Generalized Quantifiers and Dynamicity
— preliminary results —

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Abstract

We classify determiners according to the dynamic properties of the generalized quantifiers they denote. We then show how these dynamic generalized quantifiers can be defined in a continuation-based dynamic logic.

1   Introduction

Following the success of the interpretation of determiners as binary generalized quantifiers (Barwise and Cooper, 1981), on the one hand, and the success of DRT (Kamp and Reyle, 1993) and dynamic logic (Groenendijk and Stokhof, 1991), on the other hand, several authors have explored notions of dynamic generalized quantifiers (Chierchia, 1992; Fernando, 1994; Kanazawa, 1994a,b; van den Berg, 1991, 1994; van Eijck and de Vries, 1992).

In this paper, we revisit this subject in the setting of the dynamic framework introduced in (de Groote, 2006). We classify the generalized quantifiers that are denotations of determiners according to their dynamic properties (internal dynamicity, external dynamicity, and intrinsic dynamicity). We end up with three classes of dynamic generalized quantifiers, and we show how they can be formally defined in the simple theory of types (Church, 1940). To conclude, we discuss several issues raised by the proposed formalization.

2   A classification of the generalized quantifiers according to their dynamic properties

We consider binary dynamic generalized quantifiers that are used as denotations of determiners. In our dynamic setting (de Groote, 2006; Lebedeva, 2012), a quantifier belonging to this class, say $\mathscr{Q}$, is a constant (or a term) of type:

(1) $\mathscr{Q} : (\iota \to \Omega) \to (\iota \to \Omega) \to \Omega$

where ι is the type of individuals, and Ω the type of dynamic propositions. In order to form a (dynamic) proposition, $\mathscr{Q}$ must take two arguments. The first one is called its *restriction*, and the second one, its *scope*.

In their paper on dynamic predicate logic, Groenendijk and Stokhof (1991) define notions of *internal* and *external* dynamicity that apply to binary logical connectives. Extending these notions to the case of binary generalized quantifiers is straightforward.

Accordingly, we say, on the one hand, that $\mathcal{Q}$ is *internally dynamic* in case the dynamic binders occurring in its restriction have the capacity of binding material that occurs in its scope. In DRT terms, it means that the reference markers introduced by the first argument are accessible in the second one. A paradigmatic example of internal dynamicity is provided by the classical donkey sentences. Let $\mathcal{Q}$ stands for both a dynamic generalized quantifier and a determiner that denotes it. $\mathcal{Q}$ is internally dynamic in case the following utterance is felicitous:

(2) $\mathcal{Q}$ teacher who reads [a good essay]$_i$ likes it$_i$.

On the other hand, we say that $\mathcal{Q}$ is *externally dynamic* in case the dynamic binders occurring in both its arguments have the capacity of binding material that will occur in the continuation of the discourse. One may again illustrate this notion by using donkey sentences. As before, let $\mathcal{Q}$ stands for both a dynamic generalized quantifier and a determiner that denotes it. $\mathcal{Q}$ is externally dynamic in case the following utterance is felicitous:

(3)   If $\mathcal{Q}$ student writes [a good essay]$_i$, the teacher will like it$_i$.

In addition to these notions of internal and external dynamicity, we consider a third notion that we call intrinsic dynamicity. This notion is defined as follows. We say that $\mathcal{Q}$ is *intrinsically dynamic* in case it has the capacity of extending its scope on the continuation of the discourse. In DRT terms, it means that $\mathcal{Q}$ is responsible for introducing a new reference marker. Again, donkey sentences provide us with a test. $\mathcal{Q}$ is intrinsically dynamic in case the following utterance is felicitous:

(4)   If [$\mathcal{Q}$ student]$_i$ writes a good essay, she$_i$ will get a good mark.

With respect to these three properties, one could a priori define eight possible classes of dynamic generalized quantifiers. Nevertheless, based on empirical observations,[1] we conjecture that:

(i) all the determiners are internally dynamic;

(ii) every (singular) determiner that is intrinsically dynamic is also externally dynamic.[2]

Consequently, we end up with three classes of dynamic generalized quantifiers, which correspond fairly to the notions of specific (e.g., *the, this, his*), general (e.g., *a, some, another*), and quantificational determiners (e.g., *every, no*)[3]:

	internal dynamicity	*external dynamicity*	*intrinsic dynamicity*
quantificational determiners	+	−	−
specific determiners	+	+	−
general determiners	+	+	+

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[1]Mainly for French determiners.

[2]This second statement, which we think to be clear in the case of singular determiners, could be challenged in the case of plural ones. We will be back to this question in Section 4.3

[3]We are conscious that these classes of determiners are vague and that precise definitions would be needed in order to make the correspondance effective. To this end, the opposition definiteness/indefiniteness is certainly relevant. According to our formalization, a DP whose head is a specific determiner presupposes the existence of a discourse referent. This is consistent with Heim's (1983) familiarity theory. The possible correspondance between generality and indefiniteness is less clear. It seems that the relation is an inclusion rather than an equivalence. Many linguists, if not most of them, consider *no* as an indefinite determiner because it can occur in existential sentences such as *there is no good essay this week*. Test (3) and (4), however, show clearly that *no* is neither externally nor intrinsically dynamic.

3 Formalization

Following the continuation-based approach introduced in (de Groote, 2006), and developed in (Lebedeva, 2012), we let:

(5) $\Omega = \gamma \to (\gamma \to o) \to o$

where γ is the type of left contexts, o the type of static propositions, and $\gamma \to o$ the type of continuations that model right contexts. In this setting, the dynamic interpretation of an atomic proposition a is defined as follows:

(6) $\bar{a} := \lambda ek.a \wedge (ke)$

As for Groenendijk's and Stokhof's closure operator, dynamic negation, dynamic conjunction, dynamic disjonction, and dynamic implication, they are defined by the following equations:

(7)
$$\diamond A := \lambda ek.(Ae(\lambda e.\top)) \wedge (ke)$$
$$\neg A := \diamond(\lambda ek.\neg Aek)$$
$$A \wedge\!\!\!\wedge B := \lambda ek.Ae(\lambda e.Bek)$$
$$A \vee\!\!\!\vee B := \diamond(\lambda ek.(Aek) \vee (Bek))$$
$$A \Rightarrow B := \diamond(\lambda ek.\neg Ae(\lambda e.\neg Bek))$$

Then, given a static generalised quantifier:

(8) $q : (\iota \to o) \to (\iota \to o) \to o$

we define the corresponding intrinsically dynamic quantifier as follows:

(9) $\mathcal{Q}_{\mathrm{w}} AB := \lambda ek.q(\lambda x.Axe(\lambda e.\top))(\lambda x.((Ax)\wedge\!\!\!\wedge(Bx))(x::e)k)$

where "$_-::_-$" is an operator that pushes a variable (which will act as a new discourse referent) into the current context.

Following Chierchia (1992) and Kanazawa (1994a,b), Equation (9) provides the *weak* dynamic interpretation of q. The strong interpretation is then given as follows:

(10) $\mathcal{Q}_{\mathrm{s}} AB := \lambda ek.q(\lambda x.Axe(\lambda e.\top))(\lambda x.((Ax)\Rightarrow(Bx))(x::e)k)$

Now, let Q_{w} and Q_{s} stand for the weak and strong dynamic generalized quantifiers corresponding to q that are internally dynamic but neither intrinsically nor externally dynamic. These are easily obtained by applying the closure operator to $\mathcal{Q}_{\mathrm{w}}$ and $\mathcal{Q}_{\mathrm{s}}$:

(11) $Q_{\mathrm{w}} AB := \diamond(\mathcal{Q}_{\mathrm{w}} AB)$

(12) $Q_{\mathrm{s}} AB := \diamond(\mathcal{Q}_{\mathrm{s}} AB)$

Finally, dynamic generalized quantifiers that are externally dynamic but not intrinsically dynamic correspond to generalized quantifiers that do not introduce any new binding operator. For this reason, they do not obey any general scheme. Consider, for instance, the use of Russell's iota operator in the analysis of definite descriptions. It allows one to associate to the definite article the following (static) generalized quantifier:

(13) $\lambda AB.B(\iota x.Ax)$

In our dynamic setting, the dynamic generalized quantifier corresponding to (13) will be defined as follows:

(14) $\quad \lambda ABek. B\left(\mathsf{sel}\left(\lambda x. A\,x\,e\left(\lambda e. \top\right)\right)e\right)e\,k$

where "$\mathsf{sel} : (\iota \to o) \to \gamma \to \iota$" is a choice operator such that "$\mathsf{sel}\,p\,e$" selects from the current context "e" the most salient entity (acting as a discourse referent) that has property "p".

4 Discussion

4.1 Sortal reducibility

Consider the definition of the dynamic quantifier $\mathscr{Q}_{\mathrm{w}}$ (respectively, $\mathscr{Q}_{\mathrm{s}}$) as given by Equation (9) (respectively, Equation (10)). Parameter A, which stands for the restriction of the dynamic quantifier, occurs twice in the righthand side of the equation. This non linearity, which is needed in order to make the defined quantifiers internally dynamic, is harmless from a truth-conditional point of view because natural language determiners denote conservative generalized quantifiers (Barwise and Cooper, 1981). These are generalized quantifiers q such that:

(15) $\quad (q\,a\,(\lambda x. (a\,x) \wedge (b\,x))) = (q\,a\,b) = (q\,a\,(\lambda x. (a\,x) \to (b\,x)))$

From a computational point of view, however, the duplication of A means that one has to evaluate it twice. This problem may be circumvented for quantifiers such as *some* and *every* because they respectively satisfy the following properties:

(16) $\quad (some\,a\,b) = (some\,(\lambda x. \top)\,(\lambda x. (a\,x) \wedge (b\,x)))$

(17) $\quad (every\,a\,b) = (every\,(\lambda x. \top)\,(\lambda x. (a\,x) \to (b\,x)))$

Keenan (1993) defines two classes of binary generalized quantifiers that respectively satisfy properties akin to (16) and (17). In addition, he shows that there is no other conservative generalized quantifier that could be expressed in terms of a corresponding unary quantifier and an appropriate binary logical connective. These two classes of quantifiers are called the *intersective* and the *co-intersective* quantifiers, respectively.

Accordingly, if q is an intersective generalized quantifier, we define the corresponding intrinsically dynamic quantifier as follows:

(18) $\quad \mathscr{Q}\,A\,B := \lambda ek. q\,(\lambda x. \top)\,(\lambda x. ((A\,x) \mathbin{/\!\!\backslash} (B\,x))\,(x\!::\!e)\,k)$

Similarly, if q is co-intersective:

(19) $\quad \mathscr{Q}\,A\,B := \lambda ek. q\,(\lambda x. \top)\,(\lambda x. ((A\,x) \Rightarrow (B\,x))\,(x\!::\!e)\,k)$

Notice that Definitions (18) and (19) provide a weak interpretation to the intersective quantifiers and a strong interpretation to the co-intersective ones. This is consistent with Kanazawa's (1994b) analysis. It is indeed not difficult to prove that a monotonic intersective quantifiers is either ↑MON↑ or ↓MON↓. Similarly, a monotonic co-intersective quantifiers is either ↓MON↑ or ↑MON↓.

4.2 Logical constraints

In order to make sense, our dynamic interpretation of the generalized quantifiers must satisfy some logical constraints. Let φ be a formula of propositional logic. Definitions (6) and (7) allow one to give a dynamic interpretation to φ. This dynamic interpretation, say $\overline{\varphi}$, is conservative in the sense that it satisfies the following equation for every context e:

$$(20) \qquad \overline{\varphi}\, e\,(\lambda e.\top) = \varphi$$

In order to preserve this property, every generalized quantifier q whose dynamic interpretation is externally dynamic must satisfy the following constraint:

$$(21) \qquad (q\,a\,b) \wedge c = q\,a\,(\lambda x.\,(b\,x) \wedge c)$$

It is easy to show that this condition is equivalent to the following one:

$$(22) \qquad q\,a\,(\lambda x.\bot) = \bot$$

Therefore, in some sense, our formalization predict that a determiner that passes Test (3) denotes a generalized quantifier that satisfies Equation (22). It would be interesting to conduct further investigations in order to establish to which extent this prediction is correct.

4.3 Plurality

The very form in which (2), (3), and (4) have been given covers only cases of singular determiners. Adapting these tests to plural determiners is almost straightforward, but brings about some questions. Consider the following instance of (4):

(23)   If *three* students write a good essay, they will get a good mark.

The felicity of this utterance would indicate that *three* is intrinsically dynamic and that Equation (18) might be used to give an appropriate dynamic interpretation of *three*. This is indeed the case: the truth conditions that are assigned to (23) using Equation (18) conform to the intuitive meaning of the sentence. But now, consider the following variant of (23)

(24)   If *three* students write a good essay, they will meet and work together.

The formalism we have developed so far will fail in assigning any sensible truth conditions to (24). The reason is that the use of first-order variables as discourse referents enforces a distributive reading of the sentence (which makes no sense in the case of (24)).

In fact, what Example (24) suggests is that intrinsically dynamic plural determiners should introduce second-order discourse referents that stand for collection of entities. A scheme introducing such a second-order referent is the following:

$$(25) \qquad \mathscr{Q}\,A\,B = \lambda ek.\,\exists a.\,(q\,(\lambda x.A\,x\,e\,(\lambda e.\top))\,a) \wedge (\forall x.\,(a\,x) \rightarrow (((A\,x) \barwedge (B\,x))\,(a::e)\,k))$$

This is not the only possibility, and other schemes are certainly needed. In particular, (25) enforces again a distributive reading of the predicate. In fact, the dynamic interpretation of plural determiners raises several issues that have yet to be settled. These are the topic of ongoing work.

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