Var}_{\mathrm{Y}}\left(\mathbb{E}[\mathrm{L} \mid \mathrm{Y}]\right)}{\mathbb{E}_{\mathrm{Y}}\left[\mathsf{Var}(\mathrm{L} \mid \mathrm{Y})\right]}\right)}$$

$\rightarrow$ Generalizes the data complexity for correlation attack:

$$N_a^{corr} \approx \frac{28}{\rho^2}$$

Reduction to MI estimation

Estimating a Mutual Information is generally hard:

[18] Paninski, "Estimation of Entropy and Mutual Information".
[19] Masure et al., "Information Bounds and Convergence Rates for Side-Channel Security Evaluators".

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✗ *Empirical* estimator:

→ Positively biased in average ✓

→ Suffers from *curse of dimensionality*[19] ✗

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Definition (PI)

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Definition (PI)

$$\mathsf{PI}\left(\mathrm{Y}; \mathbf{L}; \theta\right) = \mathop{\mathbb{E}}_{\mathbf{L}}\left[\mathsf{D}(F(\mathbf{L}, \theta) \parallel \mathsf{p}_{\mathrm{Y}})\right] \leq \mathsf{MI}\left(\mathrm{Y}; \mathbf{L}\right)$$

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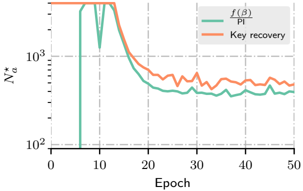
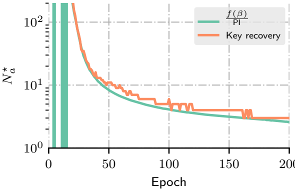
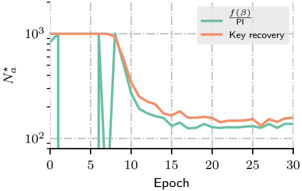
Examples



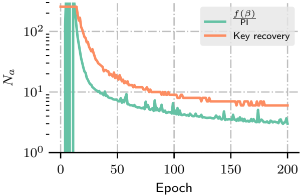
(a) AES FPGA



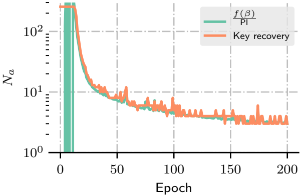
(b) AES with misalignment



(c) AES soft with masking



(d) Polymorphic AES



(e) Polymorphic AES

PI and other Surrogates

PI can be well[20] estimated/optimized over the open sample:

- Estimation error $\epsilon = \mathcal{O}\left(\frac{poly(\mathcal{H})}{N_p}\right)$, where

$\rightarrow$ $N_p = \#$profiling traces

$\rightarrow$ $\mathcal{H}$: class of models (neural network, #parameters, ...)

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- We want the estimation error $\epsilon \lessapprox \mathsf{MI}\left(\mathrm{Y}; \mathrm{L}\right) \implies$

$$N_p(\epsilon) \geq \Omega\left(\frac{poly(\mathcal{H})}{\mathsf{MI}\left(\mathrm{Y}; \mathrm{L}\right)}\right) \gtrapprox N_a(\beta)$$

 $\rightarrow$ "Profiling is a costly as attacking"

[20] Ito, Ueno, and Homma, "Perceived Information Revisited: New Metrics to Evaluate Success Rate of Side-Channel Attacks", Might suffer from inconsistencies in rare cases.

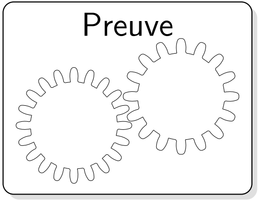
Wrap-Up

Side-Channel Analysis is a threat as powerful (but cheaper) as quantum computers

Need to assess the security level against SCA in an affordable manner
$\Longrightarrow$ evaluation shortcuts

Tomorrow: presentation of masking, how to implement it, security analysis

Preuves de Sécurité pour Masquage



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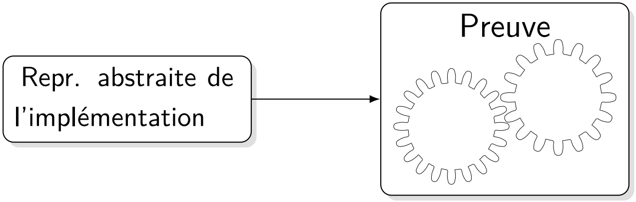

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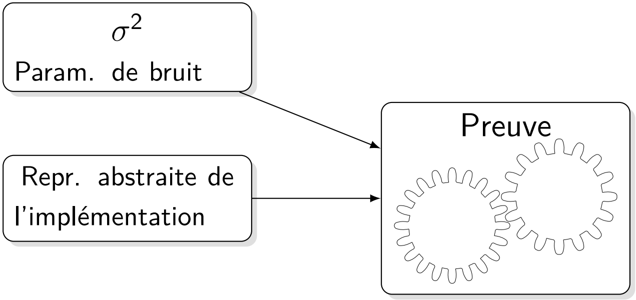

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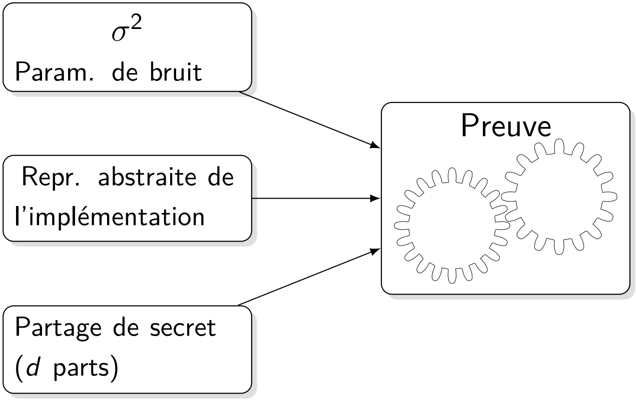

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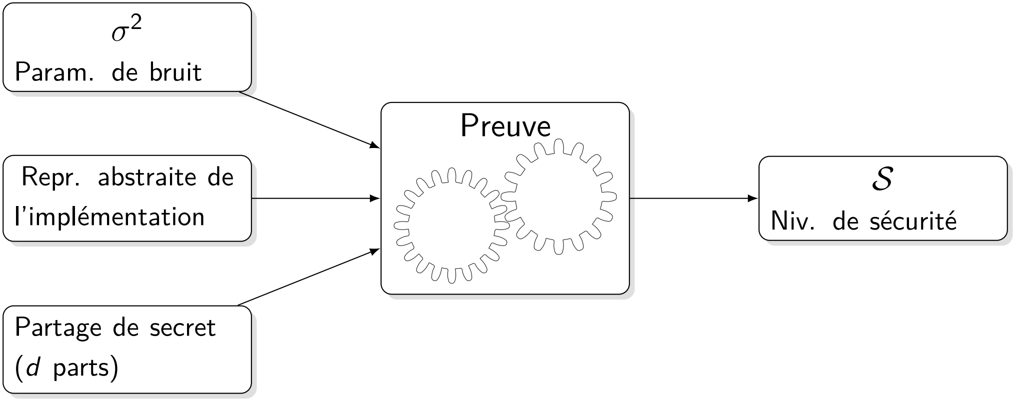

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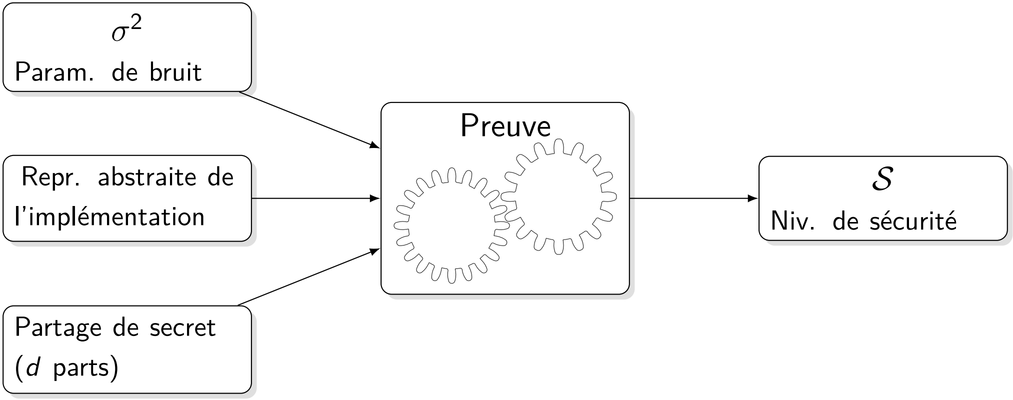

"Toute attaque nécessite $\mathcal{S}$ observations"

Content

Masking: what is that ?

Masking, a.k.a. *MPC on silicon*:[21][22] secret sharing over a finite field $(\mathbb{F}, \oplus, \otimes)$

$$\Upsilon(\text{secret})$$

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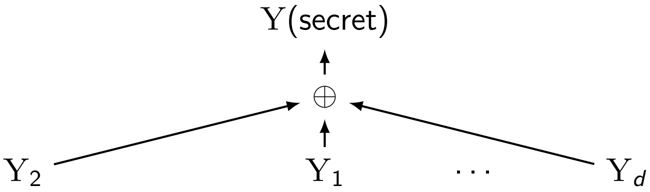


Masking: what is that ?

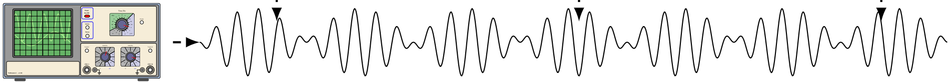
Masking, a.k.a. *MPC on silicon*:[21][22] secret sharing over a finite field $(\mathbb{F}, \oplus, \otimes)$

$$\mathrm{Y}(\text{secret})$$

$$\oplus$$

$$\mathrm{Y}_2 \qquad \mathrm{Y}_1 \qquad \cdots \qquad \mathrm{Y}_d$$

$$\mathrm{L}(\mathrm{Y}_2) = \delta(\mathrm{Y}_2) + N \qquad \mathrm{L}(\mathrm{Y}_1) = \delta(\mathrm{Y}_1) + N \qquad \mathrm{L}(\mathrm{Y}_d) = \delta(\mathrm{Y}_d) + N$$

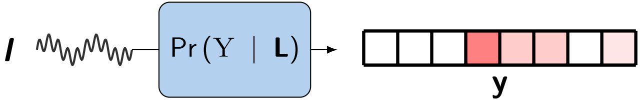


[21] Chari et al., "Towards Sound Approaches to Counteract Power-Analysis Attacks".
[22] Goubin and Patarin, "DES and Differential Power Analysis (The "Duplication" Method)".

Content

The Noisy Leakage Model

In this model, for each intermediate computation, the adversary gets a probability distribution about its operands:



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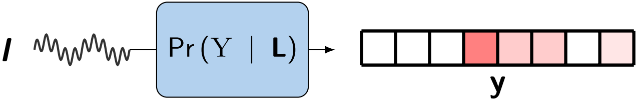
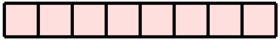


If, the adversary gets:



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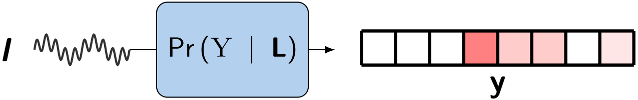

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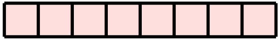

Very noisy
Sensitive computation unpredictable

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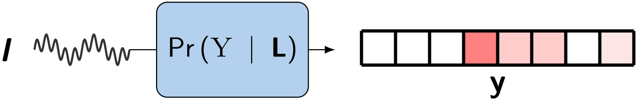

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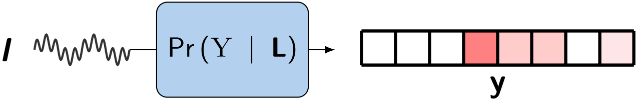


If, the adversary gets:

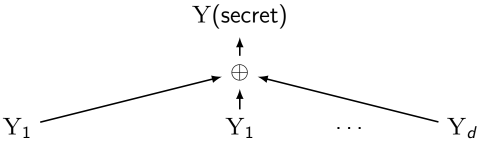
Low-noise

Exact prediction of the sensitive computation

The Effect of Masking

Y(secret)

The Effect of Masking



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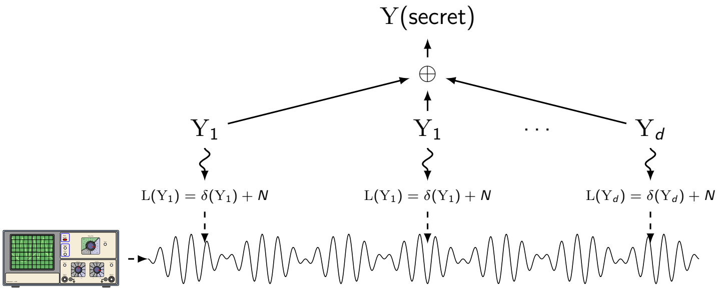

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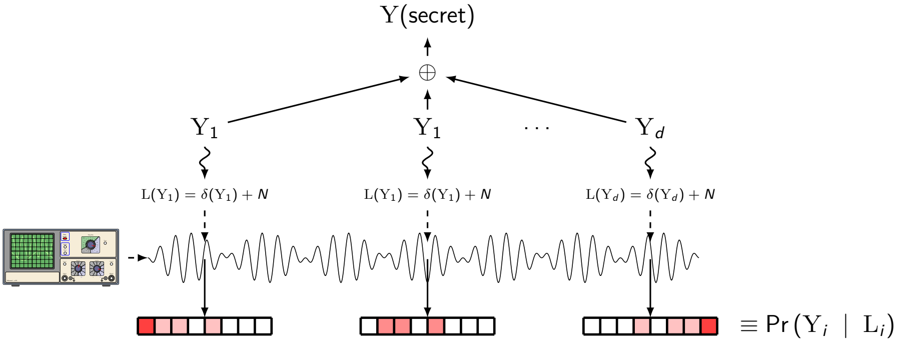

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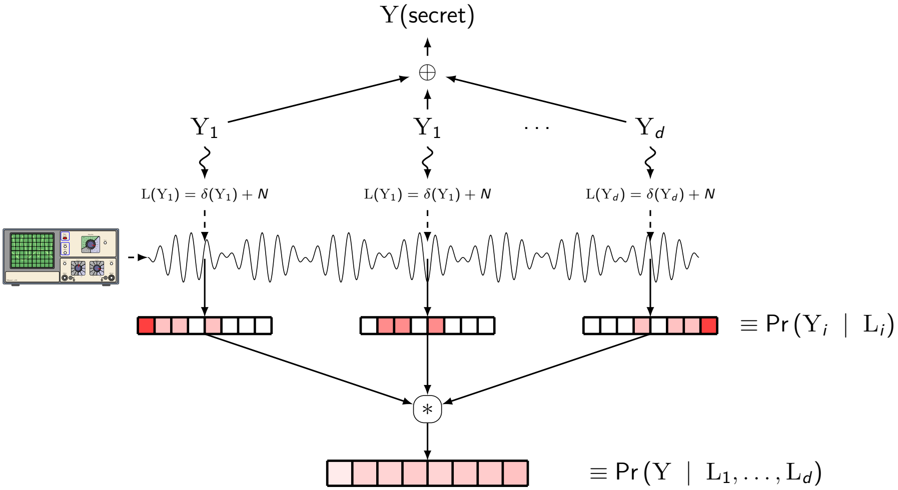

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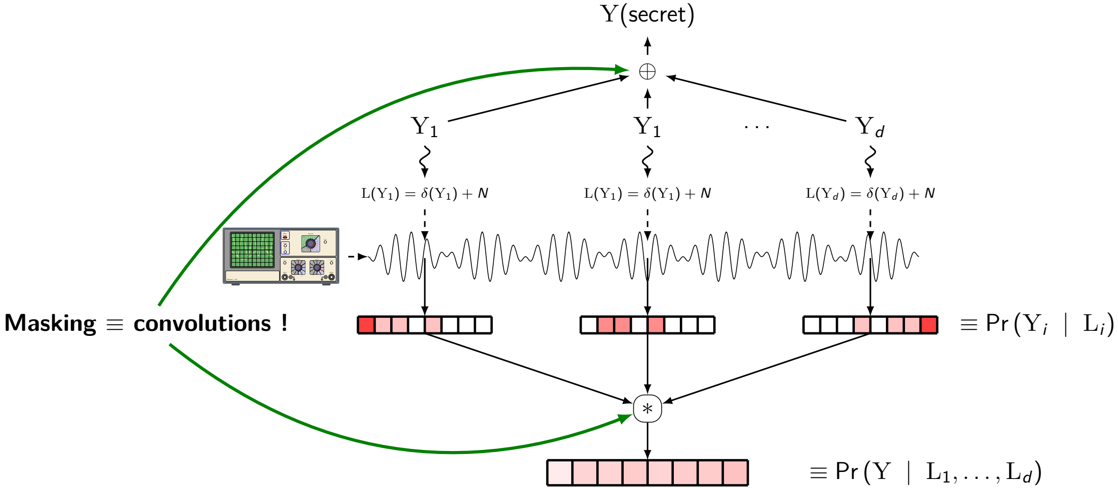
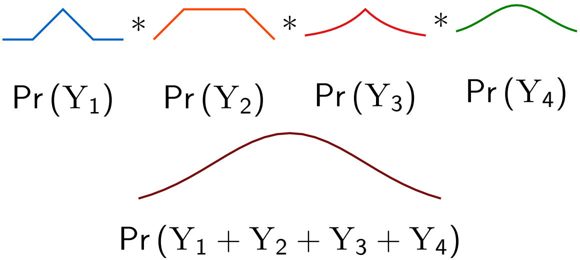


The Secret Power of Convolutions

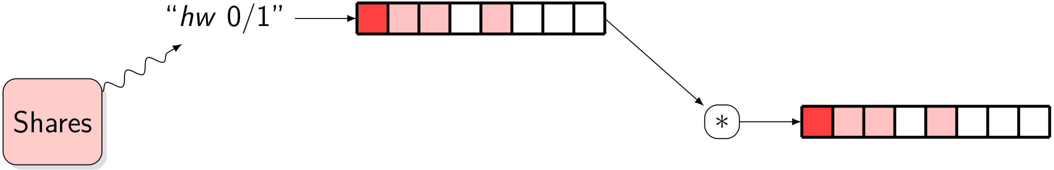
Central Limit Theorem: Assume real-valued random variables Y_i



Then the sum is (approximately) distributed like a Gaussian[23]
Interesting property of Gaussian: maximizes the entropy (*i.e.*, uncertainty)[24]

[23] With mild assumptions, but we'll get back to that ...
[24] Out of all Probability Density Functions (p.d.f.s) of same mean and variance

CLT also Works in Finite Groups/Fields !



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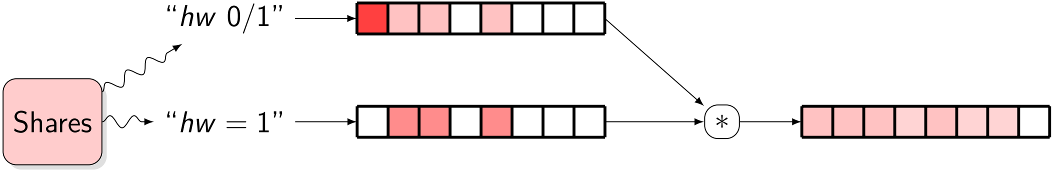

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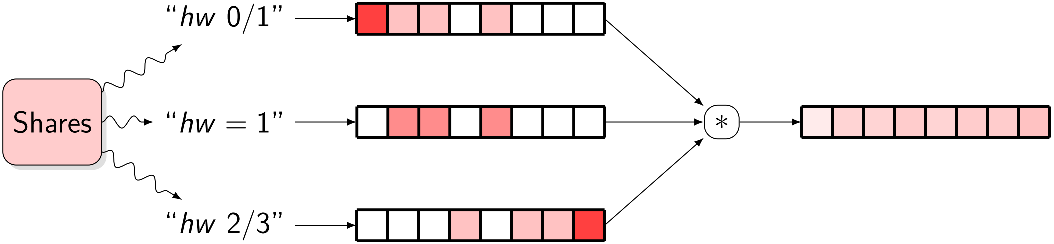

	$Y_i \in \mathbb{R}$	$Y_i \in \mathbb{F}$
Limit Entropy	Gaussian Maximized	Uniform Maximized

Fast Fourier Transform also apply over finite fields !

Quantitative version of CLT

Theorem (Mrs. Gerber's Lemma[25]*)*

Given $\mathrm{Y} = \mathrm{Y}_1 \oplus \ldots \oplus \mathrm{Y}_d$, *and each* Y_i *with (indep.) side information* $\mathrm{L}_1, \ldots, \mathrm{L}_d$, *then for* $\eta^{-1} = 2\log(2)$*:*

[25] Béguinot et al., "Removing the Field Size Loss from Duc et al.'s Conjectured Bound for Masked Encodings".

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$\rightarrow$ Security $\propto \frac{1}{\mathsf{MI}(\mathrm{Y};\mathbf{L})} \implies$ increases **exponentially fast** with d ✓

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→ Security $\propto \frac{1}{\mathsf{MI}(\mathrm{Y};\mathbf{L})} \implies$ increases **exponentially fast** with d ✓

→ Independent of the adversary ✓

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Convolution = Noise Amplification

Simulation, for $\mathbb{F}_{2^n}$: $L(Y_i) = hw(Y_i) + \mathcal{N}(0; \sigma^2)$, hw = Hamming weight

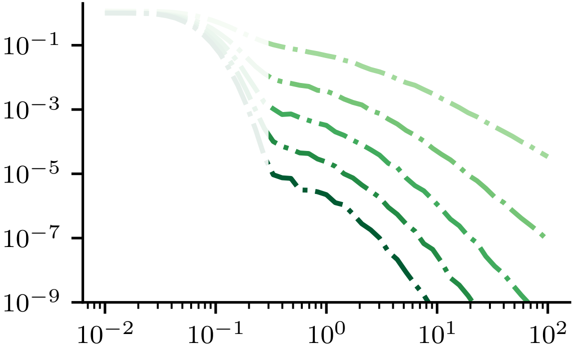

Figure: $\mathrm{MI}(\mathrm{Y}; \mathbf{L})$ vs. σ^2, $2 \leq d \leq 6$

Masking in a Low-Noise Setting

Does masking always work in a low-noise setting ?

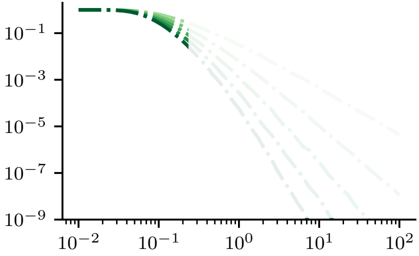

Figure: $MI(Y; Trace)$ vs. σ^2, $2 \leq d \leq 6$

Observation:
Secret always leaks > 1 bit, regardless of d
Explanation:
$\mathsf{lsb}(\mathrm{Y}_1 \oplus \ldots \oplus \mathrm{Y}_d) = \mathsf{lsb}(\mathrm{Y}_1) \oplus \ldots \oplus \mathsf{lsb}(\mathrm{Y}_d)$

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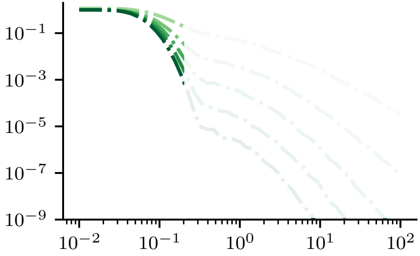

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Observation:
Secret always leaks > 1 bit, regardless of d
Explanation:
$\mathrm{hw}(\mathrm{Y}_1 \oplus \ldots \oplus \mathrm{Y}_d) = \sum_i \mathrm{hw}(\mathrm{Y}_i) - 2 \cdot \left(\ldots \right)$
Parity of $\mathrm{hw}(\mathrm{Y})$: **cosets of** $\mathbb{F}_{2^n}$
Corollary: parallelism is no cure either

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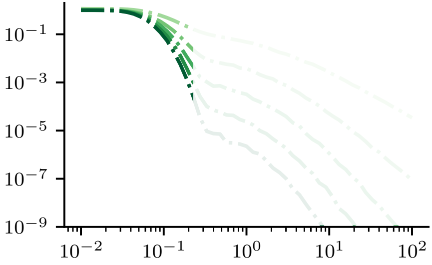

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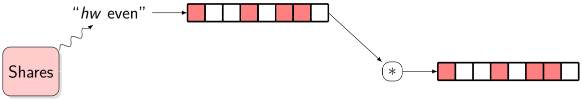

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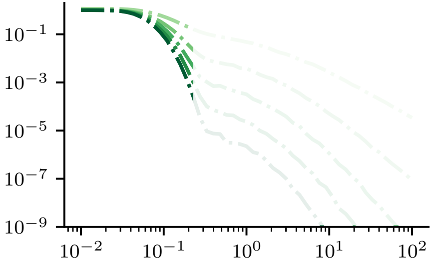

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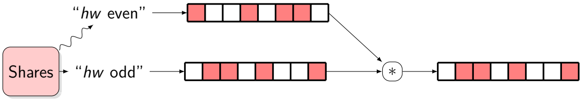

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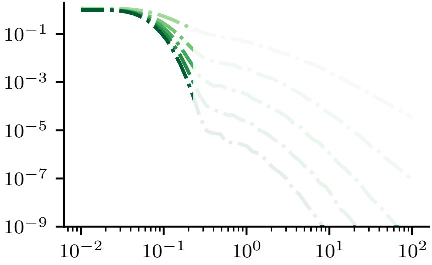

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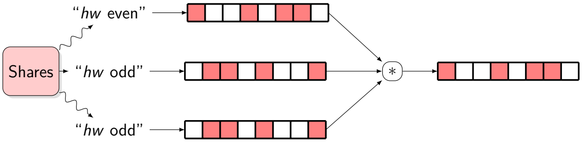
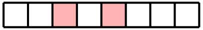


Conditions for Sound Masking

What conditions the distributions  of each share must fit?

[26] Stromberg, "Probabilities on a Compact Group".

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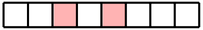
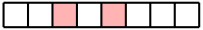
What conditions the distributions  of each share must fit?

"Central Limit Theorem" (qualitative)[26]

Conv. to uniform $\iff$ support *not* contained in any non-trivial coset of $\mathbb{F}$

[26] Stromberg, "Probabilities on a Compact Group".

Conditions for Sound Masking

What conditions the distributions  of each share must fit?

"Central Limit Theorem" (qualitative)[26]

Conv. to uniform $\iff$ support *not* contained in any non-trivial coset of $\mathbb{F}$

In $\mathbb{R}$: mild assumption

$\rightarrow$ Only $\mathbb{Z}$ and $\mathbb{Q}$ (and their respective subgroups)

$\rightarrow$ Negligible measure over $\mathbb{R}$

In finite $\mathbb{F}$: no longer mild in finite fields ...

[26] Stromberg, "Probabilities on a Compact Group".

Two Solutions

Two Solutions

Solution 1: Make sure to leak < 1 bit per share:

- Support of PMF always larger than any coset
- Work with any $\mathbb{F}$ (usually chosen to fit the cipher) ✓
- **Leakage-dependent: not always verified** ✗

Two Solutions

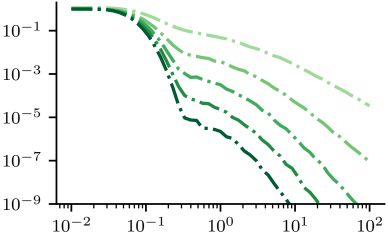
Solution 2: Choose $\mathbb{F}$ without any non-trivial subgroup, *i.e.*, $\mathbb{F}_p$, p prime:

- No assumption on the leakage ✓
- Major change of paradigm:

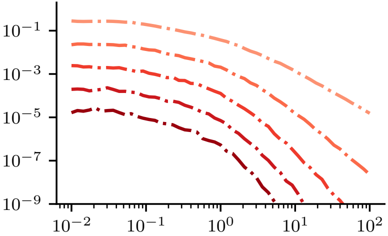
  Fix $\mathbb{F}$ masking-friendly first,

  Then build crypto upon it ✓

Comparing Binary and Prime Fields: a Simulation



(a) $\mathbb{F}_{2^7}$



(b) $\mathbb{F}_{2^7-1}$

Figure: Comparing binary and prime fields.

Content

Threshold Probing Model

Definition (t-privacy)

Any tuple of t intermediate values $\perp$ secrets

[27] Not $\iff$, see Bordes, "Security of symmetric primitives and their implementations", Example 5.5

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A set of probes $\mathcal{P}$ in a circuit $\mathbb{C}$ can be simulated with the input shares $\mathcal{I}$ if there exists an algorithm $\mathcal{S}$ (the *simulator*) such that

$$\mathcal{P} \overset{d}{=} \mathcal{S}(\mathcal{I})$$

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Q8: For a circuit with d shares, prove that d-NI $\implies$ d-privacy[27]

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$$\mathcal{P} \overset{d}{=} \mathcal{S}(\mathcal{I}) \iff \mathcal{P} \perp \text{all inputs except } \mathcal{I}$$

Definition (t-non-interference (NI))

$\mathbb{C}$ is t-NI if *any* set of t probes is simulatable by *at most t* shares of each input

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The Composition Paradigm

Idea to make a circuit NI:

[28] Ishai, Sahai, and Wagner, "Private Circuits: Securing Hardware against Probing Attacks".
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Issue: NI is not composable[30] ✗**Q8**: example on white board

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Strong Non-Interference[32]

Definition (t-Strong Non-Interference)

A gadget is t-SNI if any set of t_1 internal probes and t_2 output probes can be simulated with t_1 shares of each input sharing, and

$$t = t_1 + t_2$$

$\rightarrow$ SNI $\implies$ NI $\implies$ privacy

Other composable notions: SNIo, PINI[31], robust probing, glitch-extended, ...

[31] Cassiers and Standaert, "Trivially and Efficiently Composing Masked Gadgets With Probe Isolating Non-Interference".

[32] Barthe et al., "Strong Non-Interference and Type-Directed Higher-Order Masking".

Masked addition gadget

Inputs:

$$\begin{aligned} [\![A]\!] &= (A_1, \ldots, A_d) \\ [\![B]\!] &= (B_1, \ldots, B_d) \end{aligned}$$

Output:

$$[\![C]\!] = (C_1, \ldots, C_d)$$

such that

$$\sum_i C_i = \left(\sum_i A_i\right) \oplus \left(\sum_i B_i\right)$$

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- t-NI + t-SNI refresh $\implies$ t-SNI ✓
- Generalization: share-wise application of any affine map

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Inputs:

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Output:

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BadMult algorithm:

$$\begin{aligned} C_1 &= (A_1 \otimes B_1 \quad) \oplus (A_1 \otimes B_2 \quad) \oplus (A_1 \otimes B_3 \quad) \\ C_2 &= (A_2 \otimes B_1 \quad) \oplus (A_2 \otimes B_2 \quad) \oplus (A_2 \otimes B_3 \quad) \\ C_3 &= (A_3 \otimes B_1 \quad) \oplus (A_3 \otimes B_2 \quad) \oplus (A_3 \otimes B_3 \quad) \end{aligned}$$

Correct, but not 2-NI. **Q7**: Why ?

Masked multiplication gadget

Inputs:

$$
\begin{aligned}
[\![A]\!] &= (A_1, \ldots, A_d) \\
[\![B]\!] &= (B_1, \ldots, B_d)
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$$

Output:

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- SecMult is $(d-1)$-SNI ✓

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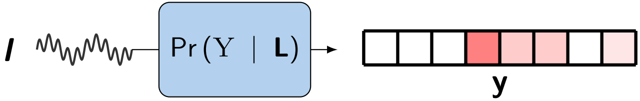
\end{aligned}
$$

- SecMult is $(d-1)$-SNI ✓
- If $[\![B]\!] = (1, 0, \ldots, 0)$, then SecMult$([\![A]\!], [\![B]\!])$ = Refresh$([\![A]\!])$ ✓

Content

Recall on Noisy Leakage Model

Yesterday:



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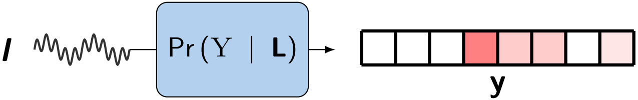
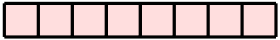


If, the adversary gets:



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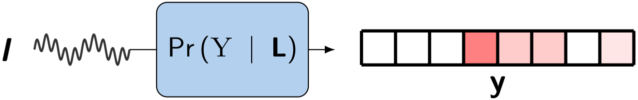

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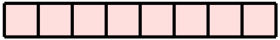

Very noisy leakage
Y indistinguishable from blind guess

Recall on Noisy Leakage Model

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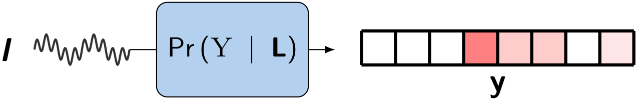

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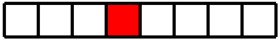

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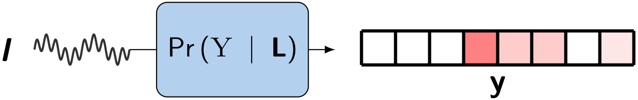

If, the adversary gets:

Low-noise leakage

Exact prediction for Y

Recall on Noisy Leakage Model

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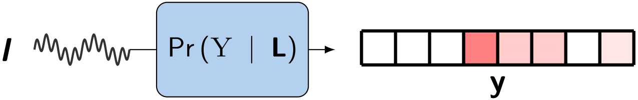

δ-NOISY ADVERSARY

Any intermediate computation Y leaks $\mathrm{L}(\mathrm{Y})$ such that:

$$\mathsf{SD}\left(\mathrm{Y};\mathrm{L}\right) = \mathop{\mathbb{E}}_{\mathrm{L}}\left[TV \left(\underbrace{\qquad\qquad}_{\mathsf{Pr}(\mathrm{Y}\mid\mathrm{L})}, \underbrace{\qquad\qquad}_{\mathsf{Pr}(\mathrm{Y})} \right)\right] \leq \delta$$

Security Proof for a Gadget

Consider a gadget with ℓ intermediate computations:

$$\boxed{\begin{matrix} X_1 \\ X_2 \\ \vdots \\ X_\ell \end{matrix}}$$

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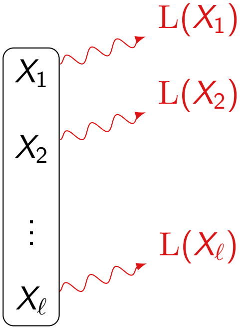
Consider a gadget with ℓ δ-noisy intermediate computations:



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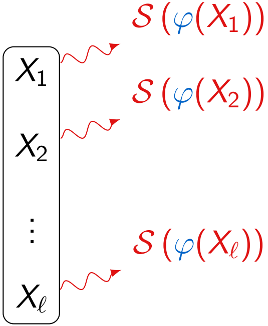

Lemma (Simulatability by RP)

The leakage function L *can be simulated from a random probing adversary:* $\varphi(x)$ *exactly reveals* x *with probability*
$\epsilon = 1 - \sum_l \min_x \Pr\left(\mathrm{L}(x) = l\right) \leq \delta \cdot |\mathbb{F}|$.[a]

[a] Duc, Dziembowski, and Faust, "Unifying Leakage Models: From Probing Attacks to Noisy Leakage".

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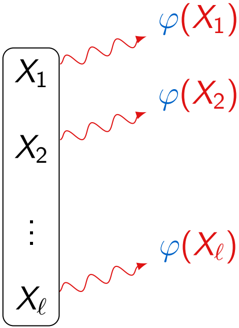

We may reduce to an adversary observing $\varphi(X)$ instead of $\mathcal{S}(\varphi(X))$ (Data Processing Inequality)

Proof of the Core Lemma (I)

Assume there exists such a simulator $\mathcal{S}$,

Proof of the Core Lemma (I)

Assume there exists such a simulator $\mathcal{S}$, we need to construct it for all inputs:

$$\begin{aligned} \Pr(\mathcal{S}(x) = l) &= \ldots, \text{ for all } x \\ \Pr(\mathcal{S}(\perp) = l) &= \ldots \end{aligned}$$

Constraints:

Proof of the Core Lemma (I)

Assume there exists such a simulator $\mathcal{S}$, we need to construct it for all inputs:

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Constraints:

$\rightarrow$ For all input x, $\Pr(\mathcal{S}(x))$ should be a p.m.f. $(2 \cdot |\mathbb{F}|$ (in)equations)

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→ For the input $\perp$, $\Pr(\mathcal{S}(\perp))$ should be a p.m.f. (2 (in)equations)

Proof of the Core Lemma (I)

Assume there exists such a simulator $\mathcal{S}$, we need to construct it for all inputs:

$$\begin{aligned} \Pr\left(\mathcal{S}\left(x\right) = l\right) &= \ldots, \text{ for all } x \\ \Pr\left(\mathcal{S}\left(\perp\right) = l\right) &= \ldots \end{aligned}$$

Constraints:

→ For all input x, $\Pr\left(\mathcal{S}\left(x\right)\right)$ should be a p.m.f. ($2 \cdot |\mathbb{F}|$ (in)equations)

→ For the input $\perp$, $\Pr\left(\mathcal{S}\left(\perp\right)\right)$ should be a p.m.f. (2 (in)equations)

→ For any x, l, $\Pr\left(\mathcal{S}\left(\varphi(x)\right) = l\right) = \Pr\left(\mathrm{L}(x) = l\right)$ ($|\mathbb{F}| \times |\mathcal{L}|$ equations)

Proof of the Core Lemma (II)

Let us start from the last constraint. For any x and any l:

$$\Pr\left(\mathrm{L}(x) = l\right) = \Pr\left(\mathcal{S}\left(\varphi(x)\right) = l\right)$$

Proof of the Core Lemma (II)

Let us start from the last constraint. For any x and any l:

$$\begin{aligned} \Pr(\mathrm{L}(x) = l) &= \Pr(\mathcal{S}(\varphi(x)) = l) \\ &= \Pr(\varphi(x) = x) \cdot \Pr(\mathcal{S}(x) = l) + \Pr(\varphi(x) = \perp) \cdot \Pr(\mathcal{S}(\perp) = l) \end{aligned}$$

Proof of the Core Lemma (II)

Let us start from the last constraint. For any x and any l:

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&= \Pr\left(\varphi(x) = x\right) \cdot \Pr\left(\mathcal{S}\left(x\right) = l\right) + \Pr\left(\varphi(x) = \perp\right) \cdot \Pr\left(\mathcal{S}\left(\perp\right) = l\right) \\
&= \epsilon \cdot \Pr\left(\mathcal{S}\left(x\right) = l\right) + (1 - \epsilon) \cdot \Pr\left(\mathcal{S}\left(\perp\right) = l\right)
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\end{aligned}$$

Hence,

$$0 \leq \Pr\left(\mathcal{S}\left(\perp\right) = l\right) = \frac{\overbrace{\Pr\left(\mathrm{L}(x) = l\right) - \epsilon \cdot \Pr\left(\mathcal{S}\left(x\right) = l\right)}^{\text{Should not depend on } X}}{1 - \epsilon}$$

Proof of the Core Lemma (II)

Let us start from the last constraint. For any x and any l:

$$\begin{aligned}
\Pr(\mathrm{L}(x) = l) &= \Pr(\mathcal{S}(\varphi(x)) = l) \\
&= \Pr(\varphi(x) = x) \cdot \Pr(\mathcal{S}(x) = l) + \Pr(\varphi(x) = \perp) \cdot \Pr(\mathcal{S}(\perp) = l) \\
&= \epsilon \cdot \Pr(\mathcal{S}(x) = l) + (1 - \epsilon) \cdot \Pr(\mathcal{S}(\perp) = l)
\end{aligned}$$

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&= \epsilon \cdot \Pr\left(\mathcal{S}\left(x\right) = l\right) + (1 - \epsilon) \cdot \Pr\left(\mathcal{S}\left(\perp\right) = l\right)
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Furthermore, summing (2) over l, by definition of probability distributions,

$$\sum_l \pi(l) = \underbrace{\sum_l \Pr\left(\mathrm{L}(x) = l\right)}_{=1} - \epsilon \cdot \underbrace{\sum_l \Pr\left(\mathcal{S}\left(x\right) = l\right)}_{=1}$$

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Hence,

$$\epsilon = 1 - \sum_{l} \pi(l) \geq 1 - \sum_{l} \min_{x} \Pr(\mathrm{L}(x) = l)$$

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Security against a Random Probing Adversary

To succeed, at least d out of ℓ wires must be revealed to the adversary:

$$\Pr\left(\text{Adv. wins}\right) \leq \Pr\left(\text{At least } d \text{ wires revealed}\right)$$

[33] Boucheron, Lugosi, and Massart, *Concentration Inequalities: A Nonasymptotic Theory of Independence*, P.24, and Ex. 2.11.

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If ℓ wires, each independently revealed with proba. ϵ:

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Q11: Prove the inequality from a particular case of Chernoff inequality[33]

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Putting all Together

In our context, $\ell \leq \mathcal{O}\left(d^2\right)$ (for $\otimes$ gadget), and $\epsilon \leq \delta \cdot |\mathbb{F}|$:

Theorem (Security Bound)

For a single gadget with $\ell \leq \mathcal{O}\left(d^2\right)$ intermediate computations:

$$\mathsf{SD}\left(k; \mathbf{L}\right) \leq \mathcal{O}\left(\left(7e \cdot d \cdot \delta \cdot |\mathbb{F}|\right)^d\right)$$

[34] *t*-Region-probing secure: NI, with *t* probes from *each* gadget

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For the whole circuit $\mathbb{C}$, $d/2$-region probing[34] security implies

$$\mathsf{SD}\left(k; \mathbf{L}\right) \leq \mathcal{O}\left(|\mathbb{C}| \left(7e \cdot d \cdot \delta \cdot |\mathbb{F}|\right)^{d/2}\right)$$

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Remarks on the Proof

- "Exponential" security ✓

[35] Brian, Dziembowski, and Faust, "From Random Probing to Noisy Leakages Without Field-Size Dependence".

[36] Belaïd, Rivain, and Taleb, "On the Power of Expansion: More Efficient Constructions in the Random Probing Model".

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→ New constructions with better (even constant) leakage rates[36] ✓

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Content

Masking Post-Quantum Cryptography: Kyber

- Basic arithmetic over $\mathbb{Z}_q$, with q prime
  - ✓ Friendly with arithmetic masking
- *Fujisaki-Okamoto* transform:
  - ✗ Needs to convert masks A2B: complexity $\mathcal{O}\left(d^2 \log(d)\right)$
  - ✗ Needs to convert masks B2A: complexity $\mathcal{O}\left(d^2\right)$
  - ✗ Needs to mask hash functions: very expensive

Slow ops unmasked (NTT) become “fast” with higher-order masking ✓
Fast ops unmasked (rejection in Dilithium) become slow with higher-order masking ✗
Alternative masking-friendly signature schemes proposed (Raccoon) ✓

Content

Conclusion

Challenges for masking generally:

$\rightarrow$ Improving the reduction from noisy leakage to probing security

$\rightarrow$ Can we prove directly in the noisy model ?

$\rightarrow$ What about non-independent leakage randomness ?

Conclusion

Challenges for masking generally:

→ Improving the reduction from noisy leakage to probing security

→ Can we prove directly in the noisy model ?

→ What about non-independent leakage randomness ?

Challenges for masking in PQC:

→ Analysis in the RP/noisy model: current implementations deviate from the arithmetic circuit ✓

→ Having masking-friendly primitives

→ Make masked BIKE affordable

→ Masking MQ-like: not thoroughly explored yet ...

Pointers

Interested ?

- Coron's keynote at CARDIS 23 on masking lattice-based cryptography
- Cassiers' keynote at COSADE 23 on masking composability
- Nicolas Bordes' thesis with nice examples of probing notions.

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