A more efficient and informed algorithm to check Weak Controllability of STNUs

(Simple Temporal Networks with Uncertainty)

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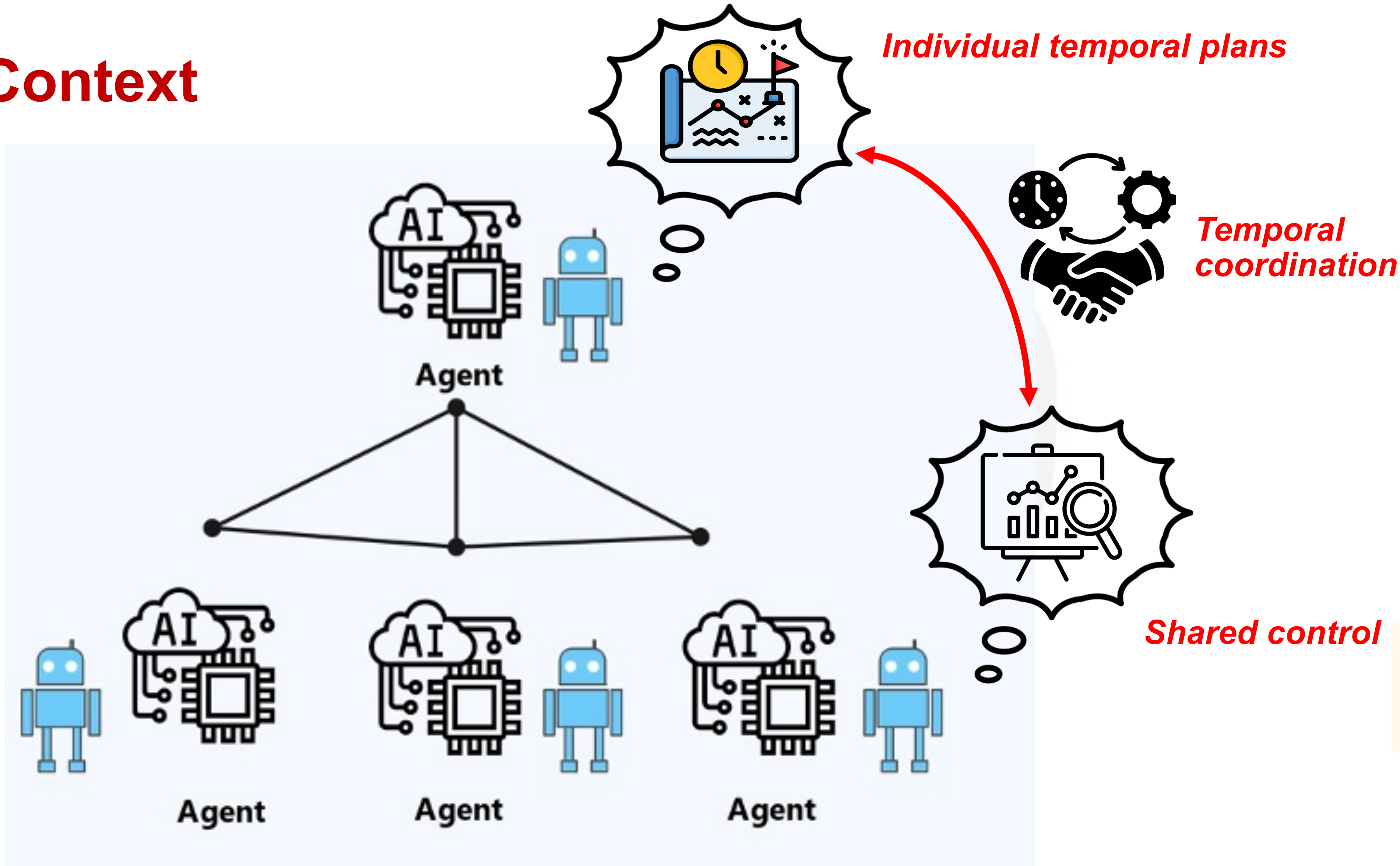
Tarbes

Context and Motivation

Context



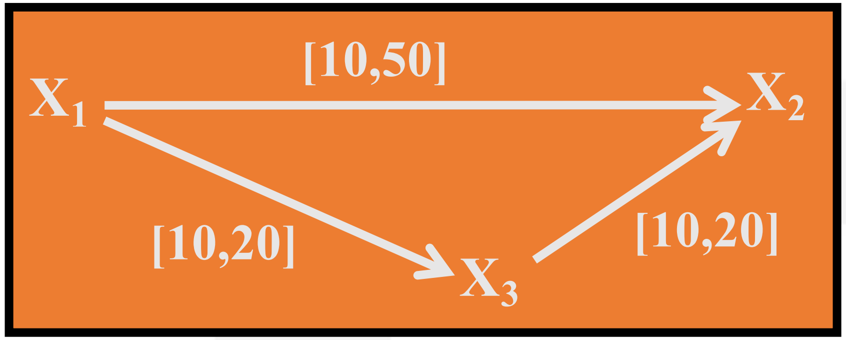
Homogeneous agents

Background

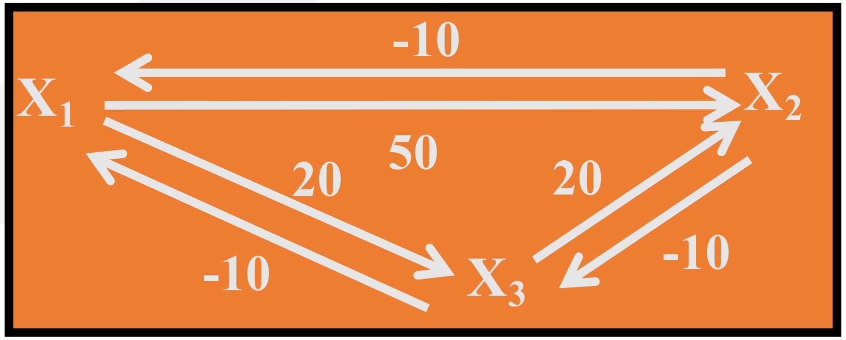
STN

STN (Simple Temporal Network) **is consistent** if there exists an assignment of the time-points that satisfies all the constraints

STN = (V, E)



Distance graph

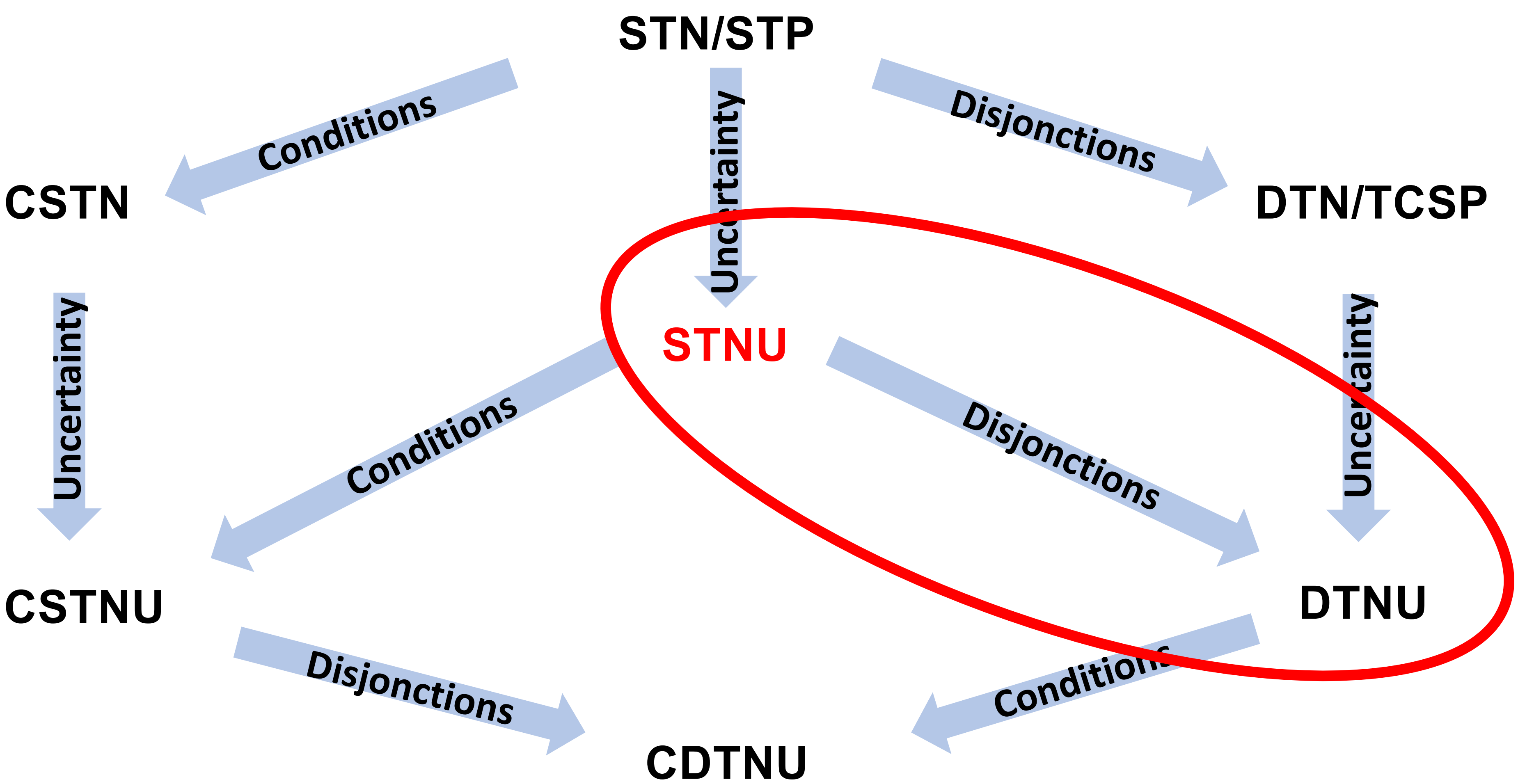


Polynomial propagation type algorithm in $O(n^3)$:

- Path consistency
- Floyd-Warshall

Negative cycle = inconsistency

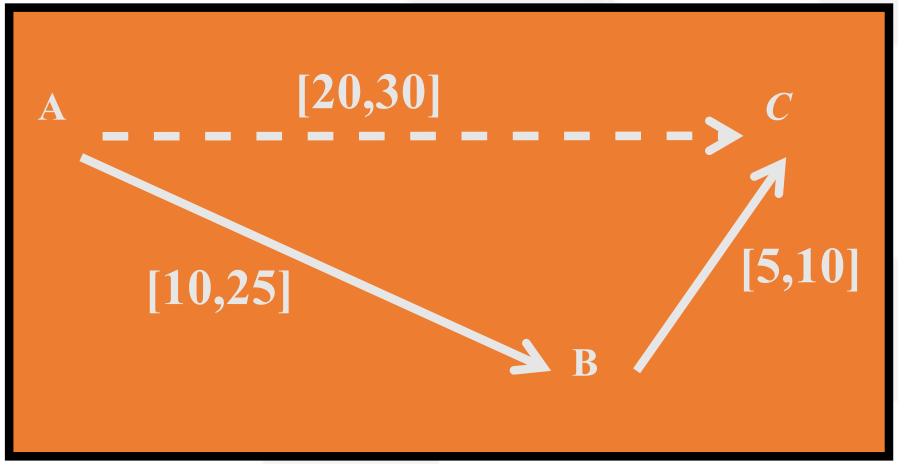
The whole family



STNU

STNU (Simple Temporal Network with Uncertainty) *[Vidal et Fargier, 1999]*

- idem STN + = intervals of possible durations : **controllable** / **contingent**



STNU X = (V, E, C)

Consistency →→→ **Controllability :**

A *situation* $\omega = \langle \omega_1 \in [L_1, U_1], \ldots, \omega_C \in [L_{|C|}, U_{|C|}] \rangle \in \Omega$

A *schedule* $\delta = \{\delta(v_1), \ldots, \delta(v_{|V_c|})\}$ *with* $\forall i, v_i \in V_c$

Clbty ≡ There exists a valid ∂ for each ω ?

3 levels of Controllability

Definition 7. *(Strong Controllability (SC)) An STNU $\mathcal{X}$ is **strongly controllable** iff $\exists\, \delta$ such that $\forall \omega \in \Omega$, δ is a solution of $\mathcal{X}_\omega$.*
Execution semantics: $\forall v_i \in V_c$, $dec(v_i) = v_0$, and the observations are free: possibly no observation ($\forall \omega_k \in \omega$, $obs(\omega_k) = \varnothing$) or observations during execution that will just update the bounds of the constraints in the network.

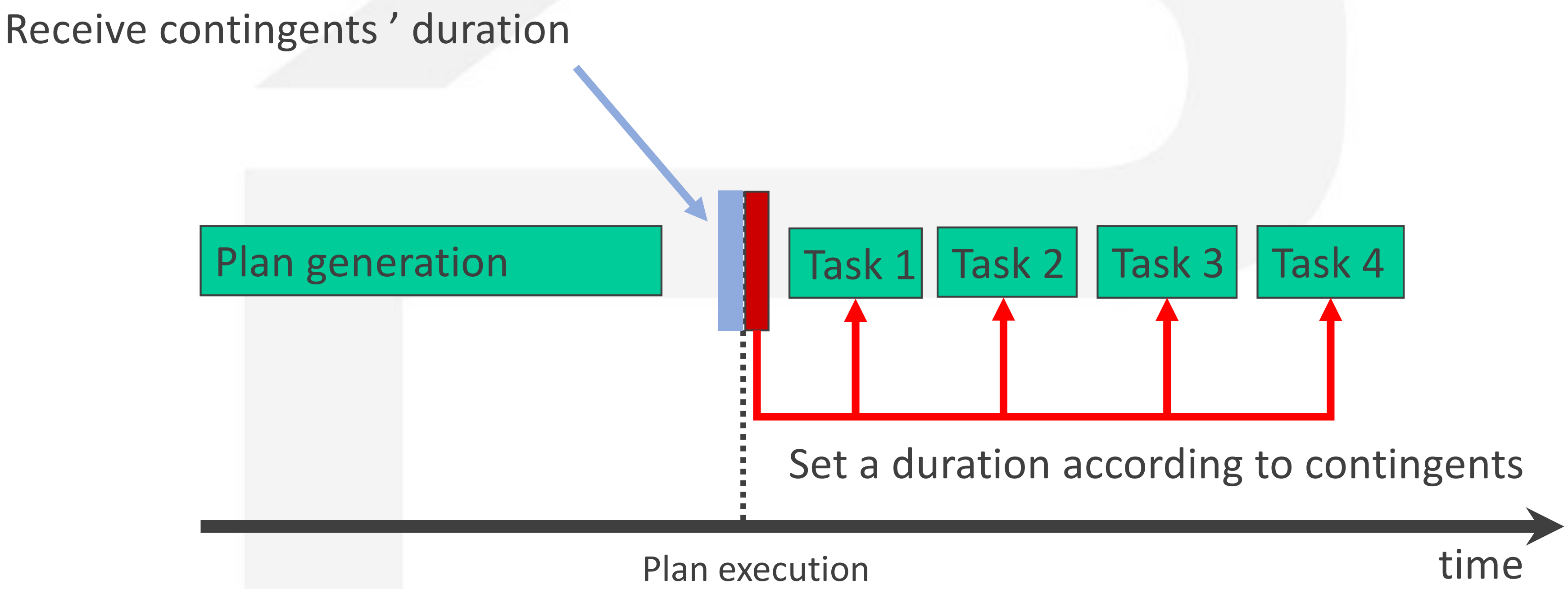
Definition 6. *(Dynamic Controllability (DC)) An STNU $\mathcal{X}$ is **dynamically controllable** iff it is weakly controllable and $\forall v_i \in V_c, \forall \omega, \omega' \in \Omega$, $\omega^{\preceq v_i} = \omega'^{\preceq v_i} \implies \delta(v_i) = \delta'(v_i)$*
Execution semantics: $\forall \omega_k \in \omega$, $obs(\omega_k) = end(c_k)$, and $\forall v_i \in V_c$, $dec(v_i) = v_i$

Definition 5. *(Weak Controllability (WC)) An STNU $\mathcal{X}$ is **weakly controllable** iff $\forall \omega \in \Omega, \exists \delta$ such that δ is a solution of $\mathcal{X}_\omega$.*
Execution semantics: $\forall \omega_k \in \omega$, $obs(\omega_k) = v_0$, and the decision policy is free: $\forall v_i \in V_c$, $dec(v_i) \leq v_i$

Strong Controllability $\implies$ Dynamic Controllability $\implies$ Weak Controllability

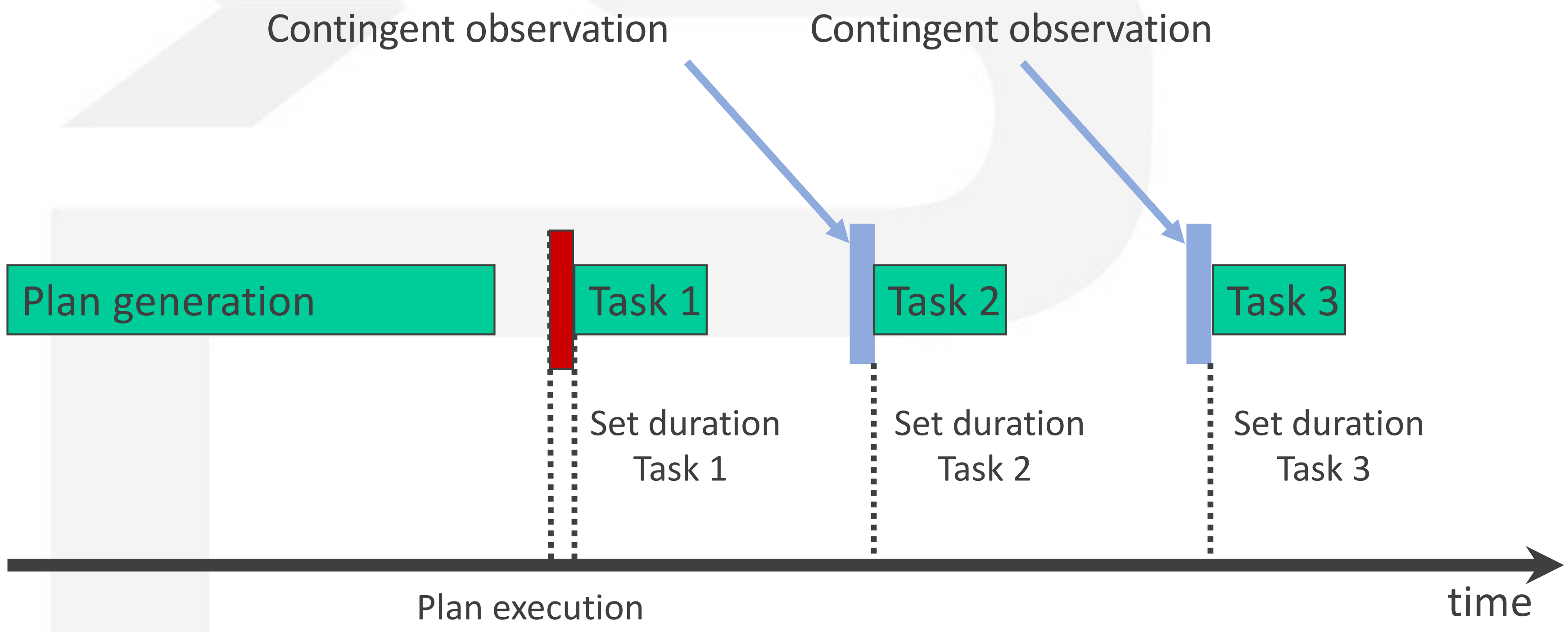
WC

- **Weak Controllability (WC):** assumes contingent durations will be known just before execution = pick up the solution that matches it.



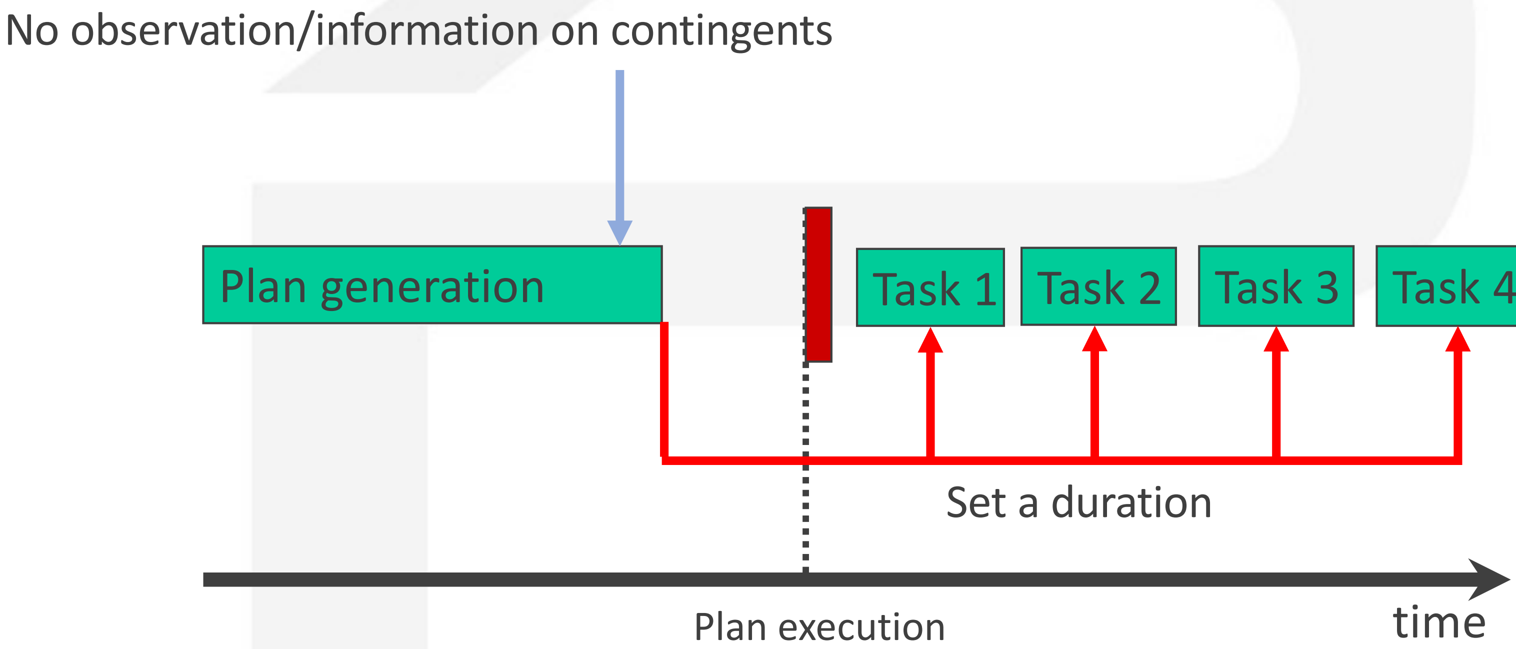
DC

- **Dynamic Controllability (DC):** assumes sequential decisions adapt to past observations.



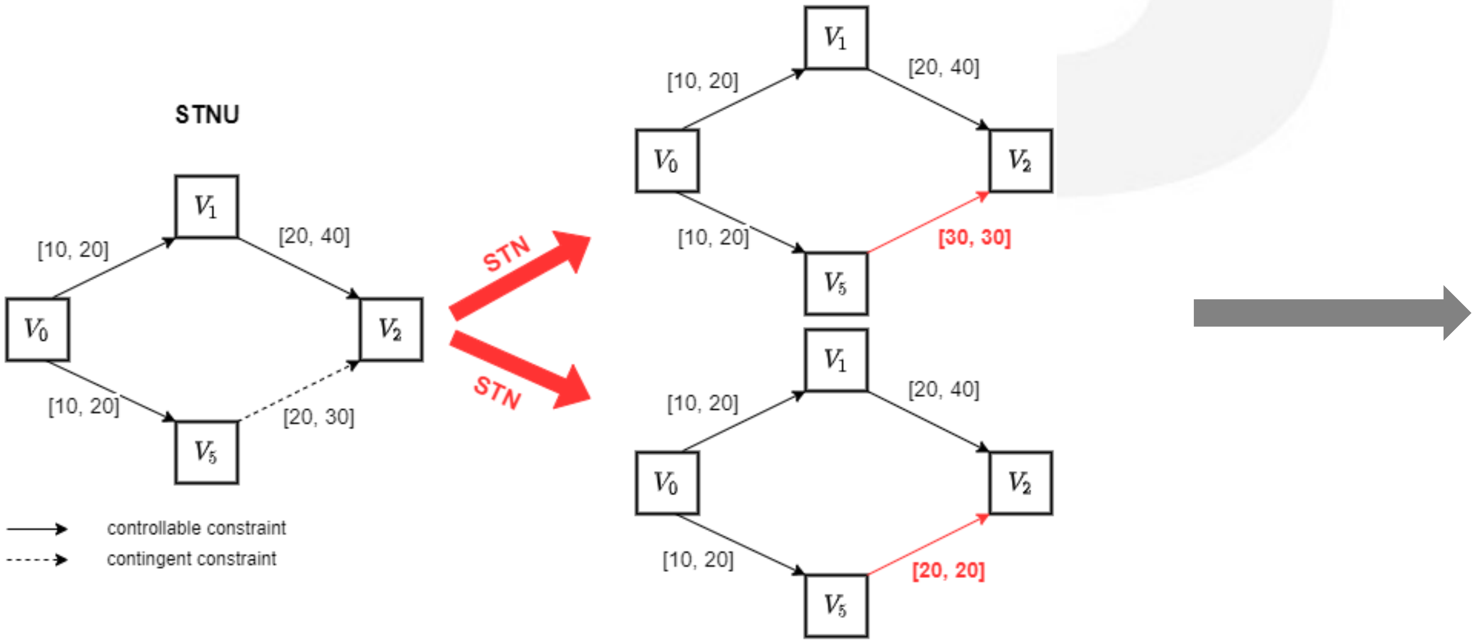
SC

- **Strong Controllability (SC):** assumes no observations, which requires a fixed schedule that satisfies the constraint, whatever the contingent durations will be.



Complexities

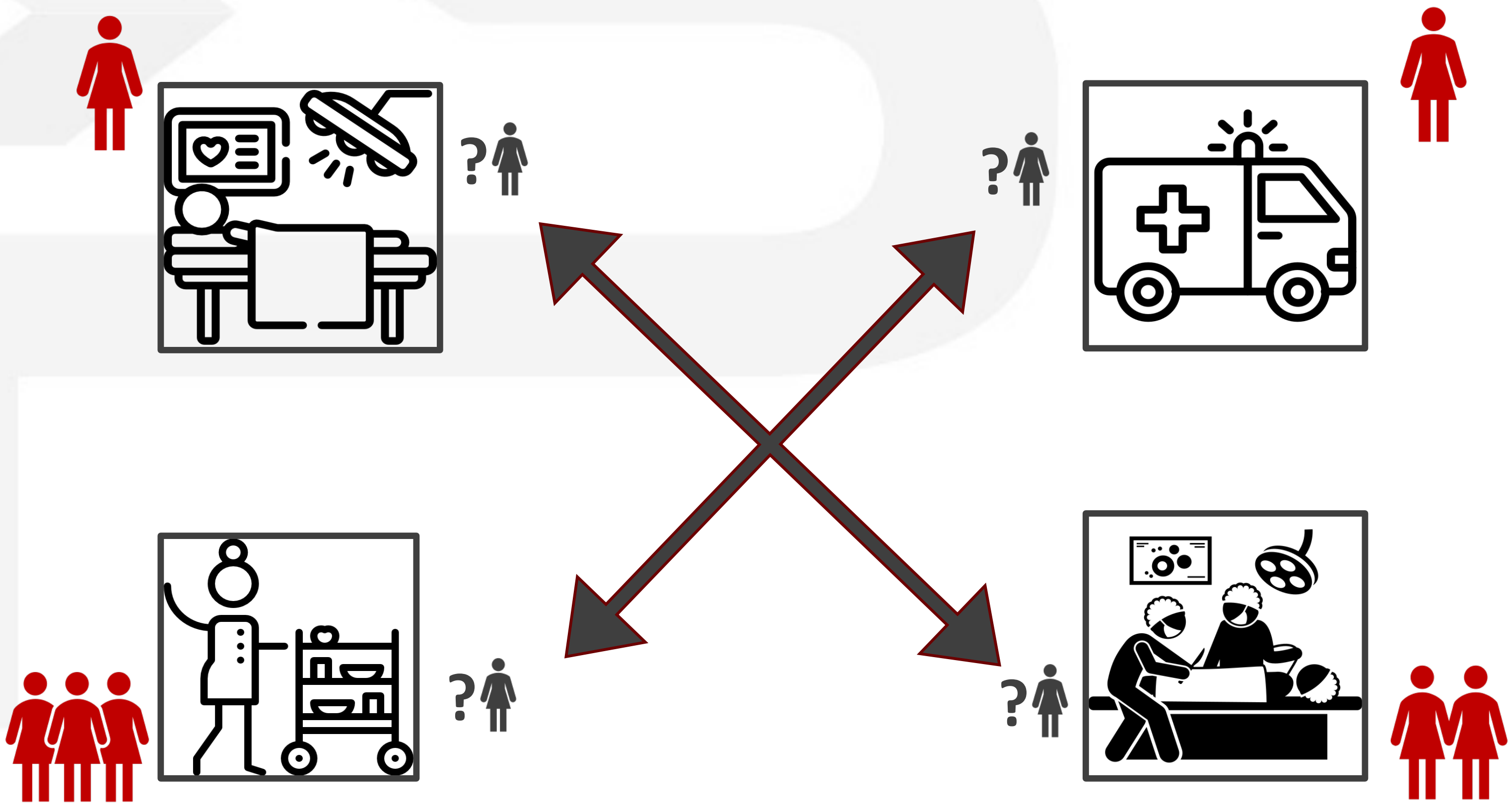
- Strong Controllability (SC) : Polynomial
- Dynamic Controllability (DC) : Polynomial
- Weak Controllability(WC) : co-NP-complete



$O(2^{|C|})$

Relevance of WC in multi-agent systems

- e.g., last minute ressource allocation



Algorithm

Consequence: new requirements for WC repair

Inform: an algorithm that is capable of explaining non WC

Efficiency: a WC checking algorithm more efficient than existing ones (exponential):

Weak Controllability: Methodology

Global Weak Controllability can be check through local checking !

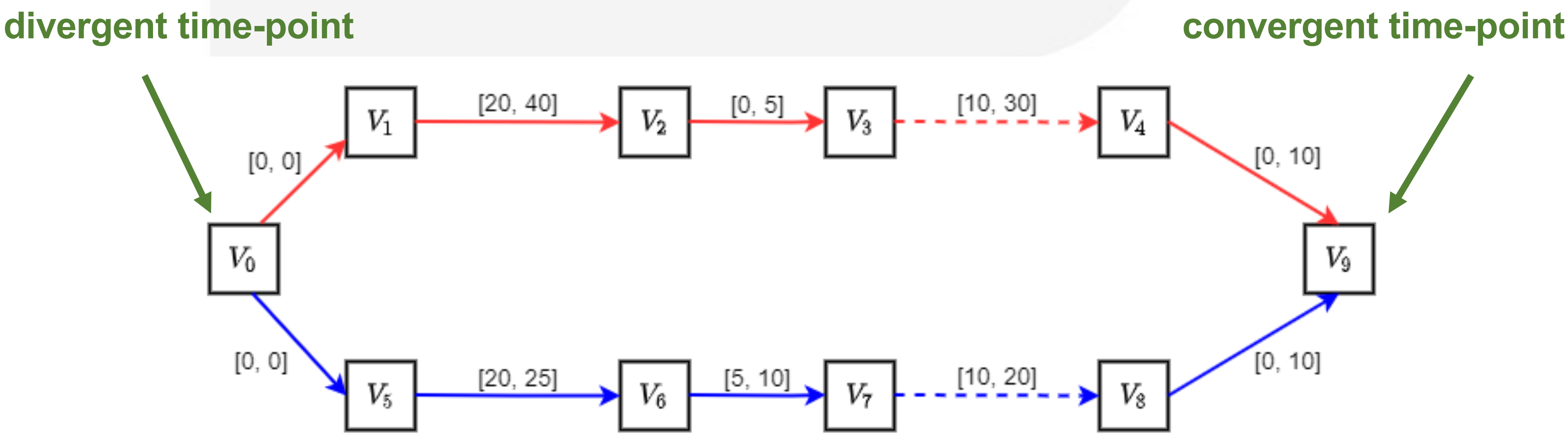
local checking: checking the cycles of an STNU (a cycle is a sub-STNU)

« A cycle is not WC if the synchronization cannot always be guareented »

Weak Controllability: Methodology

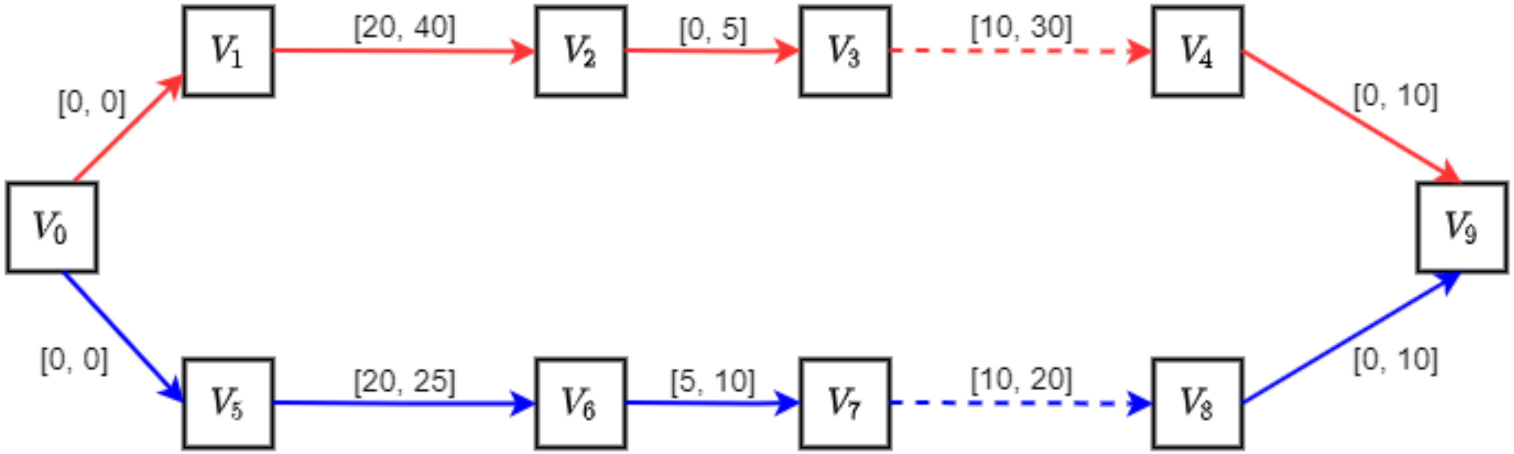
How is defined a cycle ?

Cycle: is two path that start from a divergent time-point and finishes on a convergent time-point



Weak Controllability: Methodology

How to check WC of a cycle ?



Weak Controllability: Methodology

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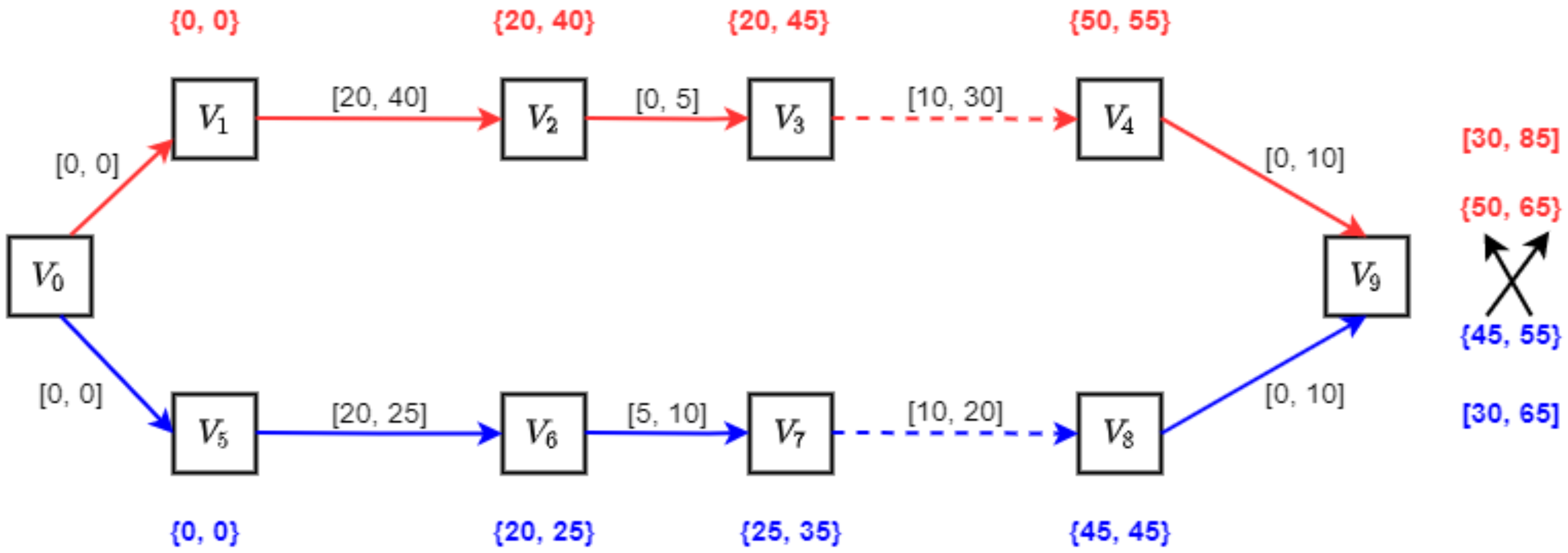
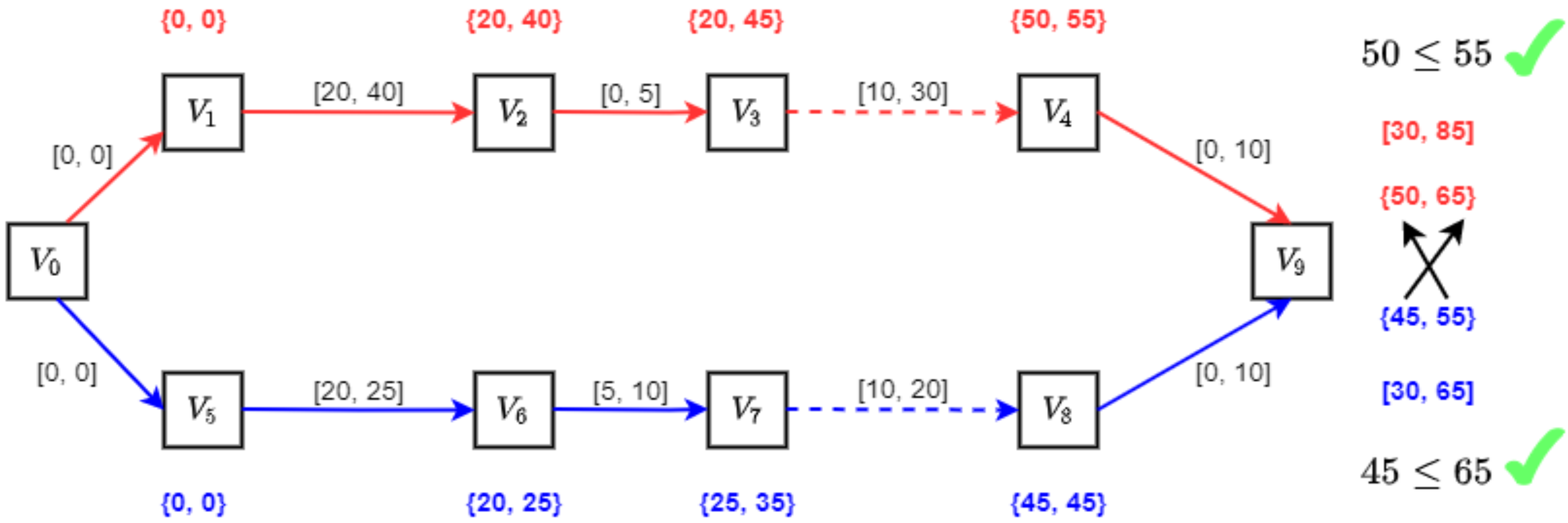


Weak Controllability: Methodology

How to check WC of a cycle ?

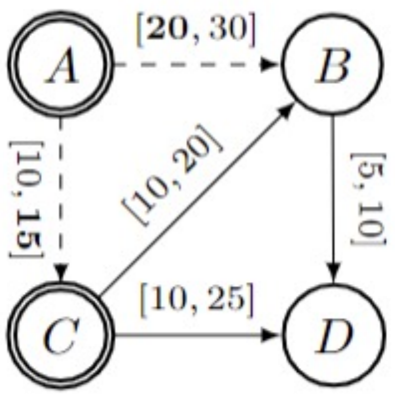


Weak Controllability: Methodology

How to globally check WC ?

Step 1: identify all divergent and convergent time-points of an STNU

Step 2: rank the time-point according to a topological ordering (use to **prune** paths)



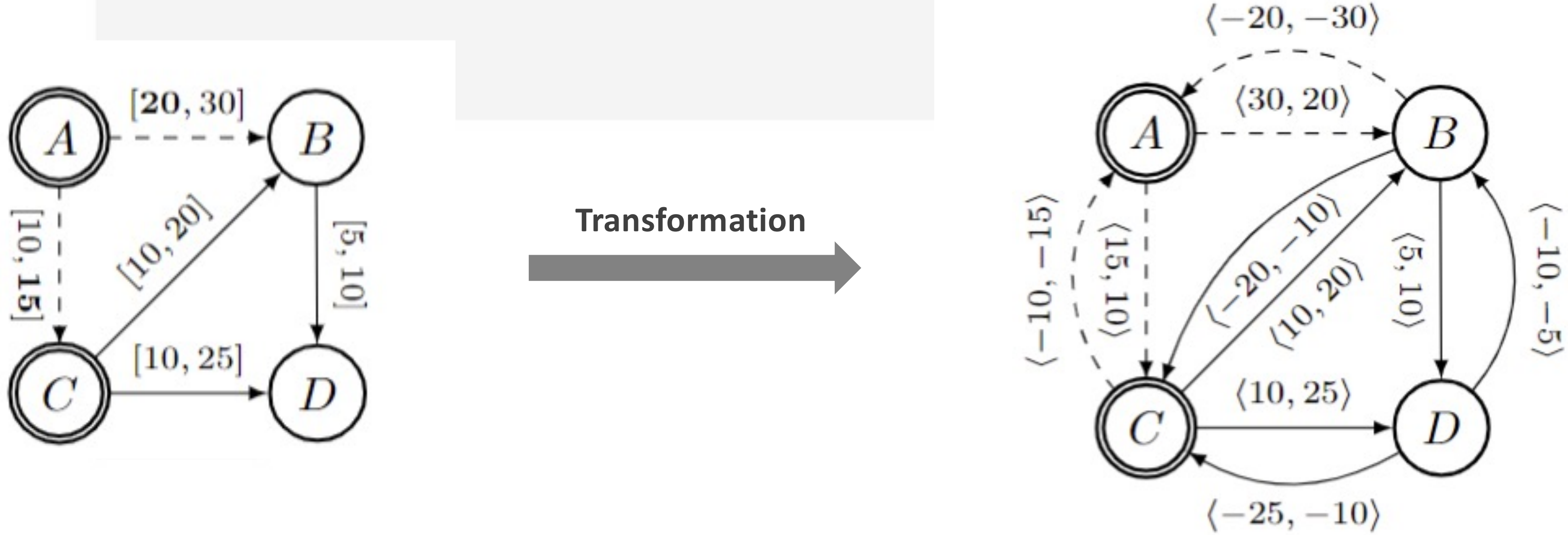
Example:

- divergent time-point (A, C) and convergent time-point (B, D)
- rank: A C B D

Weak Controllability: Methodology

How to globally check WC ?

Step 3: Find and check the cycles of each divergent time-point in the controllable bounds graph



Weak Controllability: The overall algorithm

From each divergent node

Find all couples of cycles of v_d

following a topological order

Algorithm 3 WC-Checking algorithm

Input: $\mathcal{X}$: STNU(V, v_0, V_c, V_d, E, C)

Output: Boolean

$\Pi^{ctl}_{\mathcal{X}}$ = getDistanceGraph($\mathcal{X}$)

rank = orderFromRank($\mathcal{X}$)

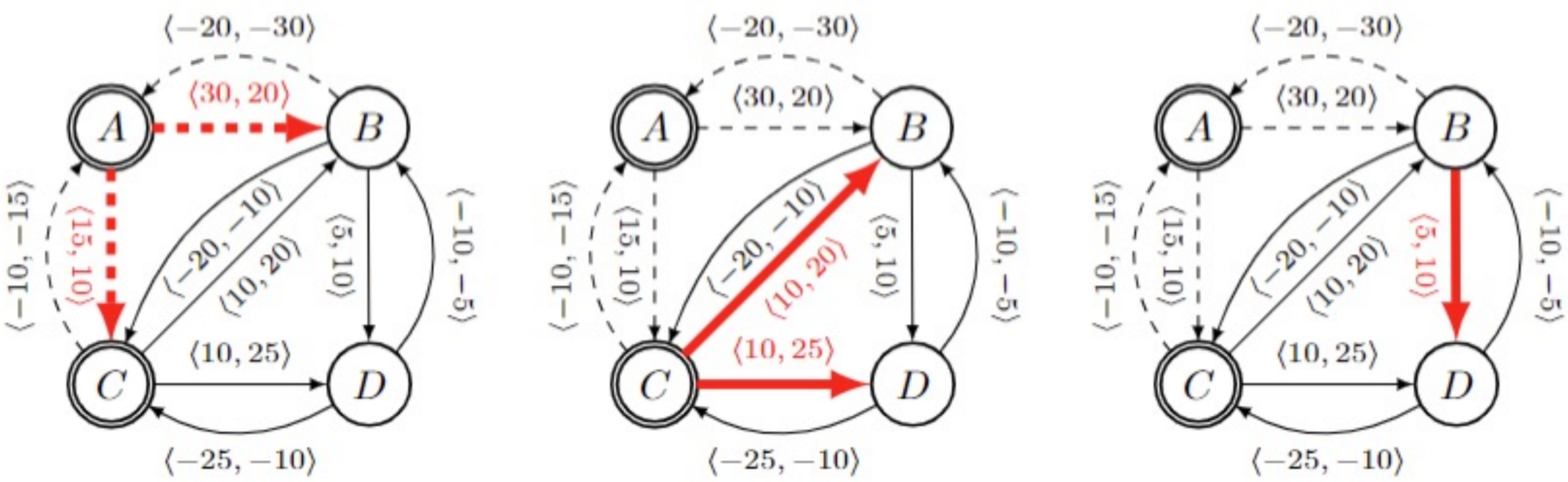
for *each v_d in V_d* **do**

  if *findDivergentCycles*(v_d, $\Pi^{ctl}_{\mathcal{X}}$, *rank*) == *False* **then**

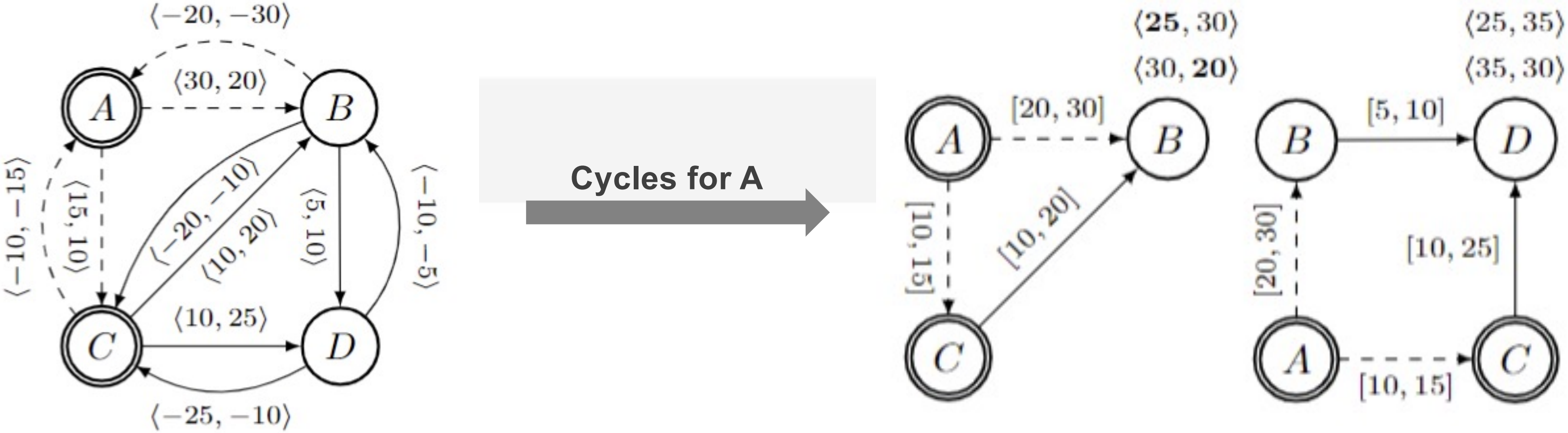
    return False `Or non-WC cycles of` v_d

return True `Or all non-WC cycles`

Weak Controllability: Example



Weak Controllability: The overall algorithm



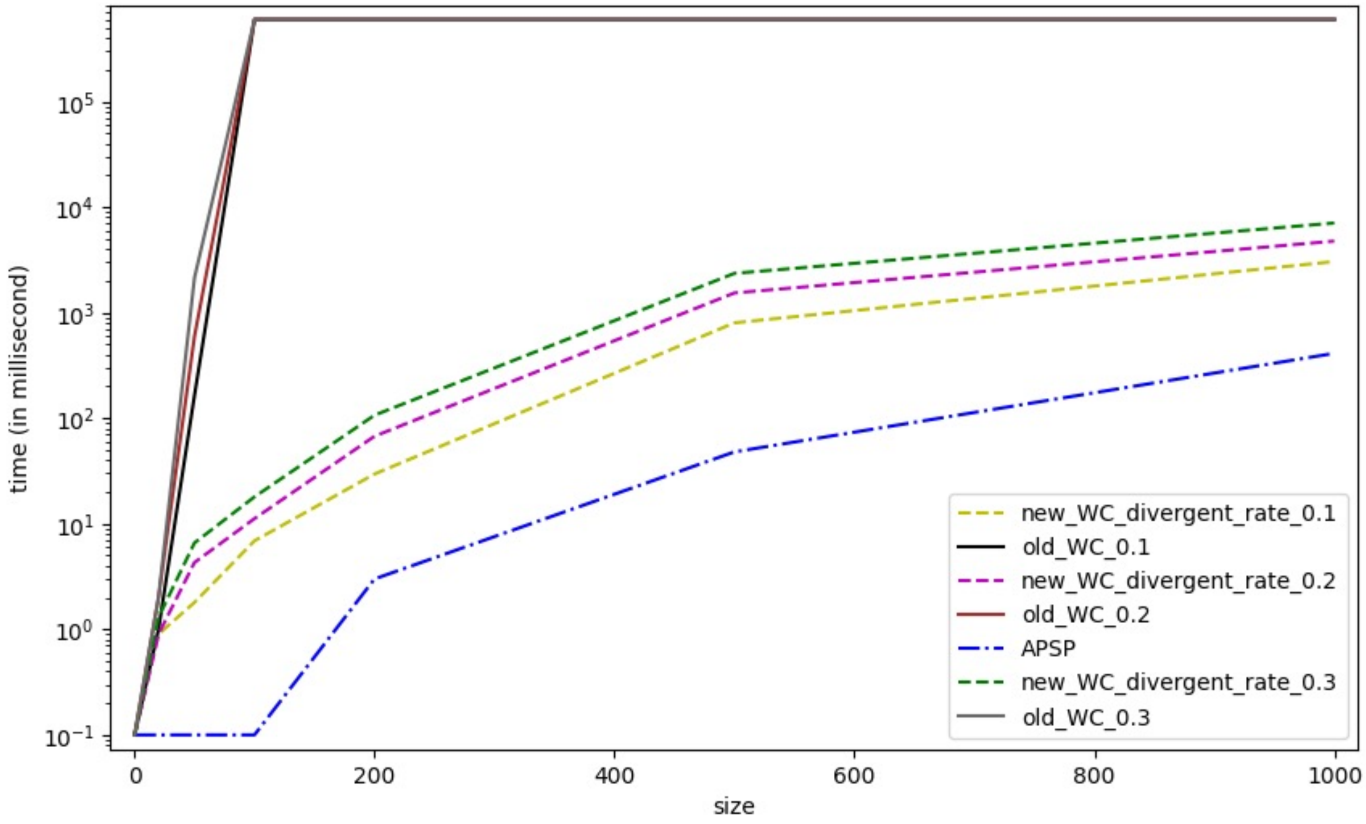
Weak Controllability: Complexity

- The algorithm is sensible to the number of cycles to check which depend on:
  - The number of divergent time-points
  - The number of succesor per divergent time-points
  - The number of contingent constraints

- In complete graph the algorithm is exponential

- We are interested in realistic graph (sequential graph): the parameters are restricted

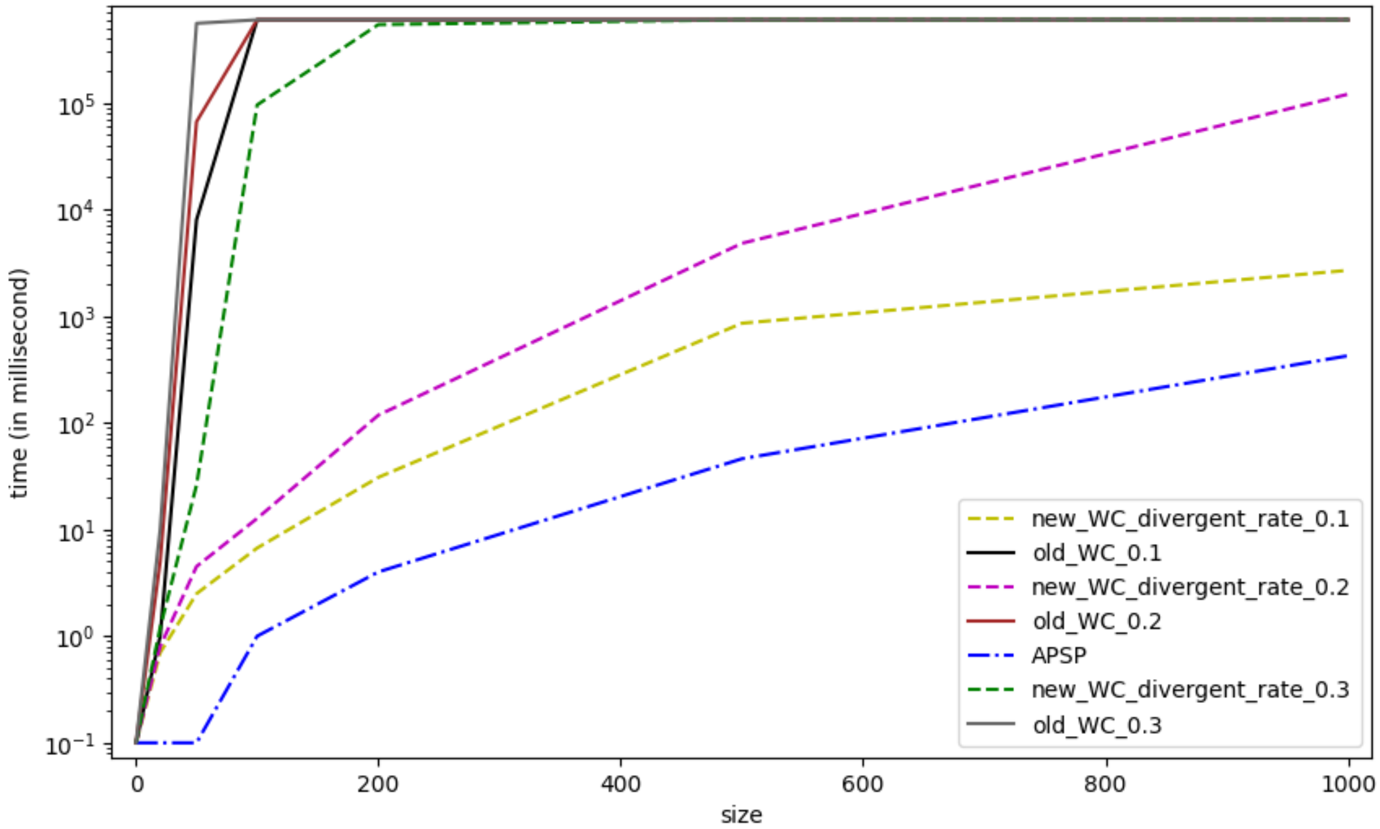
Experimentation



Settings:

- 20% of contingent
- 3 succesors per divergent time-point
- 10-30% of divergent time-points

Experimentation



Settings:

- 30% of contingent
- 3 succesors per divergent time-point
- 10-30% of divergent time-points

Conclusion

- The relevance of Weak Controllability in multi-agent system

- A new algorithm for checking Weak Controllability:
  - Inform (return the inconsistent cycles)
  - Efficient in realistic graphs but still sensible to many parameters

Thank you !

Time for questions