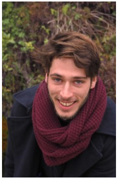
Maximizing Covered Area in the Euclidean Plane with Connectivity Constraint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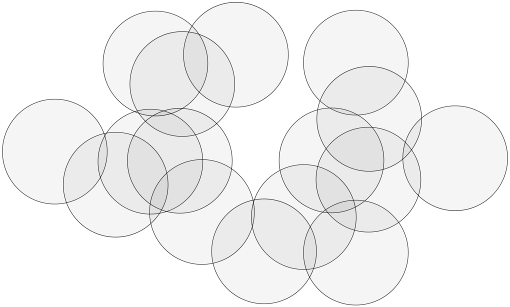
Definition: Connected Unit-disk *k*-coverage

In: A (connected) set of unit-area-disks in the Euclidean plane and an integer ***k***

Out: A connected subset *S* of size ***k***

Goal: Maximize the area covered by *S*



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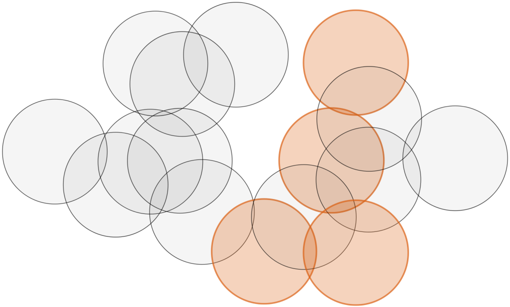

$k = 4$

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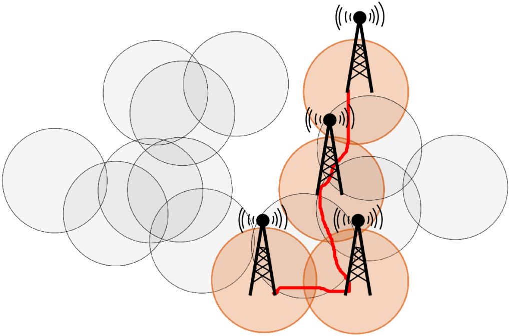


generalisations

budgeted connected dominating set: $\frac{1}{13}(1-1/e)$-approximation [Khuller, Purohit, Sarpatwar, 2014], very recently improved to $\frac{1}{7}(1-1/e)$? [Lamprou, Sigalas, Zissimopoulos, 2019]

connected *k*-coverage: $\Omega(1/\sqrt{k})$-approximation when objective function is monotone submodular. [Kuo, Lin, Tsai, 2015]

Related results

***k*-coverage:** optimal greedy $1-1/e$ approximation for monotone submodular function.
(f submodular: $f(A \cup \{x\}) - f(A) \geq f(B \cup \{x\}) - f(B), \forall A \subseteq B \subseteq X, \forall x \in X$)

unit-disk *k*-coverage: PTAS. [Chaplik, De, Ravsky, Spoerhase, 2018]

Our results

Algorithms:

(New) $1/2$-approximation

(New) PTAS with resource augmentation

Lower bounds:

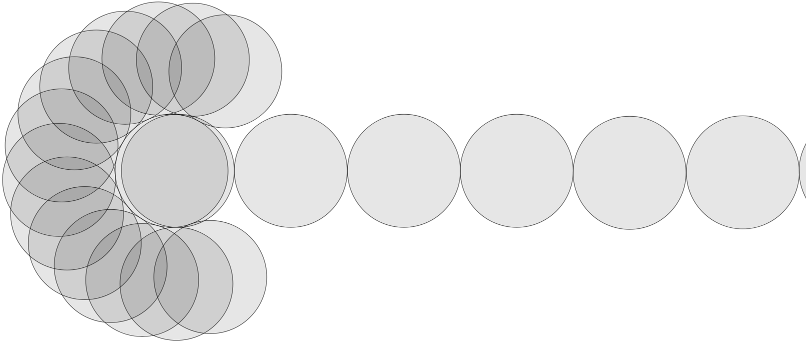
(New) NP-hardness

(New) APX-hardness with unit-area-triangles

Approximation algorithm

A bad Algorithm: The 1-by-1 Greedy algorithm

- $S = \{\text{an arbitrary disk}\}$
- While $|S| < k$, add one disk in S that maximizes the marginal area covered while maintaining S connected.



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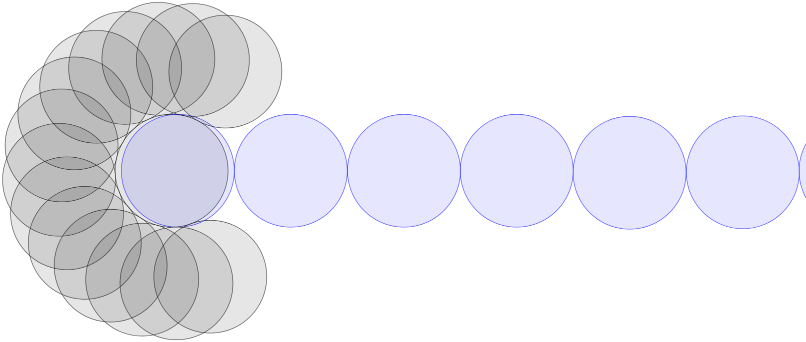

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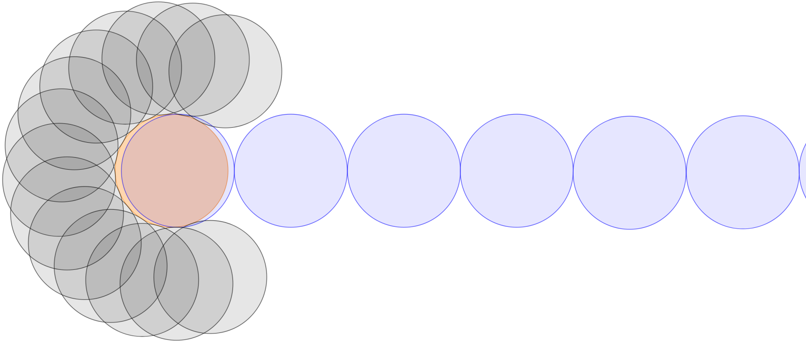

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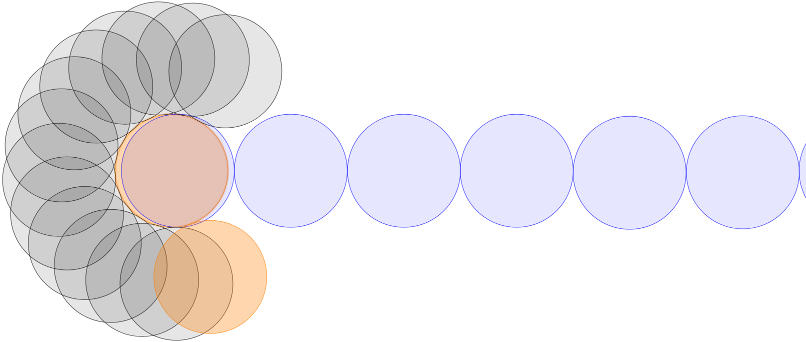

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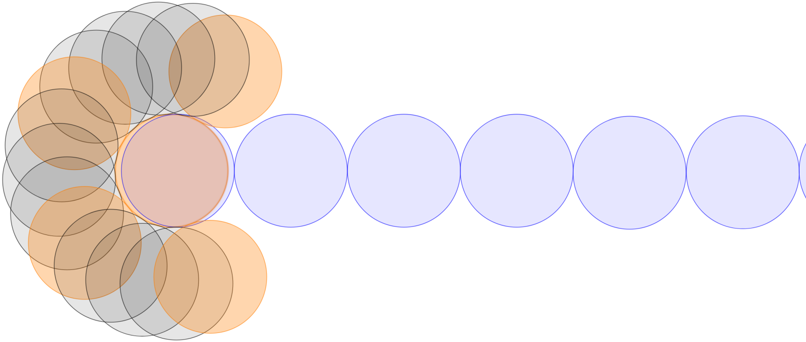

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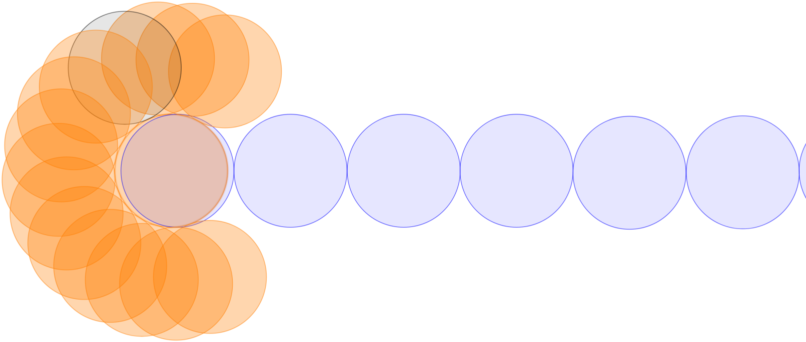

OPT$= k$ and 1-by-1 Greedy $\leq 9 \longrightarrow$ gap $= \Omega(k)$

The 2-by-2 Greedy algorithm

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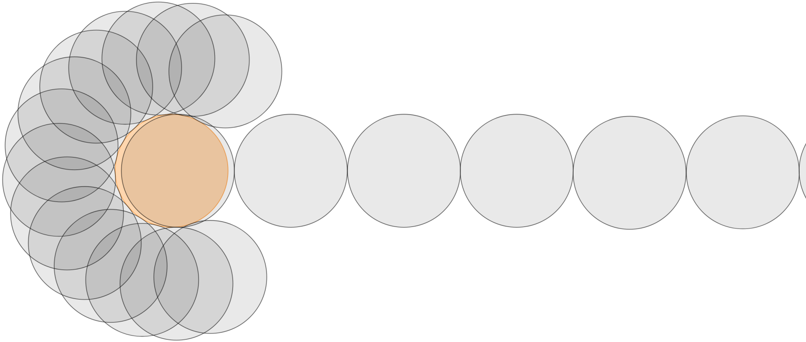

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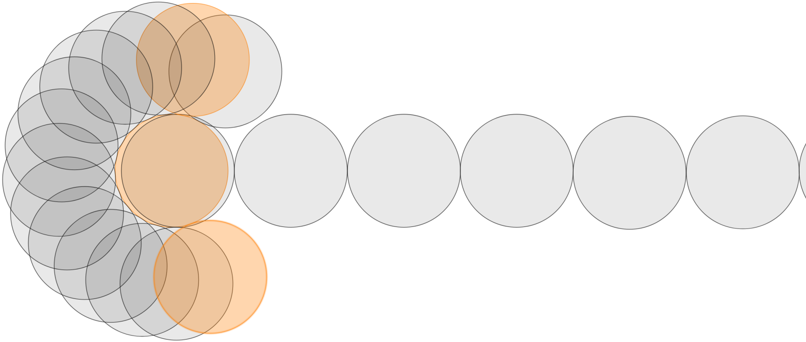

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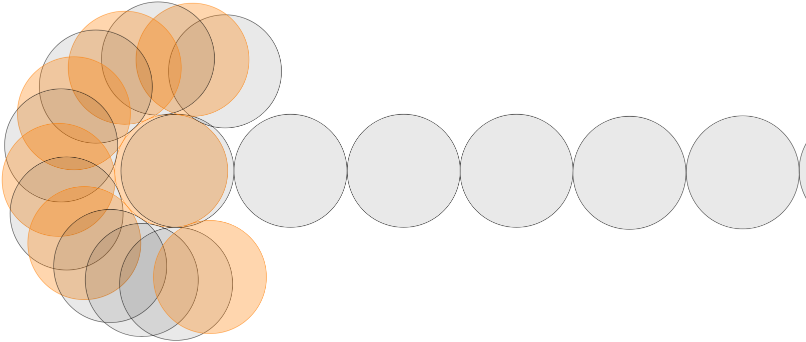

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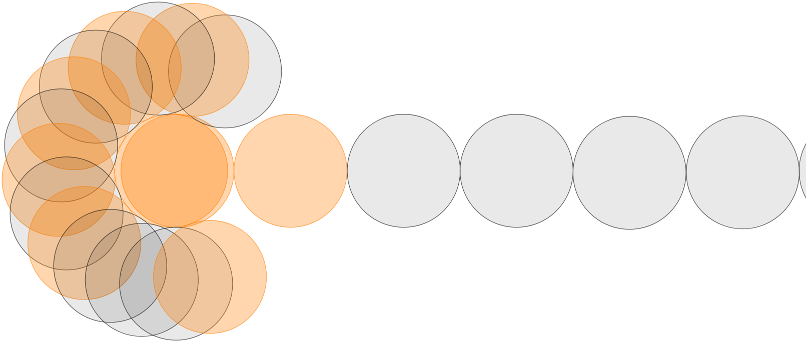

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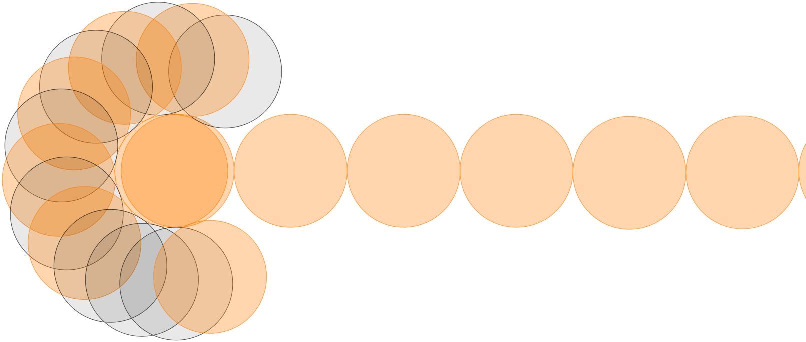

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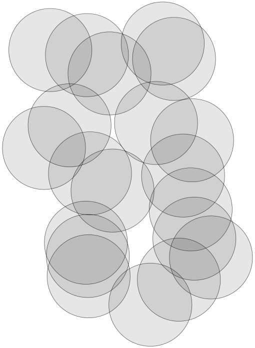
Theorem (NEW): The 2-by-2 Greedy algorithm gives a $\frac{1}{2}$-approximation of connected unit-disk k-coverage problem, and it is tight.

Proof sketch

First phase

S is not a dominating set



Proof sketch

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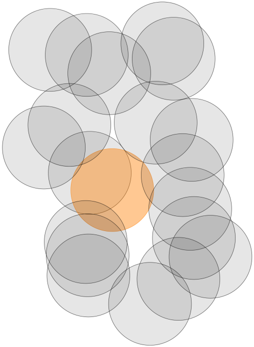

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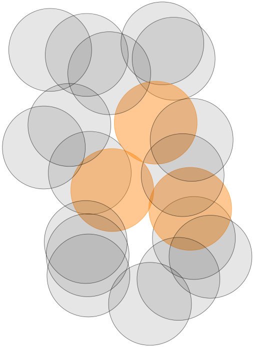

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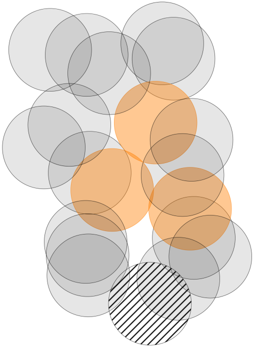

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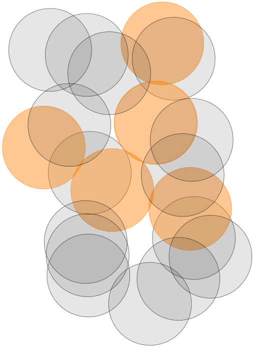

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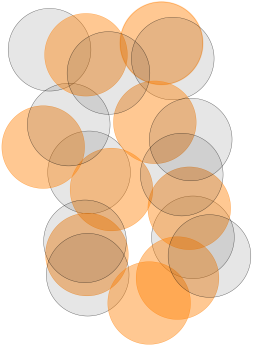

$\text{area}(S) \geq |S|/2$

Proof sketch

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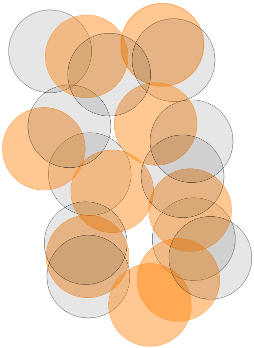

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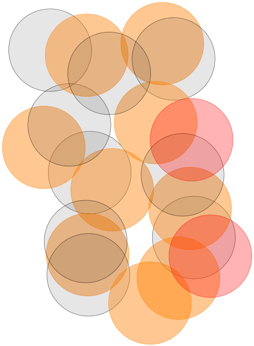
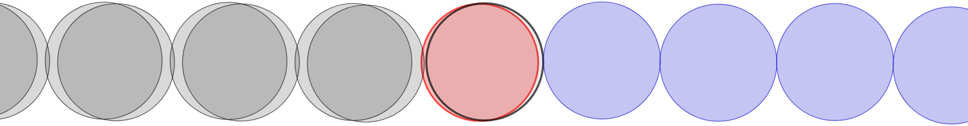
Second phase

connectivity is guaranteed



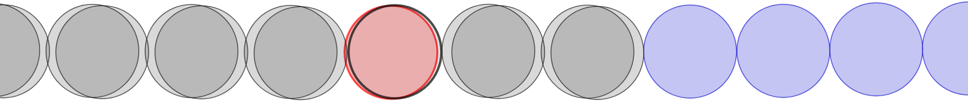
use monotone submodularity.

Theorem: The 2-by-2 Greedy algorithm gives a $\frac{1}{2}$-approximation of connected unit-disk k-coverage problem, **and it is tight**.



Going above $1/2$?

a t-by-t Greedy algorithm, with $t \geq 3$? **No.**

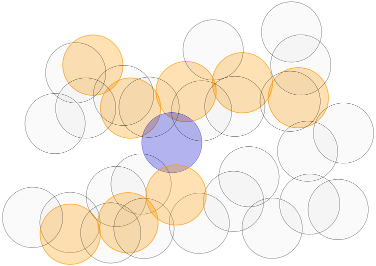


Theorem (NEW): PTAS with resource augmentation

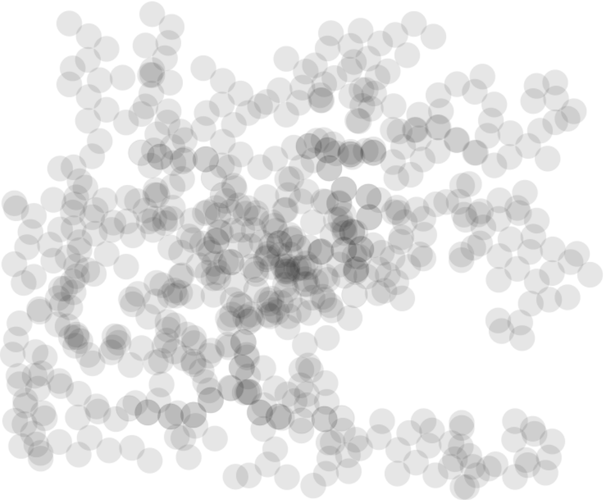
We can find in time $n^{O(1/\varepsilon)}$

- a set S of k input disks, such that area$(S) \geq (1-\varepsilon)OPT(k)$
- a set S_{add} of at most εk additional disks such that $S \cup S_{add}$ is connected.

Algorithm: Shifted quadtree (Arora), m-guillotine subdivision (Mitchell)



Proof with Arora's framework



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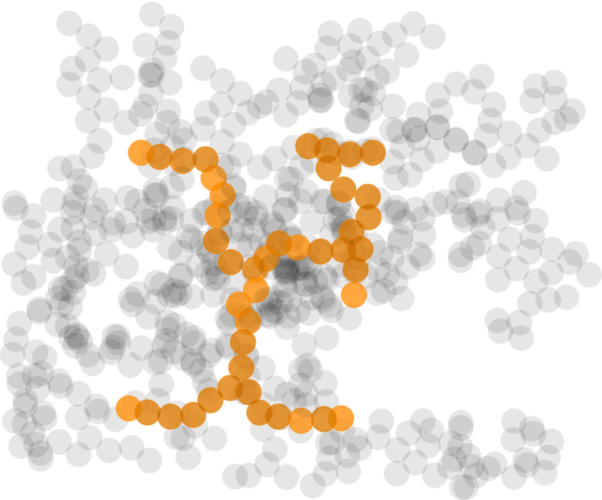

OPT

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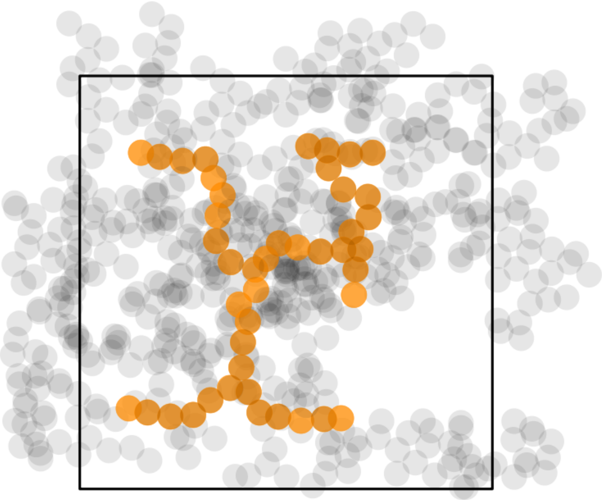

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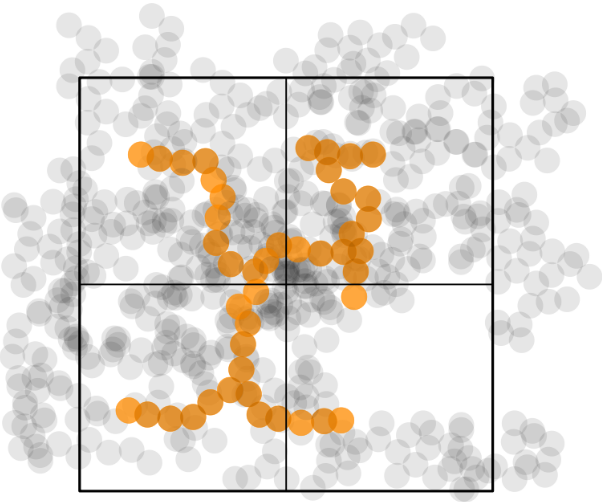

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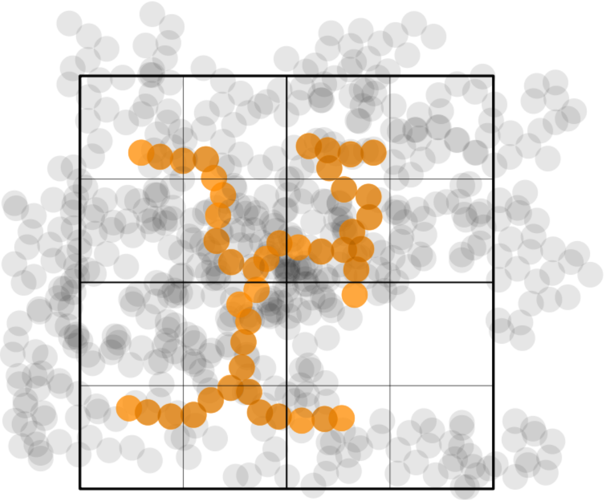

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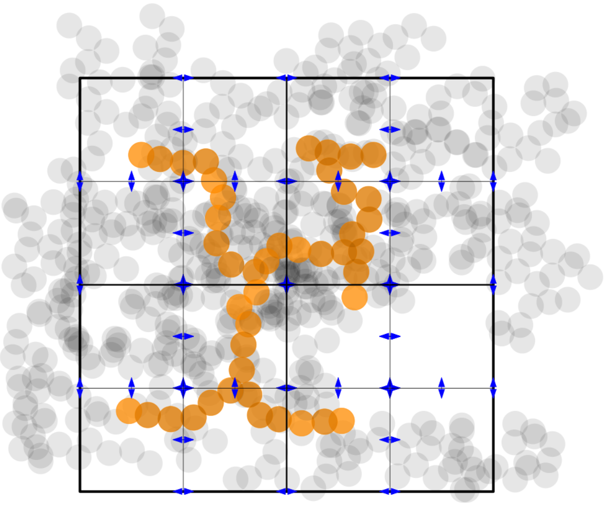

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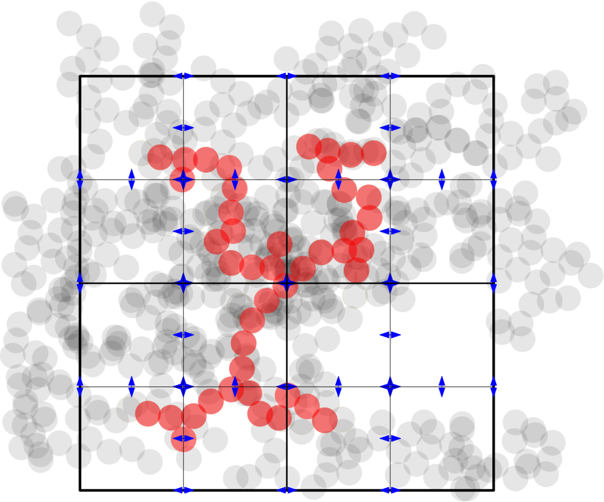
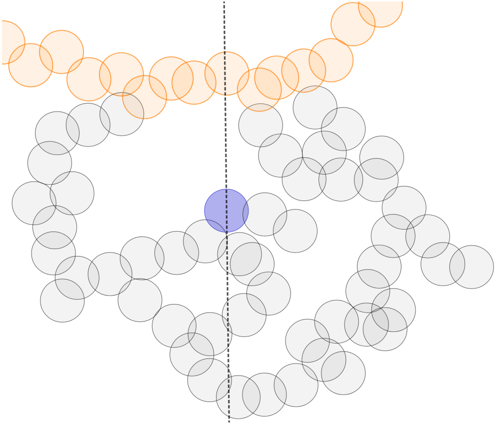


OPT $\longrightarrow \exists$ portal-respecting near-optimal solution ??

Proof with Arora's framework

Can we make short detours ?



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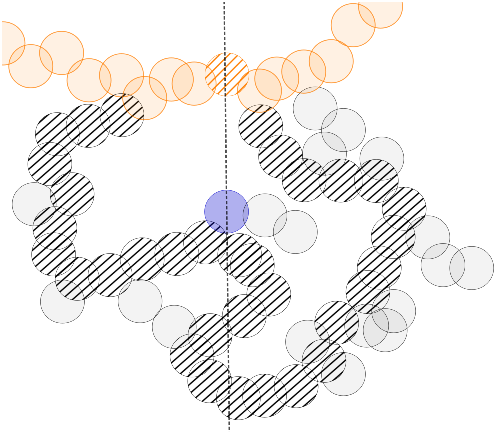
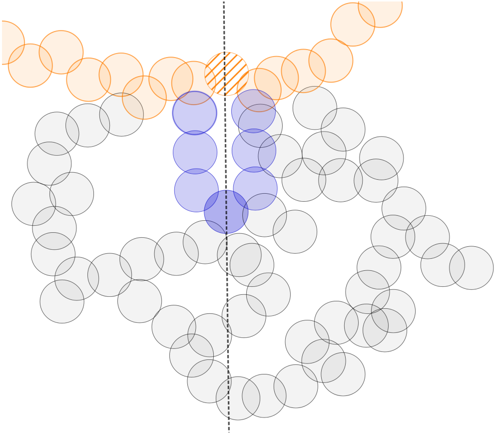


Proof with Arora’s framework

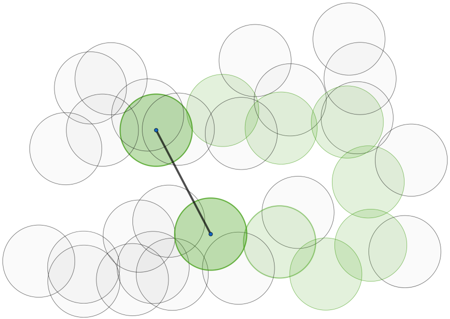
Can we make short detours ? Yes if we allow few additional disks



PTAS under resource augmentation

We can find in time $n^{O(1/\varepsilon)}$ a k-set $S \subseteq D$ and a set S_{add} of at most εk additional disks such that

1. $S \cup S_{add}$ is connected,
2. $\text{area}(S) \geq (1 - \varepsilon)OPT$



corollary

$\exists$ PTAS when

distance in intersection graph $=$ O(Euclidean distance)

Our results:

- $1/2$-approximation
- PTAS with resource augmentation
- NP-hardness
- APX-hardness with unit-area-triangles.

$$\Downarrow$$

$\exists$ PTAS for connected unit-disk k-coverage?