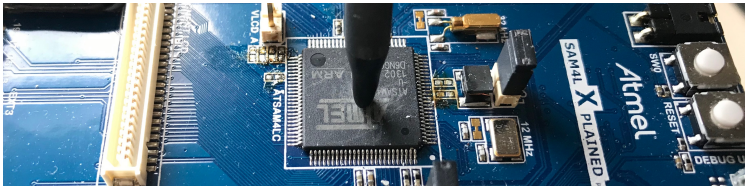

A Decade of Masking Security Proofs

Loïc Masure

WRACH, 23 Avril 2025, Roscoff

cnrs LIRMM UNIVERSITÉ DE MONTPELLIER

Agenda

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- Background & Intuitions
- Provably Secure Masking

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Tight Reduction

Conclusion

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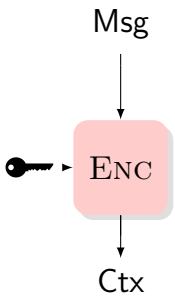

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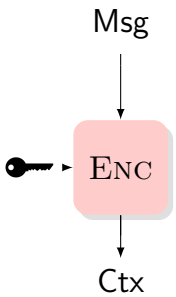

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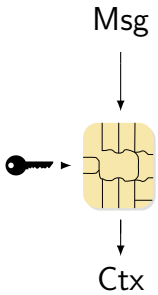

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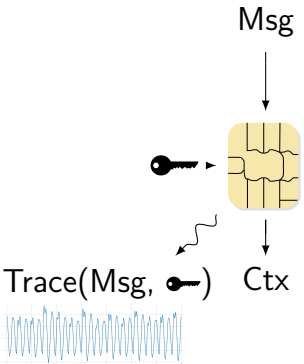

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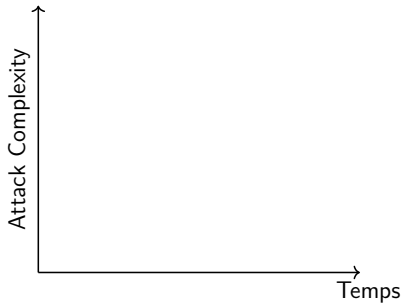

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Attack approach (industry):

[a] Shamelessly stolen to O. Bronchain

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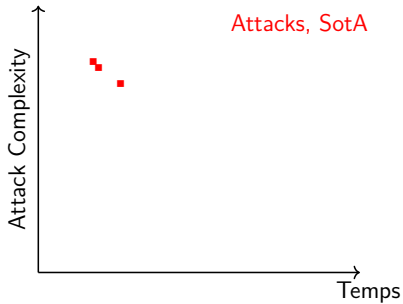

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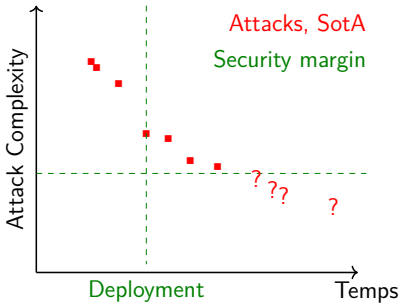

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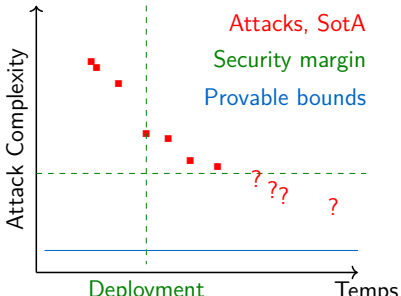

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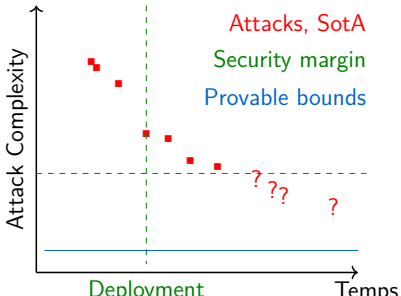

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Today's agenda: evaluation by proofs

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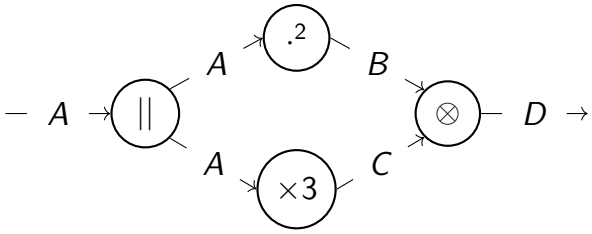
Statement of the Problem



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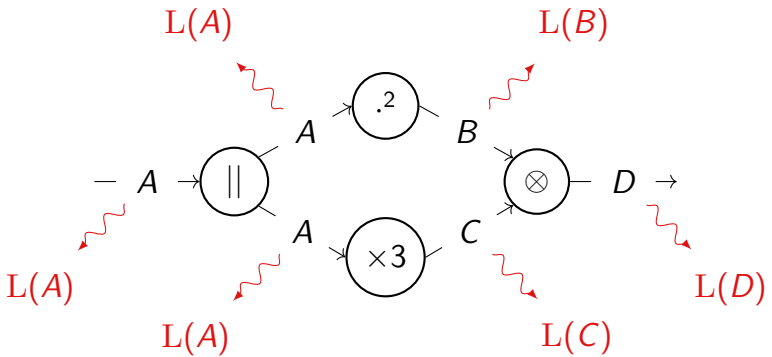

For each wire X, a leakage function $\mathrm{L}(X)$ is revealed to the adversary.

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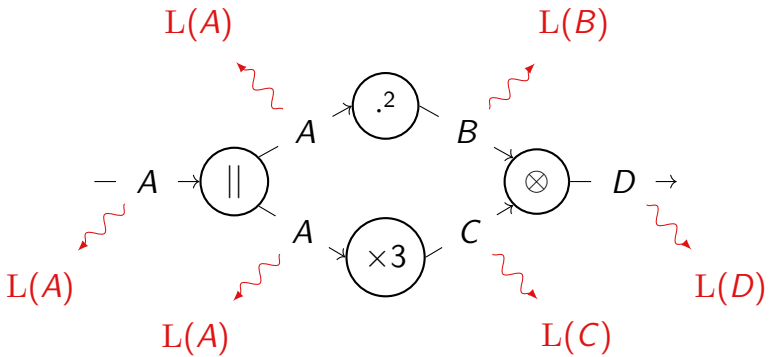

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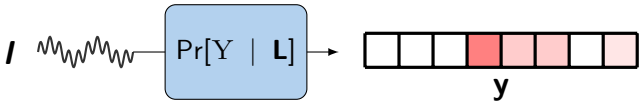
How informative **L** about A?

The Noisy Leakage Model



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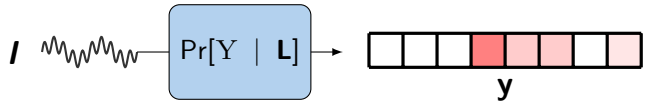
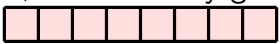


If, the adversary gets:



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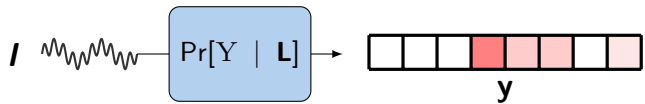

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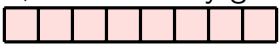

Very noisy leakage
Y indistinguishable from blind guess

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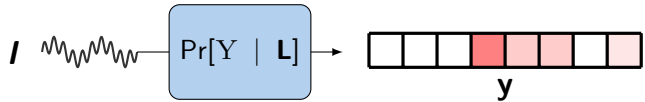

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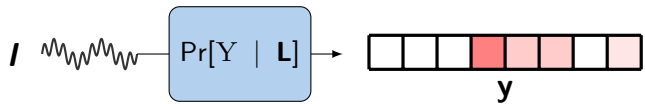
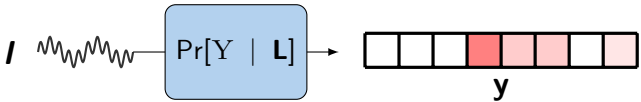


If, the adversary gets:

Low-noise leakage
Exact prediction for Y

The Noisy Leakage Model



δ-NOISY ADVERSARY

Any intermediate computation Y leaks $\mathrm{L}(\mathrm{Y})$ such that:

$$\mathsf{SD}\left(\mathrm{Y};\mathrm{L}\right) = \mathop{\mathbb{E}}_{\mathrm{L}}\left[\mathsf{TV}\left(\underbrace{}_{\Pr[\mathrm{Y}\mid\mathrm{L}]}, \underbrace{}_{\Pr[\mathrm{Y}]}\right)\right] \leq \delta$$

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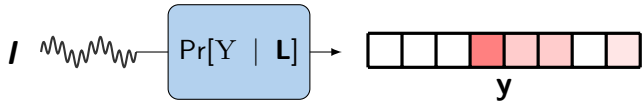

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Main assumption: every observed leakage is δ-noisy

Masking: what is that ?

Masking, a.k.a. *MPC on silicon*:[12] secret sharing over a finite field $(\mathbb{F}, \oplus, \otimes)$

$$\mathrm{Y}(\text{secret})$$

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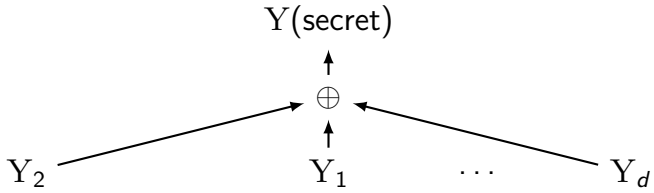

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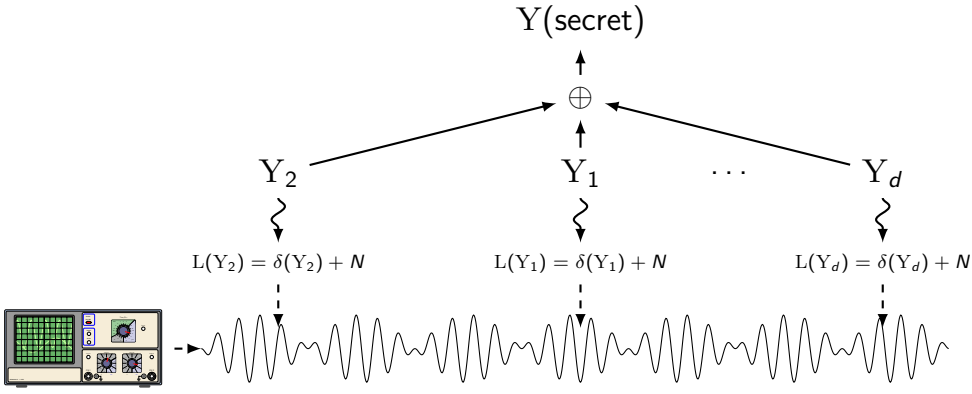

[1] Chari et al., "Towards Sound Approaches to Counteract Power-Analysis Attacks".

[2] Goubin and Patarin, "DES and Differential Power Analysis (The "Duplication" Method)".

The Effect of Masking

$\mathrm{Y}(\text{secret})$

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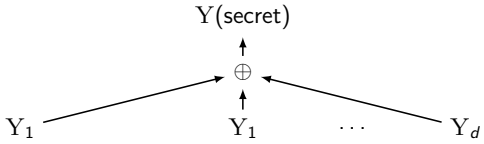

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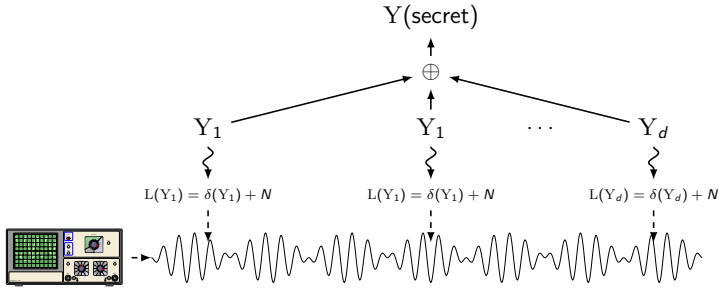

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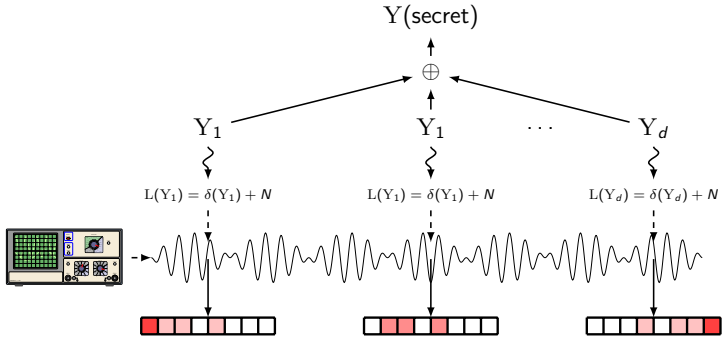

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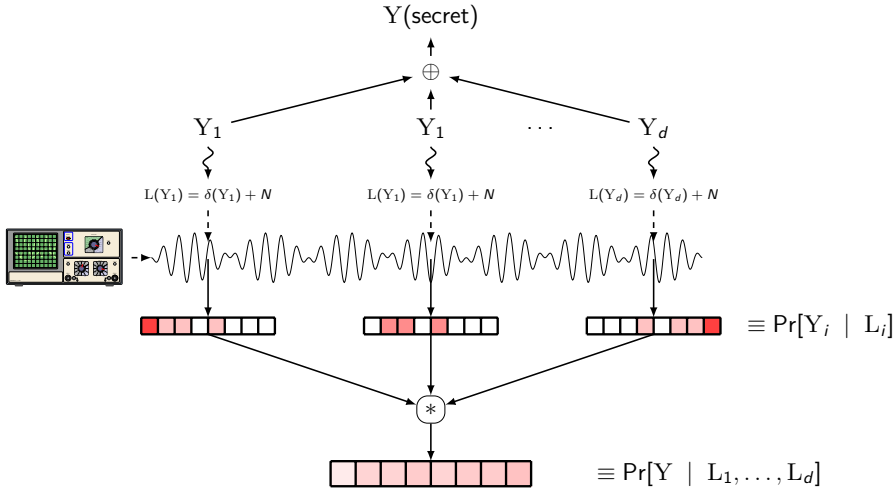

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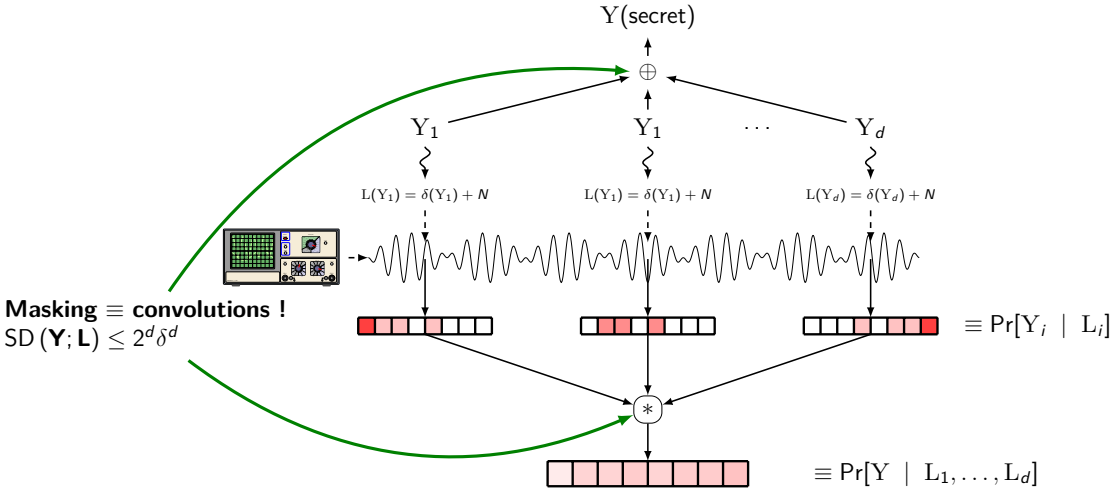

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Idea to make a masked circuit

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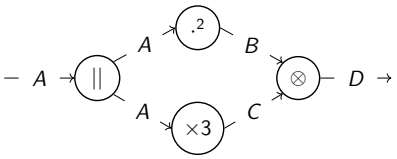

- View your algorithm as a circuit

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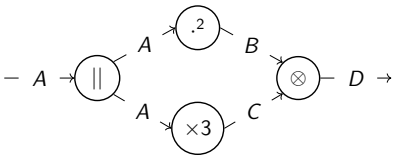

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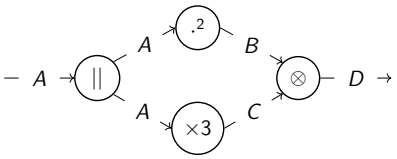

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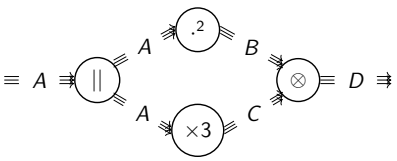

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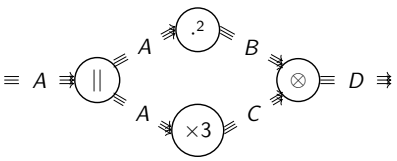

- View your algorithm as a circuit
  → Made of `not`, `and` gates [3]
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- Replace each gate by a masked *gadget*
- Et voilà !**

For now, let's assume the whole circuit to be *probing secure*: every subset of $d-1$ wires is independent from the secret.

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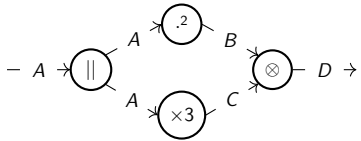
[4] Rivain and Prouff, "Provably Secure Higher-Order Masking of AES".

Security Proof for a Gadget

Consider a gadget with ℓ intermediate computations:



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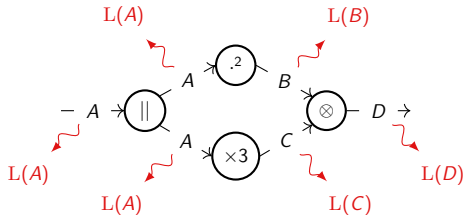

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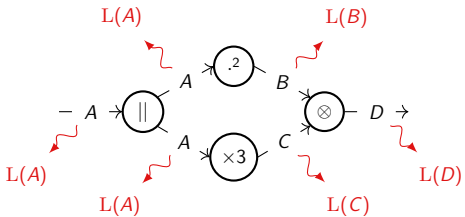

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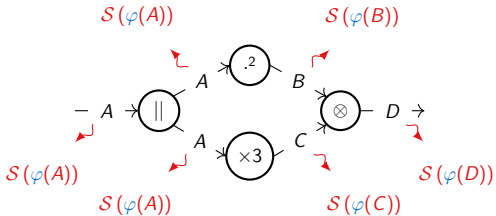

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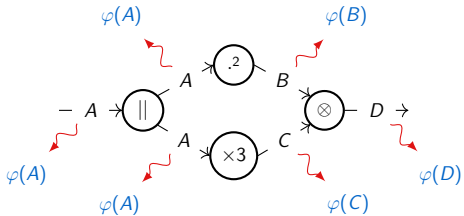

Data-processing Inequality

If for any x the leakage function $\mathrm{L}(x)$ may be expressed as $\mathcal{S}(\varphi(x))$, then: advantage from $\mathrm{L}(x) \leq$ advantage from $\varphi(x)$

Reduction from Noisy Leakage to Random Probing

Lemma (Simulatability by Random Probing)

The leakage function L *can be simulated from a random probing adversary:* $\varphi(x)$ *reveals* x *with probability* $\epsilon = 1 - \sum_l \min_x \Pr[\mathrm{L}(x) = l] \leq \delta \cdot |\mathbb{F}|$.[5]

Random probing model: easier to analyze for leakage from computations

[5] Duc, Dziembowski, and Faust, "Unifying Leakage Models: From Probing Attacks to Noisy Leakage".

Security against a Random Probing Adversary

To succeed, at least d out of ℓ wires must be revealed to the adversary:

$$\Pr[\text{Adv. learns sth}] \leq \Pr[\text{At least } d \text{ wires revealed}]$$

[6] Boucheron, Lugosi, and Massart, *Concentration Inequalities: A Nonasymptotic Theory of Independence*, P.24, and Ex. 2.11.

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Theorem (Chernoff Concentration Inequality[6])

If ℓ wires, each independently revealed with proba. ϵ:

$$\Pr[\textit{At least } d \textit{wires revealed}] \leq \left(\frac{e \cdot \ell \cdot \epsilon}{d}\right)^d$$

[6] Boucheron, Lugosi, and Massart, *Concentration Inequalities: A Nonasymptotic Theory of Independence*, P.24, and Ex. 2.11.

Putting all Together

In our context, $\ell \leq \mathcal{O}\left(d^2\right)$ (for $\otimes$ gadget), and $\epsilon \leq \delta \cdot |\mathbb{F}|$:

Theorem (Security Bound)

For a single gadget with $\ell \leq \mathcal{O}\left(d^2\right)$ intermediate computations:

$$\mathsf{SD}\left(k; \mathbf{L}\right) \leq \left(\mathcal{O}\left(d\right) \cdot \delta \cdot |\mathbb{F}|\right)^d$$

Putting all Together

In our context, $\ell \leq \mathcal{O}\left(d^2\right)$ (for $\otimes$ gadget), and $\epsilon \leq \delta \cdot |\mathbb{F}|$:

Theorem (Security Bound)

For a single gadget with $\ell \leq \mathcal{O}\left(d^2\right)$ intermediate computations:

$$\mathsf{SD}\left(k; \mathbf{L}\right) \leq \left(\mathcal{O}\left(d\right) \cdot \delta \cdot |\mathbb{F}|\right)^d$$

For the whole circuit $\mathbb{C}$,

$$\mathsf{SD}\left(k; \mathbf{L}\right) \leq \left(|\mathbb{C}| \cdot \mathcal{O}\left(d\right) \cdot \delta \cdot |\mathbb{F}|\right)^d$$

Main Challenge

For the whole circuit $\mathbb{C}$,

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A few numbers:

$$d(2, 3, 4, ..., 16) \ll |\mathsf{C}|(\approx 10^3, 10^5), |\mathbb{F}|(256, 2^{23}, 2^{50})$$

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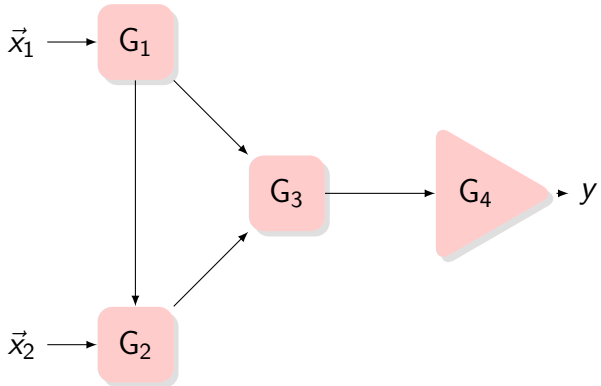

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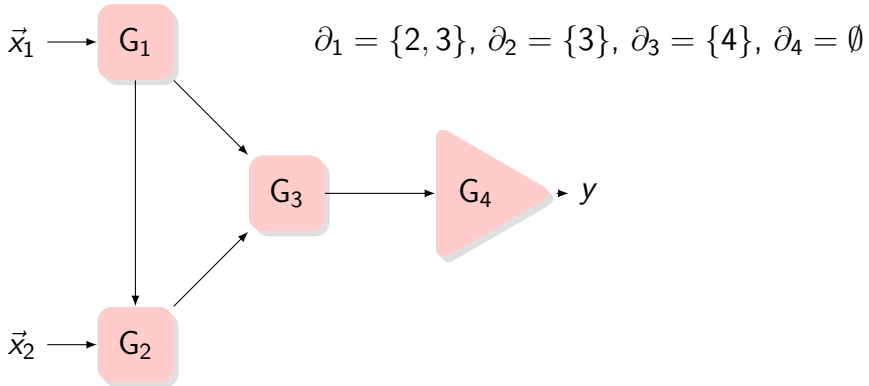

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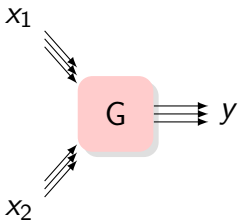
∂_i: set of all subsequent gadgets linked to G_i

Strong Non-Interference[8]

Definition (t-Strong Non-Interference)

A gadget G is t-SNI



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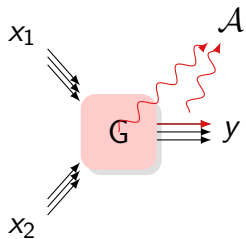
A gadget G is t-SNI if any set W^{G} of internal probes and any set J^{G} of output probes such that $\left|W^{\mathsf{G}}\right| + \left|J^{\mathsf{G}}\right| \leq t$



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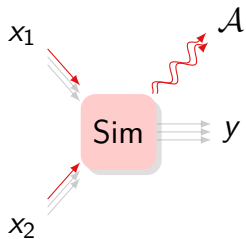

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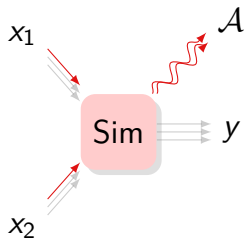

→ Composable : circ. SNI iff all gadgets SNI
→ SNI $\implies$ probing security
→ Extends to multiple outputs[7]

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Non-Interference with Public Outputs[10]

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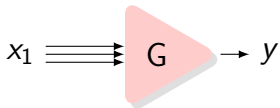

[9] Coron et al., *High-order Polynomial Comparison and Masking Lattice-based Encryption*
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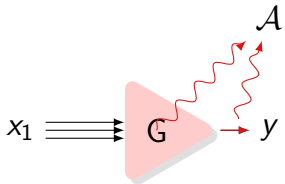

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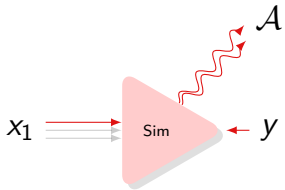

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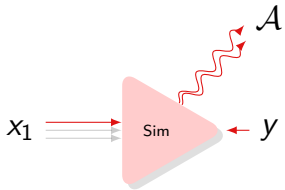

$\rightarrow$ Output assumed to be public anyway
$\rightarrow$ Built from strong Refreshing [9]

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Composition Theorem

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Assume: (1) Each output gadget $(d-1)$*-Nlo;*

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Assume: (1) Each output gadget $(d-1)$-NIo; (2) Each internal gadget t_i-SNI; (3) Each copy gadget connected to different gadgets; then, C is secure with proba $\geq 1-\eta$ such that:

$$\eta \leq \sum_{\substack{i=1 \\ G_i \text{ not output}}}^{|C|} \left(e \cdot \frac{|G_i| + \sum_{j \in \partial_i} |G_j|}{t_i + 1} \cdot \epsilon \right)^{t_i+1}.$$

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Corollary

The d-share ISW compiler is $|\mathsf{C}| \cdot (\mathcal{O}(d) \cdot |\mathbb{F}| \cdot \delta)^d$-noisy leakage secure

Proof Sketch

Apply SNI simulator gadget-wise, in reversed order, until complete or failure

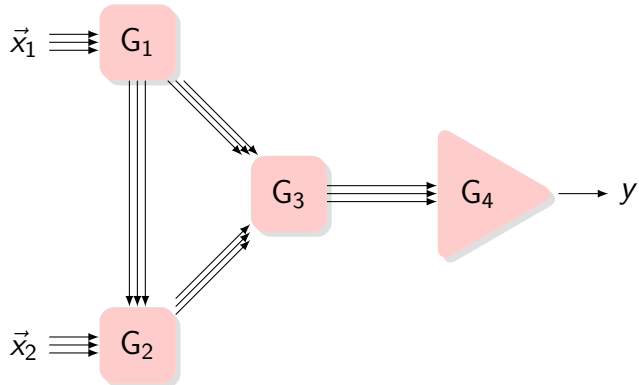

Figure: G_1: SNI copy gadget, G_2, G_3: SNI gadgets, G_4: NIo gadget. $\partial_1 = \{2, 3\}$, $\partial_2 = \{3\}$, $\partial_3 = \{4\}$, $\partial_4 = \emptyset$

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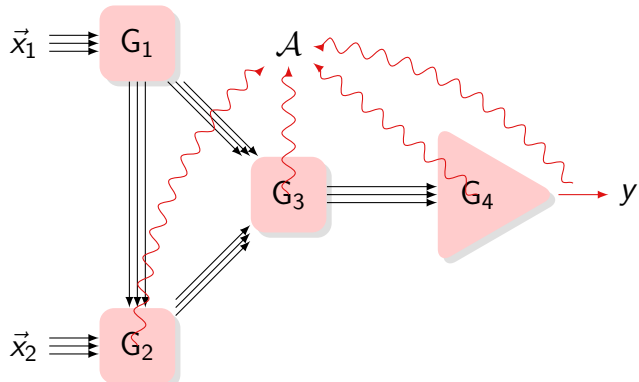

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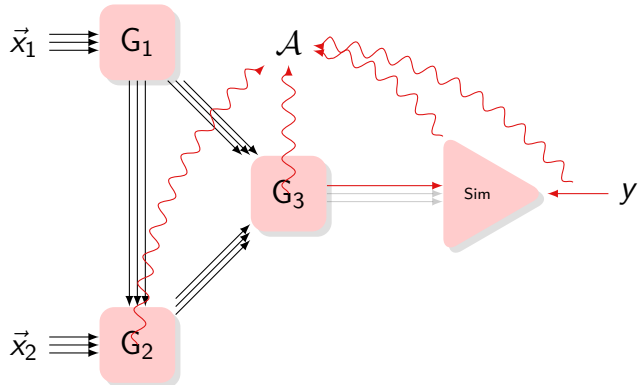

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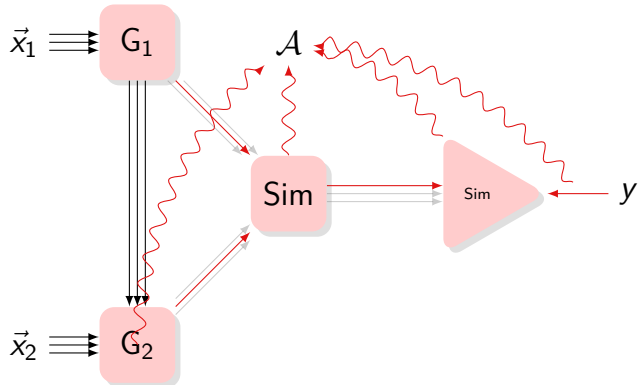

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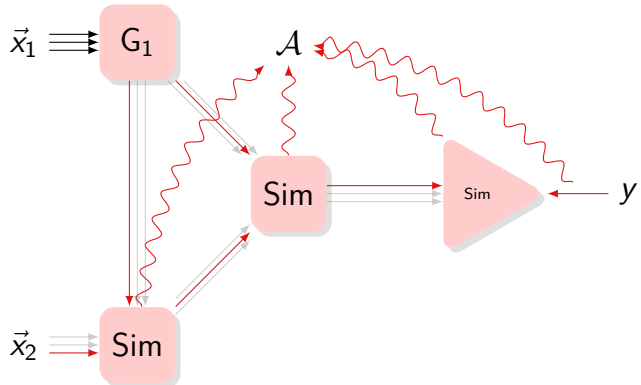

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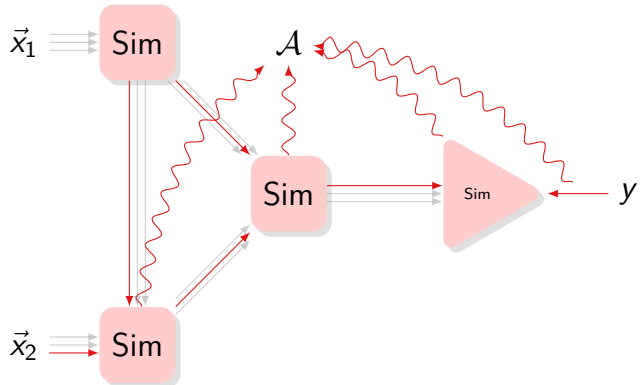

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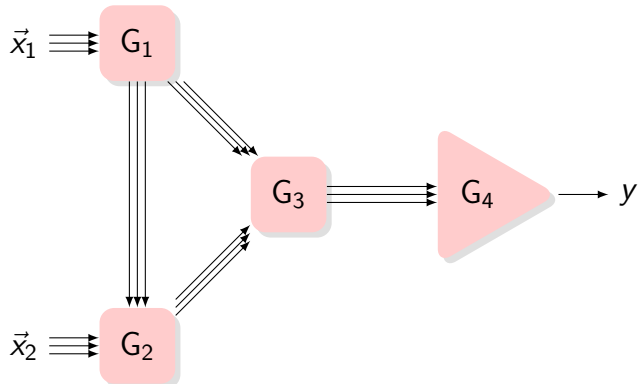

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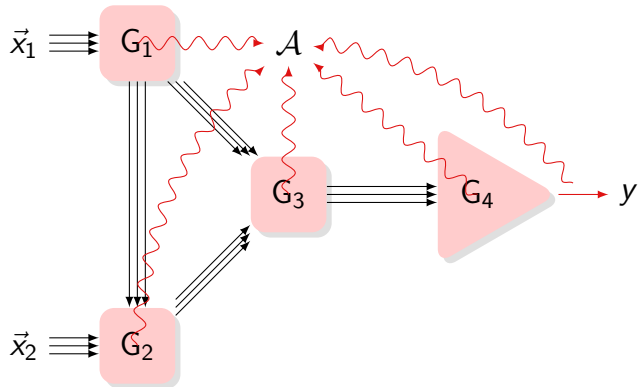

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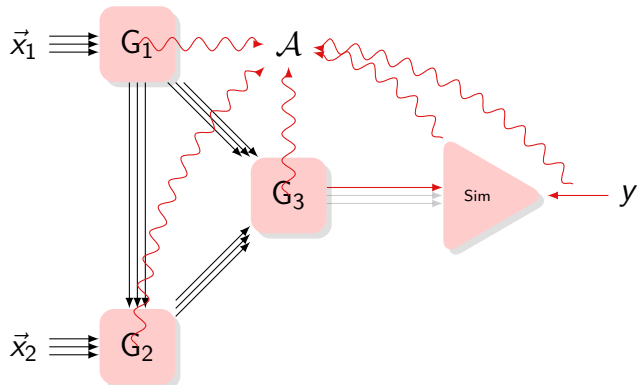

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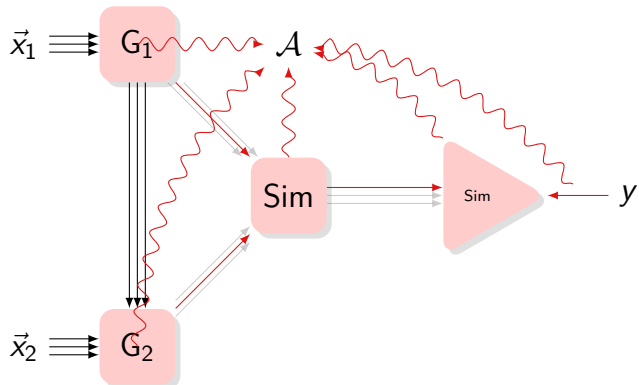

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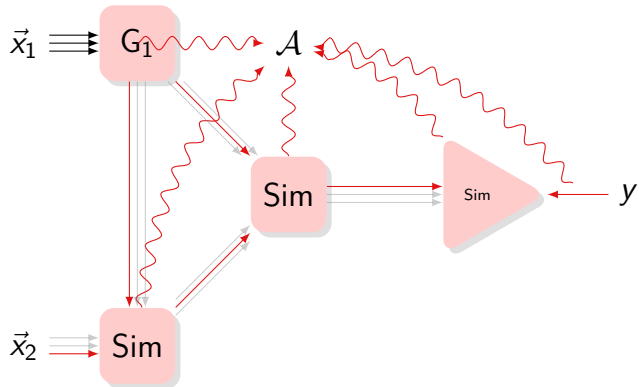

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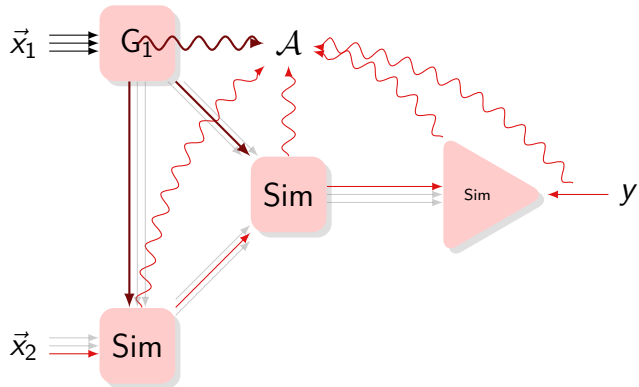

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Hence,

$$\Pr[\mathsf{bad}_i] \leq \Pr\left[\left|W^{G_i}\right| + \sum_{j \in \partial_i} \left|W^{G_j}\right| \geq t_i\right]$$

Using the union bound:

$$\eta = \sum_{\substack{i=1 \\ G_i \text{not output}}}^{|\mathsf{C}|} \Pr[\mathsf{bad}_i]$$

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Concluding the Proof

Using Chernoff:

$$\Pr\left[\left|W^{G_i}\right| + \sum_{j\in\partial_i}\left|W^{G_j}\right| > t_i\right] = \Pr\left[\left|W^{G_i} \cup \left(\bigcup_{j\in\partial_i} W^{G_j}\right)\right| \geq t_i + 1\right]$$
$$\leq \left(e \cdot \frac{|G_i| + \sum_{j\in\partial_i}|G_j|}{t_i + 1} \cdot \epsilon\right)^{t_i+1}.$$

Comparison with Previous Works

So far, trade-off was needed (see next talk):

$\rightarrow$ Duc *et al.*:[12] $|\mathsf{C}| \cdot (\mathcal{O}(d) \cdot |\mathbb{F}| \cdot \delta)^{d/2}$

$\rightarrow$ Belaïd *et al.*:[13] $|\mathsf{C}| \cdot (\mathcal{O}(1) \cdot |\mathbb{F}| \cdot \delta)^{\approx d/3}$

$\rightarrow$ Next talk: tighter and more generic way to compose

[12] Duc, Dziembowski, and Faust, "Unifying Leakage Models: From Probing Attacks to Noisy Leakage".

[13] Taleb, "Secure and Verified Cryptographic Implementations in the Random Probing Model. (Implémentations cryptographiques sûres et vérifiées dans le modèle random probing)".

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Lemma (Simulatability by Random Probing[14]*)*

The leakage function L *can be simulated from a random probing adversary:* $\varphi(x)$ *reveals* x *with probability* $\epsilon = 1 - \sum_l \min_x \Pr[\mathrm{L}(x) = l] \leq \delta \cdot |\mathbb{F}|$.

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→ Holds for any $x \in \mathbb{F}$ $\iff$ holds for any arbitrarily distributed r.v. $X \leftarrow\!\!\$\ \mathbb{F}$

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→ Holds for any $x \in \mathbb{F}$ $\iff$ holds for any arbitrarily distributed r.v. $X \leftarrow\!\!\$\ \mathbb{F}$

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[14] Duc, Dziembowski, and Faust, "Unifying Leakage Models: From Probing Attacks to Noisy Leakage".

Noisy-to-Random Reduction

Lemma (Simulatability by Random Probing[14]*)*

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Is it too much ?

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The Average Random Probing Model

The Average Random Probing Model

NL

RP

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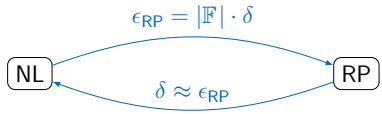
The Average Random Probing Model



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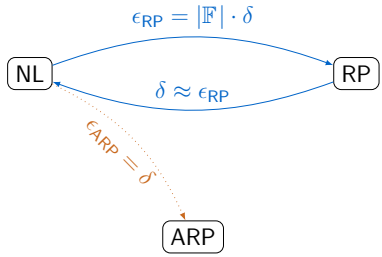

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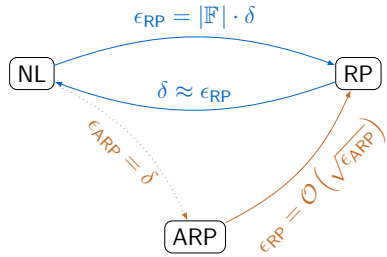

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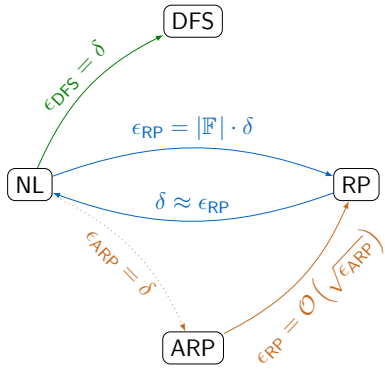

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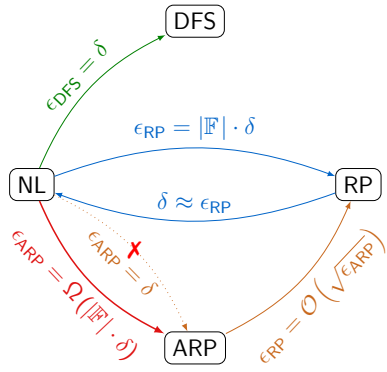

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Technical Results

Theorem (ARP-Simulability)

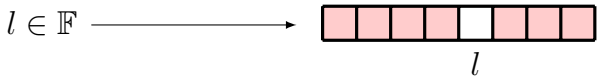
L *is simulable in the* $\{\epsilon_x\}_x$ *average random probing model iff*[15]

$$1 \leq \sum_l \min_{x' : \epsilon_{x'} < 1} \left\{ \frac{\Pr[\mathrm{L}(x') = l]}{1 - \epsilon_{x'}} \right\}$$

Remark: if ϵ_x constant, we get back the RP lemma
Proof: see appendix

[15] One needs at least one $\epsilon_x < 1$ for non-trivial simulation

The Catastrophic Channel, a.k.a., the *evil* function[16]



Equivalently:

$$\Pr[\mathrm{L}(x) = l] = \begin{cases} 0 & \text{if } x = l, \\ \frac{1}{|\mathbb{F}|-1} & \text{otherwise} \end{cases}$$

Here, $\delta = \frac{1}{|\mathbb{F}|}$, but $\epsilon = \mathop{\mathbb{E}}_{x}[\epsilon_x] \geq \frac{1}{2}$

$$\text{Hence, } \frac{\epsilon}{\delta} \geq \Omega\left(|\mathbb{F}|\right) \text{ ✗}$$

[16] Thus named by Gianluca Brian, as it appears as a worst-case in another of their works

Deterministic Leakage Functions

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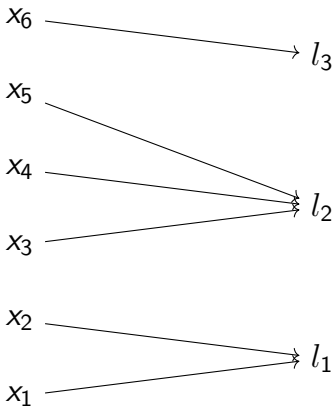

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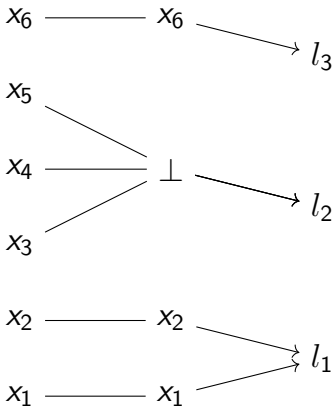

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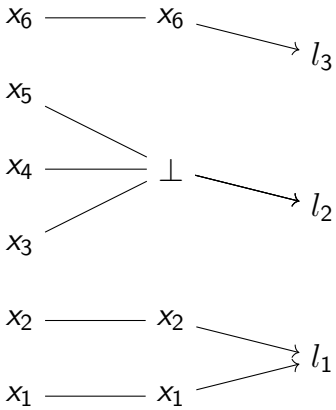

$$\epsilon = \mathop{\mathbb{E}}_{x}[\epsilon_x] = 1 - \frac{\max_l |\{x : \mathrm{L}(x) = l\}|}{|\mathbb{F}|}$$
$$\leq 1 - \frac{\mathop{\mathbb{E}}_{l}[|\{x : \mathrm{L}(x) = l\}|]}{|\mathbb{F}|}$$
$$= \delta$$

Content

Context: SCA & Security Evaluation

Masking

- Background & Intuitions
- Provably Secure Masking

Composition in the Random Probing Model

Tight Reduction

Conclusion

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Research strategy in masking security proofs:

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- Always good to start tackling a problem by simpler sides

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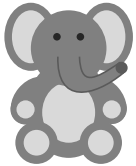

Main priority (IMHO):

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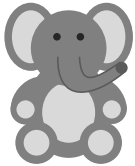

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No easy fix currently ...

Join us in this line of research !

References I

- Barthe, G. et al. "Masking the GLP Lattice-Based Signature Scheme at Any Order". In: *J. Cryptol.* 37.1 (2024), p. 5. DOI: 10.1007/S00145-023-09485-Z. URL: https://doi.org/10.1007/s00145-023-09485-z.
- Barthe, G. et al. "Strong Non-Interference and Type-Directed Higher-Order Masking". In: *Proceedings of the 2016 ACM SIGSAC Conference on Computer and Communications Security*. CCS '16. Vienna, Austria: Association for Computing Machinery, 2016, 116–129. ISBN: 9781450341394. DOI: 10.1145/2976749.2978427. URL: https://doi.org/10.1145/2976749.2978427.

References II

- Boucheron, S., G. Lugosi, and P. Massart. *Concentration Inequalities: A Nonasymptotic Theory of Independence*. Oxford University Press, 2013. ISBN: 9780191747106. URL: https://books.google.fr/books?id=O3yoAQAACAAJ.
- Brian, G., S. Dziembowski, and S. Faust. "From Random Probing to Noisy Leakages Without Field-Size Dependence". In: *Advances in Cryptology - EUROCRYPT 2024 - 43rd Annual International Conference on the Theory and Applications of Cryptographic Techniques, Zurich, Switzerland, May 26-30, 2024, Proceedings, Part IV*. Ed. by M. Joye and G. Leander. Vol. 14654. Lecture Notes in Computer Science. Springer, 2024, pp. 345–374. DOI: 10.1007/978-3-031-58737-5_13. URL: https://doi.org/10.1007/978-3-031-58737-5_13.

References III

- Chari, S. et al. "Towards Sound Approaches to Counteract Power-Analysis Attacks". In: *Advances in Cryptology - CRYPTO '99, 19th Annual International Cryptology Conference, Santa Barbara, California, USA, August 15-19, 1999, Proceedings*. Ed. by M. J. Wiener. Vol. 1666. Lecture Notes in Computer Science. Springer, 1999, pp. 398–412. ISBN: 3-540-66347-9. DOI: 10.1007/3-540-48405-1_26. URL: https://doi.org/10.1007/3-540-48405-1_26.
- Coron, J.-S. et al. *High-order Polynomial Comparison and Masking Lattice-based Encryption*. Cryptology ePrint Archive, Paper 2021/1615. 2021. URL: https://eprint.iacr.org/2021/1615.

References IV

- Duc, A., S. Dziembowski, and S. Faust. "Unifying Leakage Models: From Probing Attacks to Noisy Leakage". In: *J. Cryptology* 32.1 (2019), pp. 151–177. DOI: 10.1007/s00145-018-9284-1. URL: https://doi.org/10.1007/s00145-018-9284-1.
- Dziembowski, S., S. Faust, and M. Skorski. "Noisy Leakage Revisited". In: *Advances in Cryptology - EUROCRYPT 2015 - 34th Annual International Conference on the Theory and Applications of Cryptographic Techniques, Sofia, Bulgaria, April 26-30, 2015, Proceedings, Part II*. Ed. by E. Oswald and M. Fischlin. Vol. 9057. Lecture Notes in Computer Science. Springer, 2015, pp. 159–188. DOI: 10.1007/978-3-662-46803-6_6. URL: https://doi.org/10.1007/978-3-662-46803-6_6.

References V


Goubin, L. and J. Patarin. "DES and Differential Power Analysis (The "Duplication" Method)". In: *Cryptographic Hardware and Embedded Systems, First International Workshop, CHES'99, Worcester, MA, USA, August 12-13, 1999, Proceedings*. Ed. by Ç. K. Koç and C. Paar. Vol. 1717. Lecture Notes in Computer Science. Springer, 1999, pp. 158–172. DOI: 10.1007/3-540-48059-5_15. URL: https://doi.org/10.1007/3-540-48059-5_15.

References VI


Ishai, Y., A. Sahai, and D. A. Wagner. "Private Circuits: Securing Hardware against Probing Attacks". In: *Advances in Cryptology - CRYPTO 2003, 23rd Annual International Cryptology Conference, Santa Barbara, California, USA, August 17-21, 2003, Proceedings*. Ed. by D. Boneh. Vol. 2729. Lecture Notes in Computer Science. Springer, 2003, pp. 463–481. DOI: 10.1007/978-3-540-45146-4_27. URL: https://doi.org/10.1007/978-3-540-45146-4_27.

References VII

- Rivain, M. and E. Prouff. "Provably Secure Higher-Order Masking of AES". In: *Cryptographic Hardware and Embedded Systems, CHES 2010, 12th International Workshop, Santa Barbara, CA, USA, August 17-20, 2010. Proceedings*. Ed. by S. Mangard and F. Standaert. Vol. 6225. Lecture Notes in Computer Science. Springer, 2010, pp. 413–427. DOI: 10.1007/978-3-642-15031-9_28. URL: https://doi.org/10.1007/978-3-642-15031-9_28.
- Taleb, A. R. "Secure and Verified Cryptographic Implementations in the Random Probing Model. (Implémentations cryptographiques sûres et vérifiées dans le modèle random probing)". PhD thesis. Sorbonne University, Paris, France, 2023. URL: https://tel.archives-ouvertes.fr/tel-04457258.

Reduction from Noisy Leakage to Random Probing (I)

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$\rightarrow$ For any x, l, $\Pr[\mathcal{S}(\varphi(x)) = l] = \Pr[\mathrm{L}(x) = l]$

Reduction from Noisy Leakage to Random Probing (II)

Let us start from the last constraint. For any x and any l:

$$\Pr[\mathrm{L}(x) = l] \;=\; \Pr[\mathcal{S}(\varphi(x)) = l]$$

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Hence,

$$0 \leq \Pr[\mathcal{S}(\bot) = l] = \frac{\overbrace{\Pr[\mathrm{L}(x) = l] - \epsilon \cdot \Pr[\mathcal{S}(x) = l]}^{\text{Should not depend on } X}}{1 - \epsilon}$$

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Furthermore, summing (3) over l, by definition of probability distributions,

$$\sum_l \pi(l) = \underbrace{\sum_l \Pr[\mathrm{L}(x) = l]}_{=1} - \epsilon \cdot \underbrace{\sum_l \Pr[\mathcal{S}(x) = l]}_{=1}$$

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Hence,

$$\epsilon = 1 - \sum_l \pi(l) \geq 1 - \sum_l \min_x \Pr[\mathrm{L}(x) = l]$$

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$$0 \leq \pi(l) \leq \min_x \Pr[\mathrm{L}(x) = l]$$

Furthermore, summing (3) over l, by definition of probability distributions,

$$\sum_l \pi(l) = \underbrace{\sum_l \Pr[\mathrm{L}(x) = l]}_{=1} - \epsilon \cdot \underbrace{\sum_l \Pr[\mathcal{S}(x) = l]}_{=1} = 1 - \epsilon$$

Hence, to have the smallest ϵ,

$$\epsilon = 1 - \sum_l \pi(l) = 1 - \sum_l \min_x \Pr[\mathrm{L}(x) = l]$$

Reduction to Average Random Probing (I)

For any x and any l:

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And (3) also gives

$$0 \leq \pi(l, x) \leq (1 - \epsilon_x) \cdot \min_{x' : \epsilon_{x'} < 1} \left\{ \frac{\Pr[\mathrm{L}(x') = l]}{1 - \epsilon_{x'}} \right\}$$

Characterization of ARP-simulable Leakages

Furthermore, summing (3) over l, by definition of probability distributions,

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Hence, the following result

Theorem (ARP-Simulability)

L *is simulable in the* $\{\epsilon_x\}_x$ *average random probing model iff*[17]

$$1 \leq \sum_l \min_{x':\epsilon_{x'}<1} \left\{ \frac{\Pr[\mathrm{L}(x')=l]}{1-\epsilon_{x'}} \right\}$$

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