A New Intersection Model and Improved Algorithms for Tolerance Graphs

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WG 2009

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Overview

- Preliminaries on tolerance graphs.
- A new intersection model.
- A canonical representation and applications of this model.
    - Minimum Coloring: $O(n \log n)$ (optimal) [previous result: $O(n^2)$].
    - Maximum Clique: $O(n \log n)$ (optimal) [previous result: $O(n^2)$].
    - Maximum Weighted Independent Set: $O(n^2)$ [previous result: $O(n^3)$].
- Open problems.

Intersection graphs

Definition

An undirected graph $G = (V, E)$ is called an intersection graph, if each vertex $v \in V$ can be assigned to a set S_v, such that two vertices of G are adjacent if and only if the corresponding sets have a nonempty intersection, i.e. $E = \{uv \mid S_u \cap S_v \neq \emptyset\}$.

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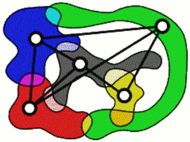

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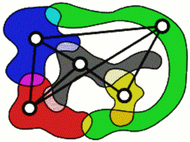

Lemma

Every undirected graph G has a (trivial) intersection model, based on adjacency relations.

Interval and permutation graphs

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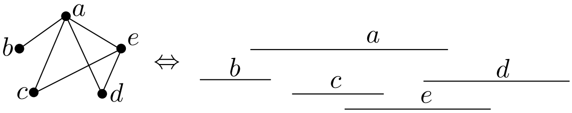
A graph G is called an interval graph, if G is the intersection graph of a set of intervals on the real line.



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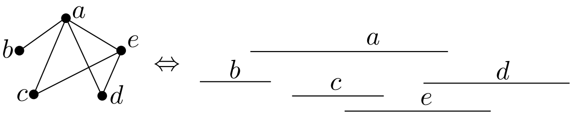
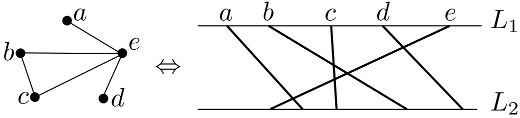


Definition

A graph G is called a permutation graph, if G is the intersection graph of a set of line segments between two parallel lines.



Tolerance and bounded tolerance graphs

Both interval and permutation graphs can be generalized as follows:

Definition (Golumbic, Monma, 1982)

A graph $G = (V, E)$ is called a tolerance graph, if there is a set $I = \{I_v \mid v \in V\}$ of intervals and a set $t = \{t_v \mid v \in V\}$ of positive numbers, such that $uv \in E$ if and only if $|I_u \cap I_v| \geq \min\{t_u, t_v\}$.

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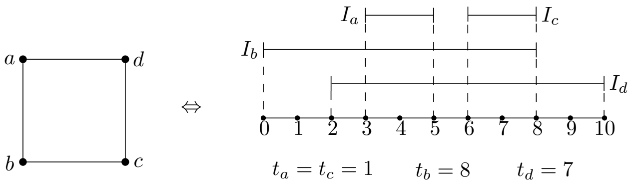

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Tolerance graphs find important applications [Golumbic, *Tolerance graphs*, 2004]:

- biology and bioinformatics (DNA sequences),
- temporal reasoning,
- resource allocation,
- scheduling ...

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Bounded tolerance graphs have a (non-trivial) intersection model of parallelograms between two parallel lines.
[Langley, PhD thesis, 1993; Bogart et al., 1995]

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Why? A succinct intersection model enables us to design efficient algorithms.

Parallelogram and bounded tolerance graphs

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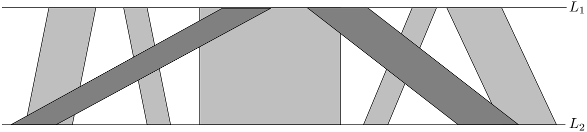
A graph G is called a parallelogram graph, if G is the intersection graph of a set of parallelograms between two parallel lines.



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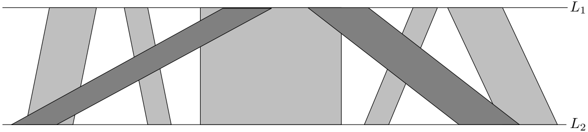

Theorem (Langley; Bogart et al.)

Bounded tolerance graphs coincide with parallelograms graphs.

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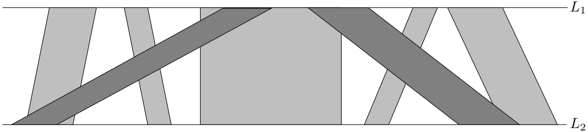

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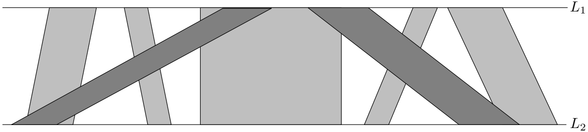

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Clearly:

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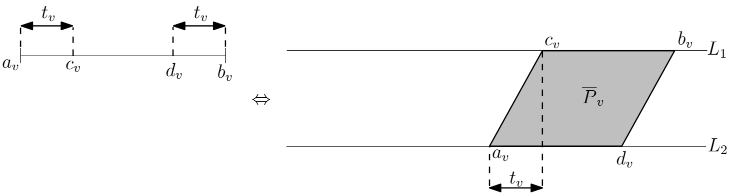
- permutation graphs are bounded tolerance graphs (lines are trivial parallelograms).

Parallelogram and bounded tolerance graphs

Idea (for bounded vertices):



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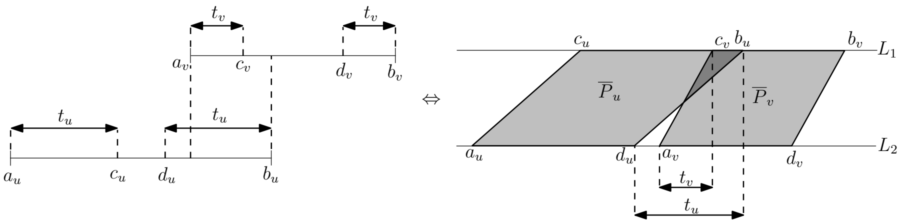

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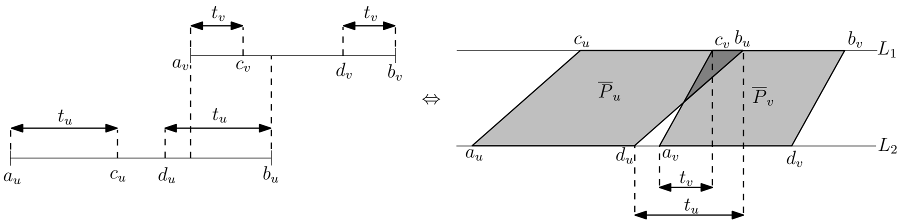

Question: How do we deal with the unbounded vertices?

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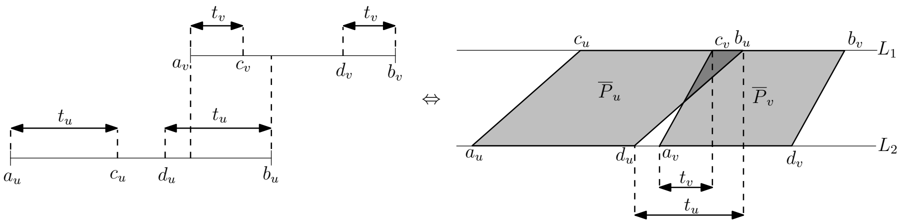

Question: How do we deal with the unbounded vertices?

Answer: We exploit the third dimension.

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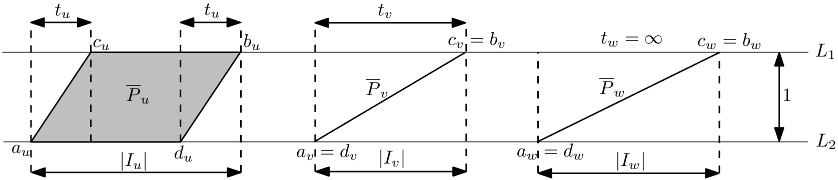
The slopes of the parallelograms

- We extend the construction of the parallelograms $\overline{P}_v$ to the case of unbounded vertices v.



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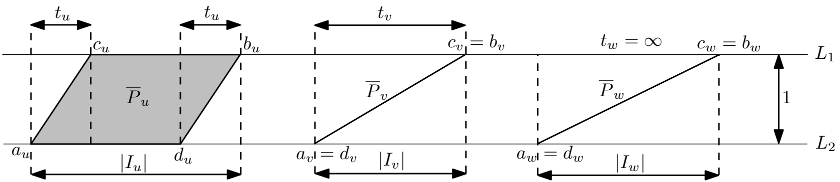

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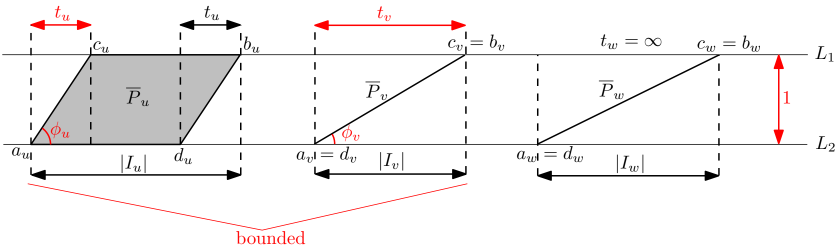

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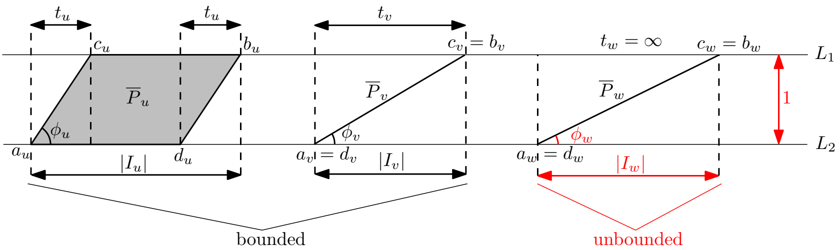

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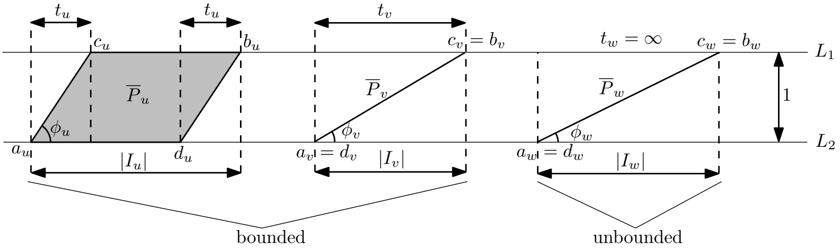
The slopes ϕ_v capture essential information about the structure.



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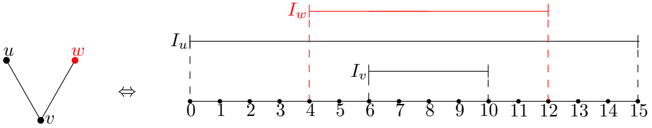

$t_u = 13 < |I_u| = 15$ (bounded)
$t_v = 2 < |I_v| = 4$ (bounded)
$t_w = 100 > |I_w| = 8$ (unbounded)

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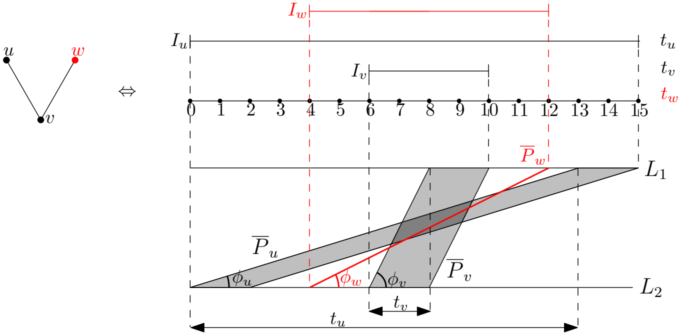

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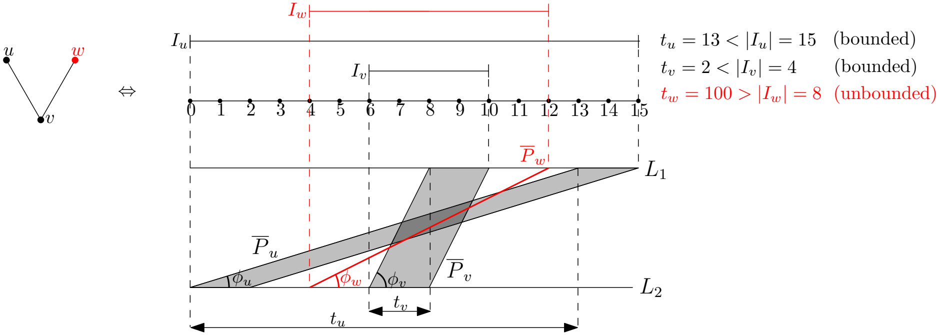

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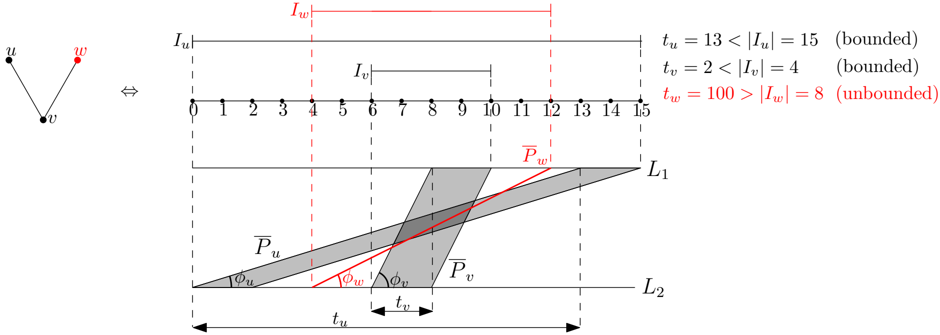

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However, $wv \in E$, but $wu \notin E$.

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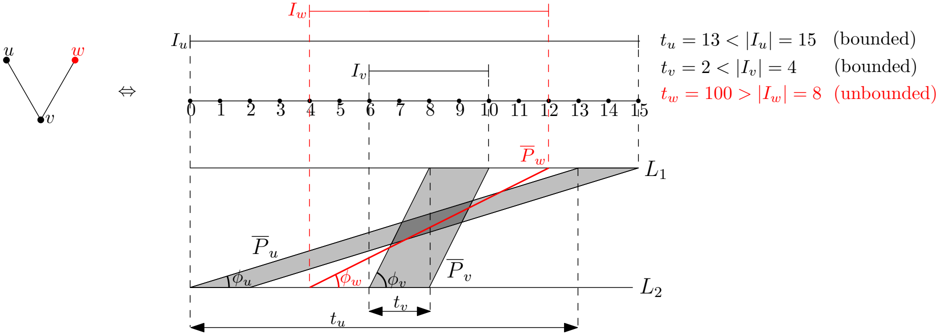

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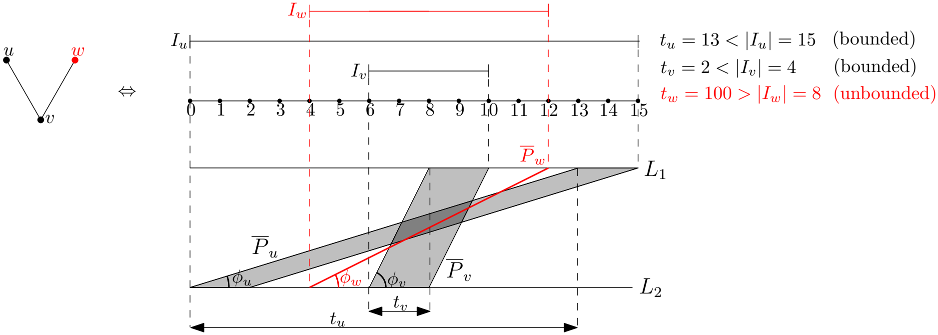

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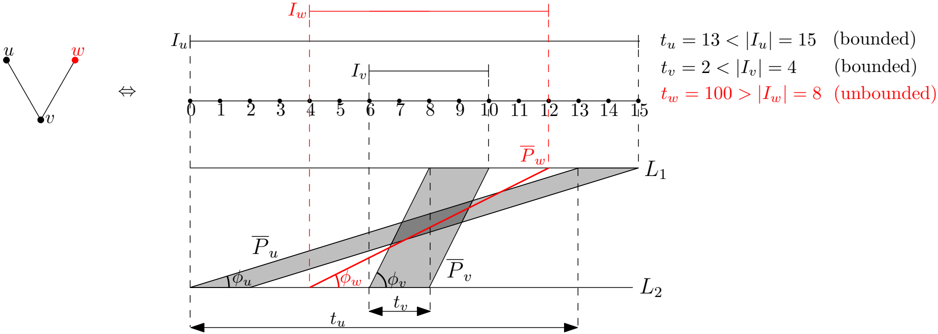

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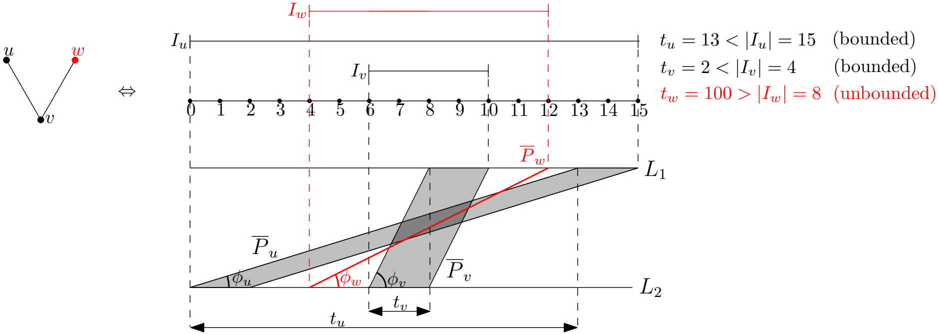

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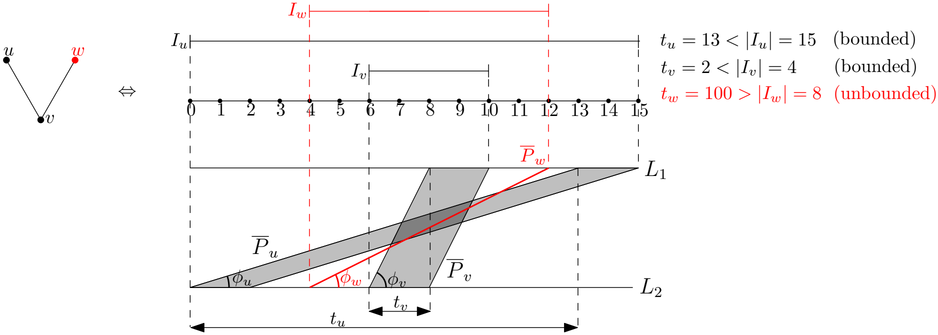

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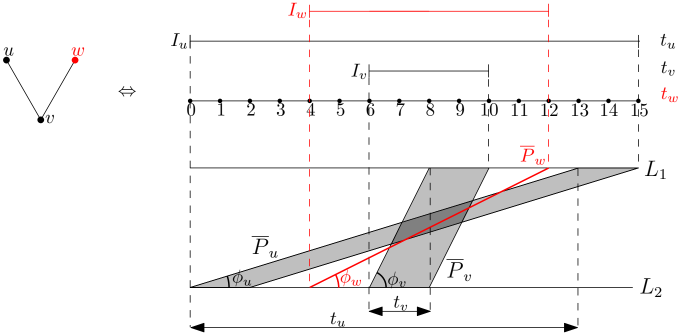

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Intersection of 3D-parallelepipeds

We define the parallelepipeds P_v as follows:

- If v is bounded, then
  $P_v = \{(x, y, z) \in \mathbb{R}^3 \mid (x, y) \in \overline{P}_v,\ 0 \leq z \leq \phi_v\}$.

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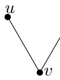
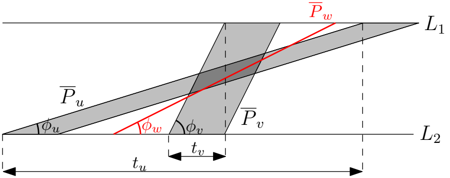
The parallelepiped representation: (an intersection model for general tolerance graphs)





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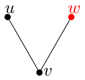

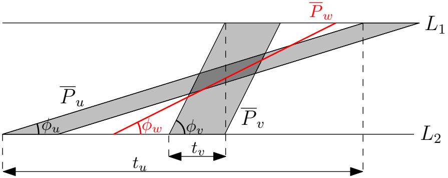

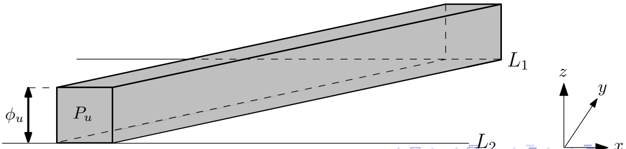

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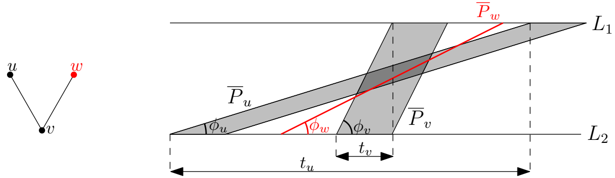

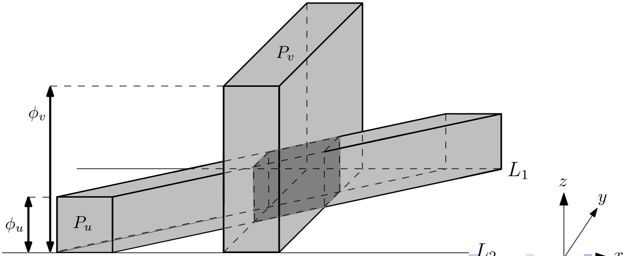

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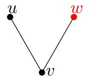

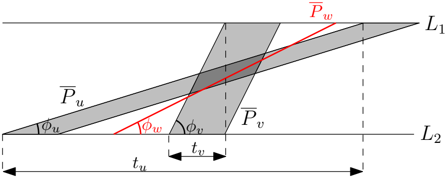

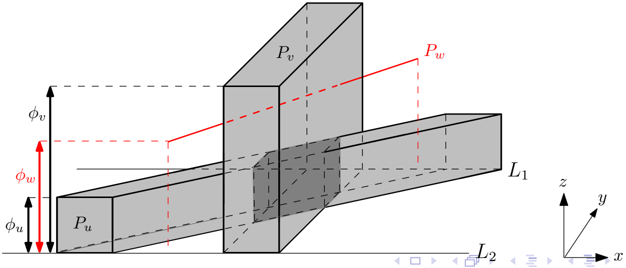

Intersection of 3D-parallelepipeds

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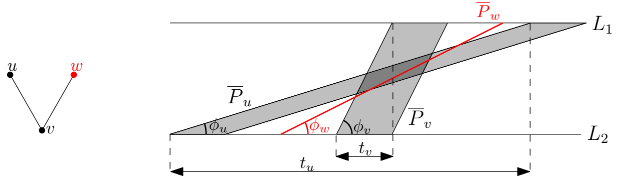
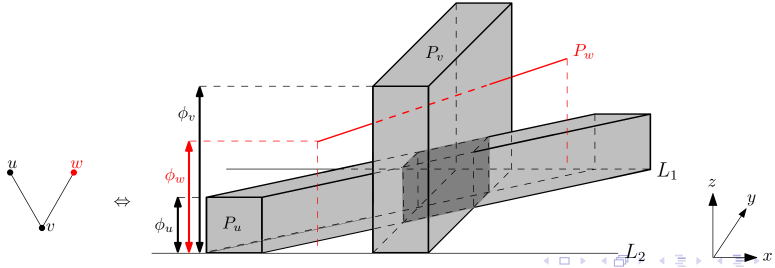
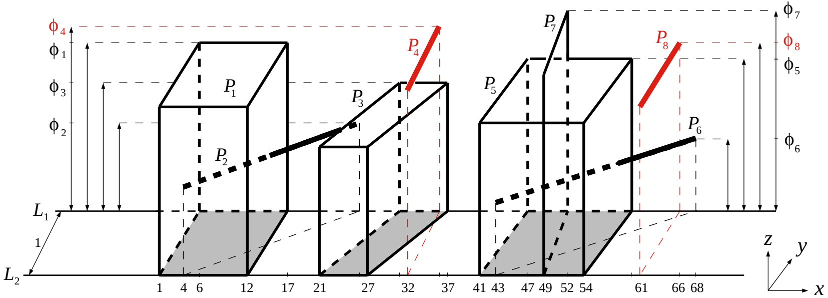




The parallelepiped representation

Definition

An unbounded vertex v of a tolerance graph G (in a certain parallelepiped representation) is called inevitable, if replacing P_i with $\{(x,y,z) \mid (x,y) \in P_v, 0 \leq z \leq \phi_v\}$ creates a new edge in G.



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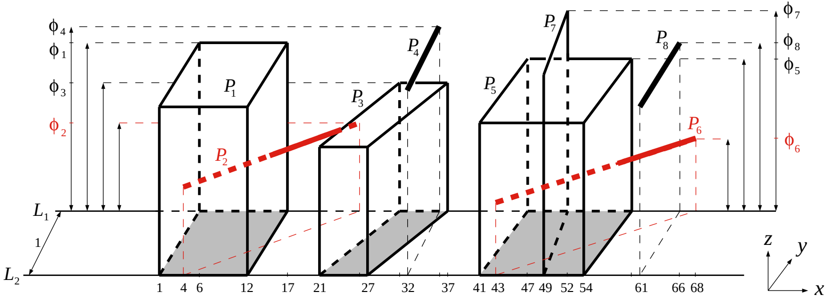

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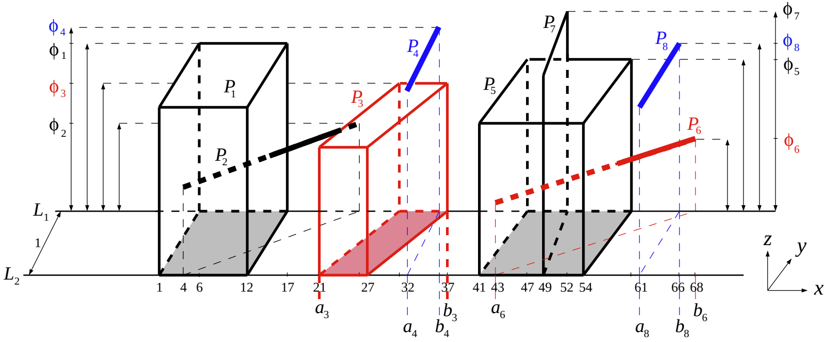
Definition

Let v be an inevitable unbounded vertex of a tolerance graph G (in a certain parallelepiped representation). A vertex u is called a hovering vertex of u if $\phi_u < \phi_v$, $a_u < a_v$, and $b_u > b_v$.



Overview

- Preliminaries on tolerance graphs.
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- A canonical representation and applications of this model.
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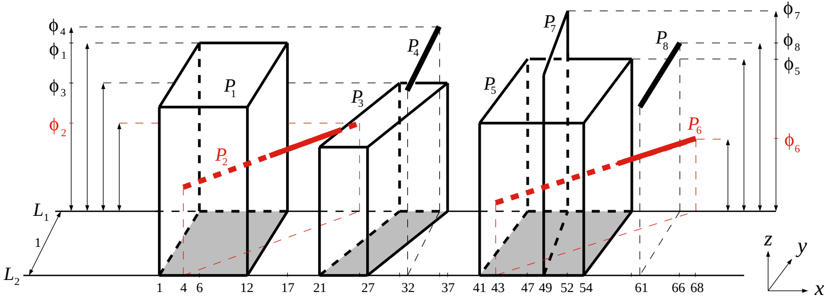
A parallelepiped representation of a tolerance graph G is called canonical if every unbounded vertex is inevitable.



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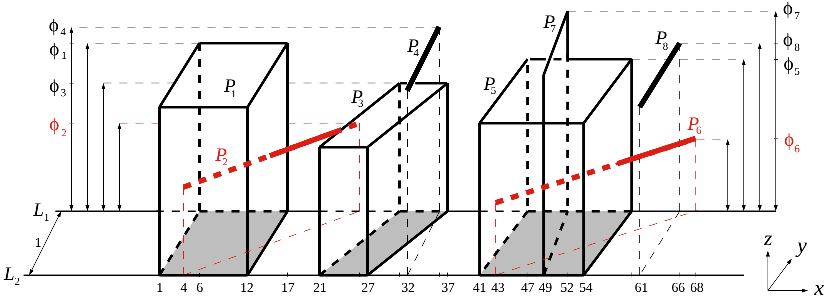
The canonical tolerance representation of a tolerance graph G can be defined similarly [Golumbic, Siani, 2002].



The canonical (parallelepiped) representation

Lemma (Golumbic, Siani, 2002)

Given a tolerance representation of a tolerance graph G, a canonical tolerance representation of G can be computed in $O(n^2)$ time.

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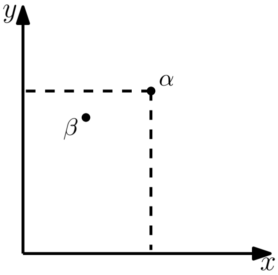
Idea: binary search

Definition

Let $\alpha = (x_\alpha, y_\alpha)$ and $\beta = (x_\beta, y_\beta)$ be two points in the plane. Then α dominates β if $x_\alpha > x_\beta$ and $y_\alpha > y_\beta$.



The canonical (parallelepiped) representation

Sketch of the algorithm

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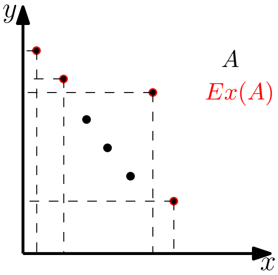
Let $\alpha = (x_\alpha, y_\alpha)$ and $\beta = (x_\beta, y_\beta)$ be two points in the plane. Then α dominates β if $x_\alpha > x_\beta$ and $y_\alpha > y_\beta$. Given a set A of points, the point $\gamma \in A$ is called an extreme point of A if there is no point $\delta \in A$ that dominates γ. $Ex(A)$ is the set of the extreme points of A.



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- let $A_i = \{p_1, p_2, \ldots, p_i\}$ be the first i points.

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Let v_i be an unbounded vertex of a tolerance graph G. Then:

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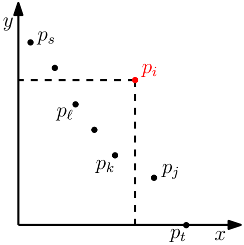

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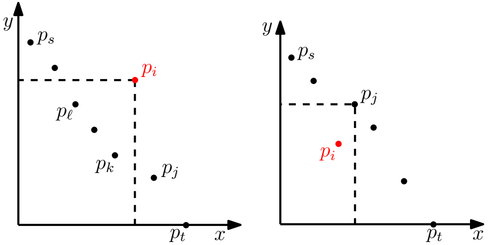

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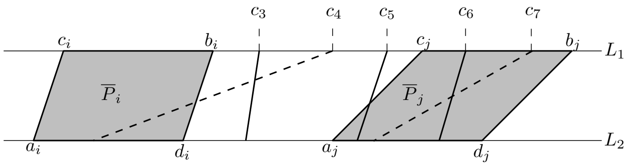

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- Recognition of proper / unit tolerance graphs ?

Thank you for your attention!