An algorithmic study of switch graphs

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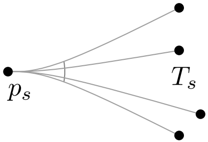
What is a switch?

Definition (Switch)

A *switch* on a vertex set V is a pair (p_s, T_s).

- $p_s \in V$ is the *pivot* vertex
- $\emptyset \neq T_s \subseteq V$ is the set of *target* vertices

We may enable exactly one of the edges $\{\{p_s, t_s\} \mid t_s \in T_s\}$



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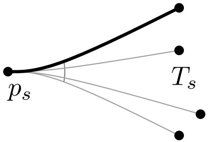

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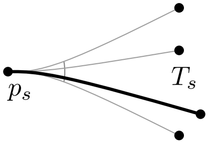

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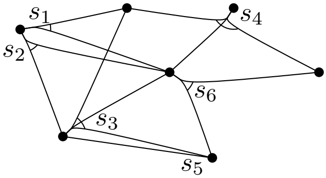
Switch graph $G = (V, S)$: S set of switches on vertex set V.



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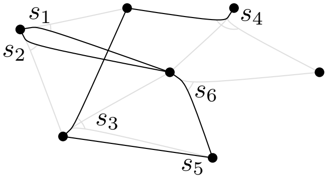

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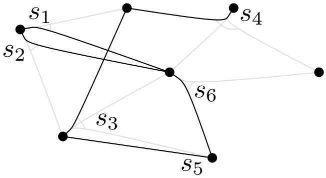

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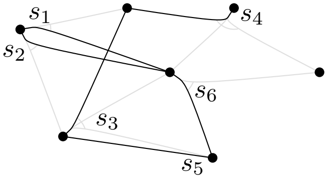

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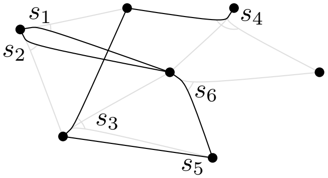

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A switch graph represents a population of graphs, a configuration describes a concrete member of that family.

Problems on switch graphs

Every graph property $\mathcal{P}$ yields problem on switch graphs:

Problem Switch-$\mathcal{P}$

Input: Switch graph $G = (V, S)$ with $n := |V|$, $m := |S|$ and fan-out $k := \max_{s \in S} |T_s|$.
Question: Does G have a configuration c such that G_c has property $\mathcal{P}$?

- Bipartiteness
- Global Connectivity
- Local Connectivity (given vertices a, b are connected)
- Biconnectivity
- Planarity
- Eulerianness
- Acyclicity

Related work

Cook introduced switch graphs to study certain stable configurations of Rule 101: [Cook '03]

- $O(n^2)$ algorithm detecting a configuration containing a cycle.
- *binary* switches, only vertices with degree 3 may have a switch

Groote and Ploeger study complexity of certain graph properties on switch graphs with binary switches: [Groote, Ploeger '08]

- mostly *forward directed* switch graphs (pivot $\rightarrow$ target)
- $O(n)$ algorithm for finding configuration with a directed a–b path, for given a, b.
- Several open questions, our work is heavily inspired by these.

Reinhardt gives $O(n^4)$ algorithm for finding a configuration with an a–b path in a model similar to Cook's. [Reinhardt '09]

Our results.

The following problems are NP-hard for switch graphs:

- SwitchBipartite
- SwitchTriangleFree
- SwitchPlanar
- SwitchBiconnected
- SwitchEulerian (directed and undirected)
- SwitchStronglyConnected
- SwitchMinCycles

We give efficient algorithms for the following problems:

- SwitchDirectedAcyclic: $O(n + km)$ time
- Global connectivity: $O(km + kn^2)$ time
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SwitchPlanar

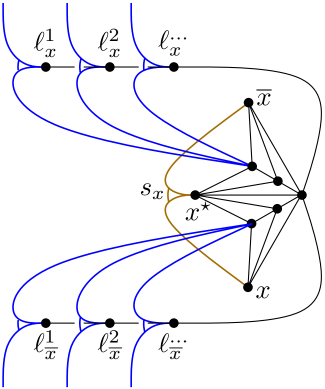
Theorem

Given a switch graph $G = (V, S)$ it is NP-hard to decide whether there exists a configuration c such that G_c is planar.

Proof: Gadget proof, reduction from a variant of Planar 3Sat.

Hardness of SWITCHPLANAR

Gadget proof:



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Hardness of SwitchPlanar

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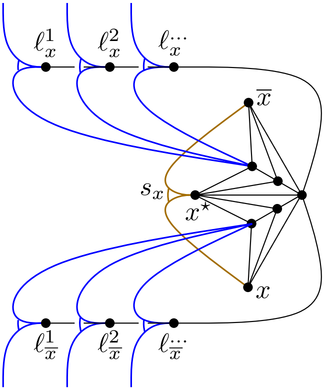

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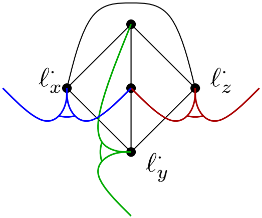
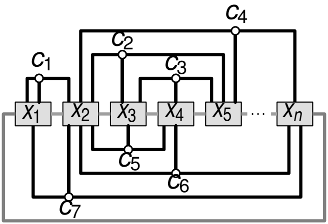


Clause

Monotone Planar 3SAT

Planar 3SAT:



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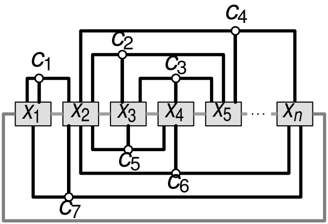

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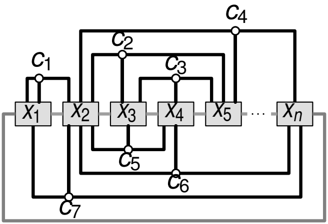

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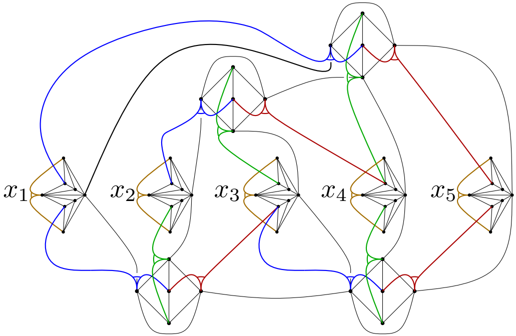
Theorem (de Berg, Koshravi, submitted)

Monotone Planar 3Sat *is NP-hard.*

SwitchPlanar *(example)*



$$(x_1 \vee x_4 \vee x_5) \wedge (x_2 \vee x_3 \vee x_4) \wedge$$
$$(\overline{x}_1 \vee \overline{x}_2 \vee \overline{x}_3) \wedge (\overline{x}_3 \vee \overline{x}_4 \vee \overline{x}_5)$$

Global Connectivity

Definition (SwitchConnect)

Given a switch graph $G = (V, S)$. Is there a configuration c of the switches such that G_c is connected?

A polynomial-time algorithm

Switch graph $G = (V, S)$,
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Connectivity via matroid intersection

Define two matroids $(E, \mathcal{I}_1)$, $(E, \mathcal{I}_2)$:

- $E' \in \mathcal{I}_1 \Leftrightarrow E'$ is acyclic
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$\rightsquigarrow$ polynomial algorithm $\approx O(n^4)$

A new matroid

A matroid of switches

Define new matroid $(S, \mathcal{I}_3)$:
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G YES-Instance of SWITCHCONNECT $\Leftrightarrow$ $(S, \mathcal{I}_3)$ has independent set of size $n-1$

To solve SWITCHCONNECT we need to steps:

1. Show that $(S, \mathcal{I}_3)$ is a matroid.
2. Give a fast independence test for $(S, \mathcal{I}_3)$.

A new matroid

Theorem

$(S, \mathcal{I}_3)$ is a matroid.

Proof:

- $\emptyset \in \mathcal{I}_3$.
- $A \in \mathcal{I}_3$, $B \subseteq A \Rightarrow B \in \mathcal{I}_3$.

Remains to show: $A, B \in \mathcal{I}_3$, $|A| < |B| \Rightarrow \exists x \in B \setminus A : A \cup \{x\} \in \mathcal{I}_3$.

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- Let c_A, c_B acyclic configurations of A, B with max. similarity, i.e., $|\{s \in A \cap B \mid c_A(s) = c_B(s)\}|$ maximum.
  Let E_A, E_B be the corresponding acyclic edge sets.
- $|E_A| < |E_B| \Rightarrow$ ex. $e_b \in E_B \setminus E_A$ with $E_A \cup \{e_b\}$ acyclic. Let $x \in B$ the corresponding switch.
- Now $A \cup \{x\}$ is independent and $x \notin A$, otherwise could make c_A, c_B more similar.

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Let $S' \subseteq S$. *Then* $S' \in \mathcal{I}_3 \quad \Rightarrow \quad \forall S'' \subseteq S' \colon |S''| < |V(S'')|$.

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We show that S' contains a subset with few vertices.

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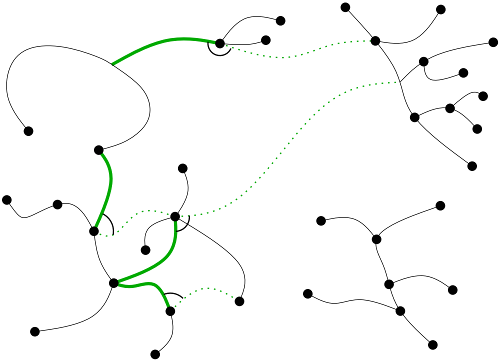

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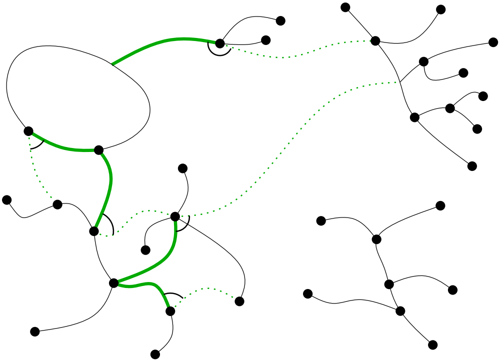

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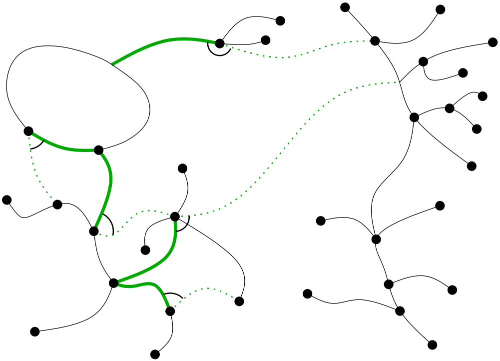

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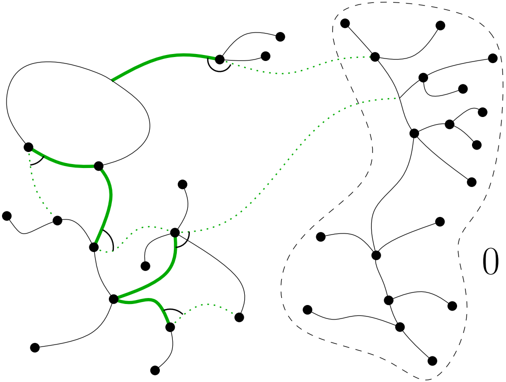

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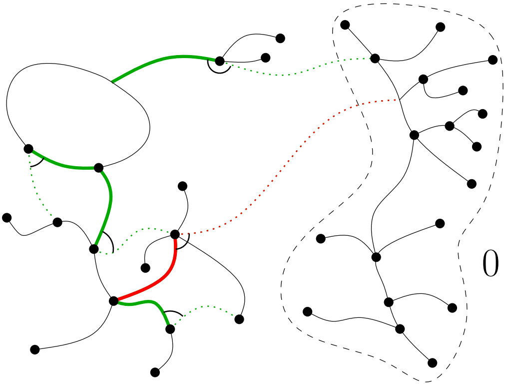

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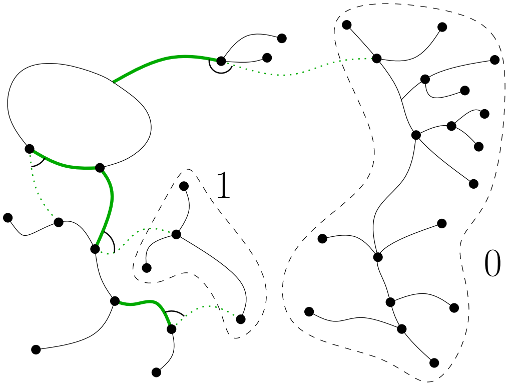

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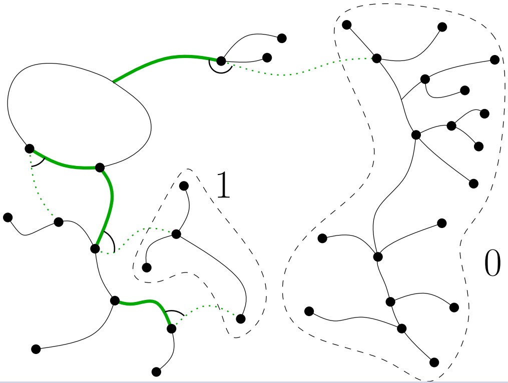

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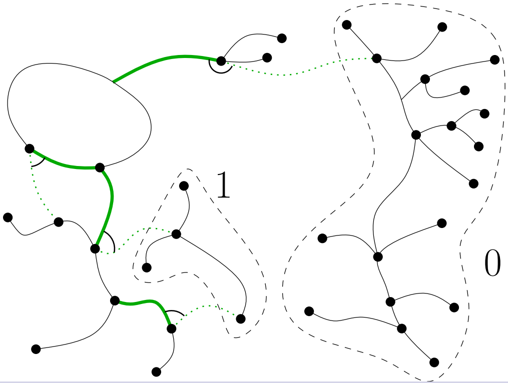

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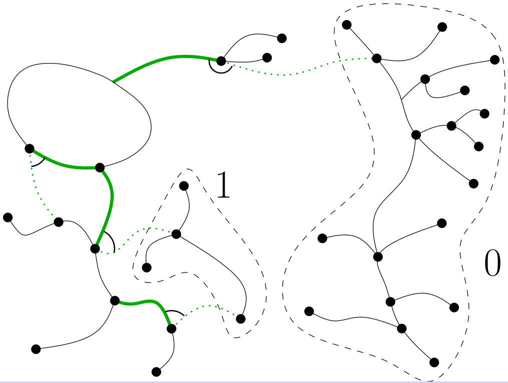

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1. $!\exists$ selectable switch $\Rightarrow$ set of vertices with too many switches
2. After removal of $s_1, \ldots, s_k$ remaining graph is acyclic

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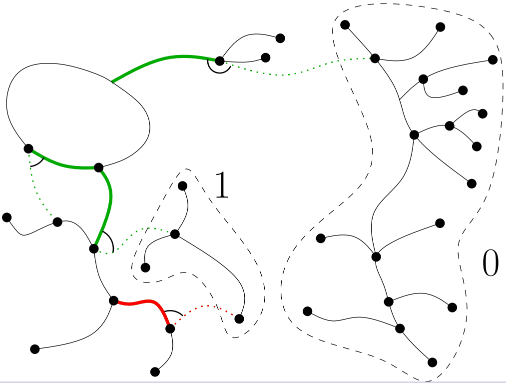

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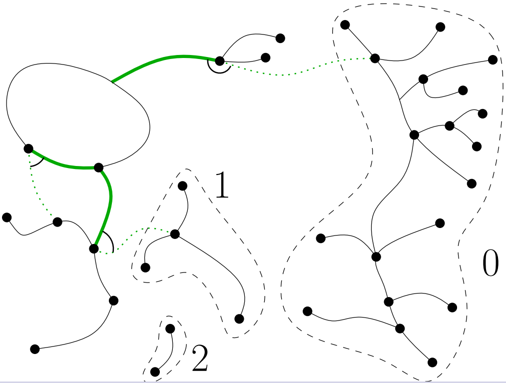

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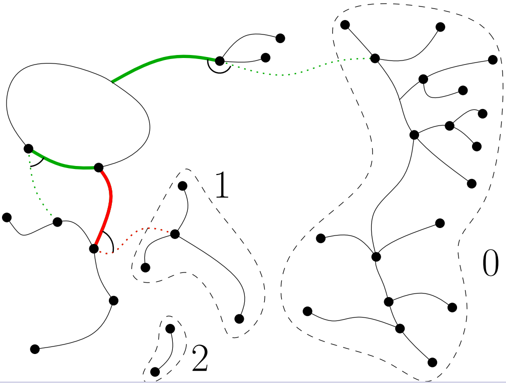

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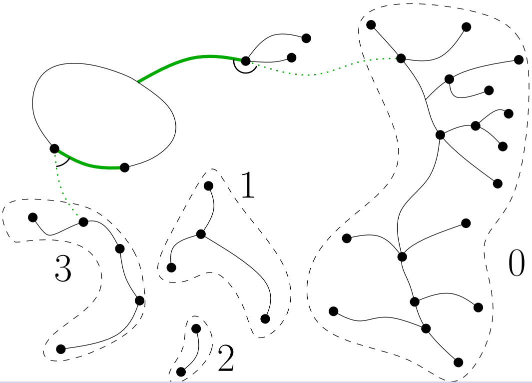

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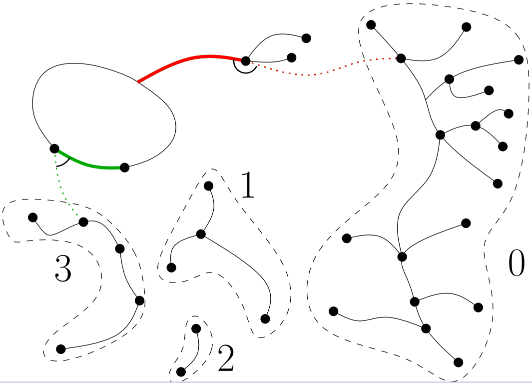

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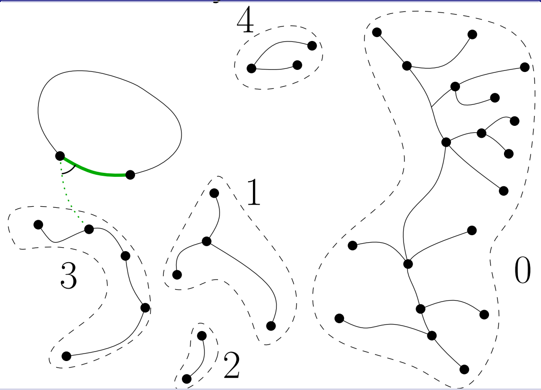

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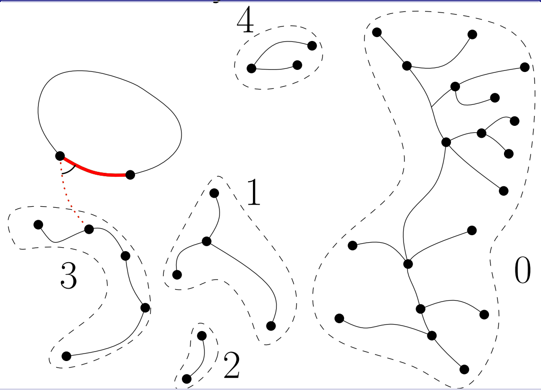

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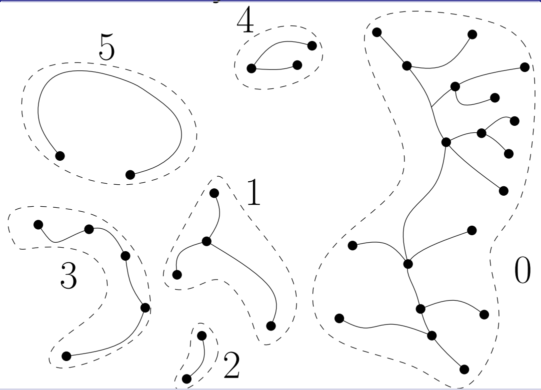

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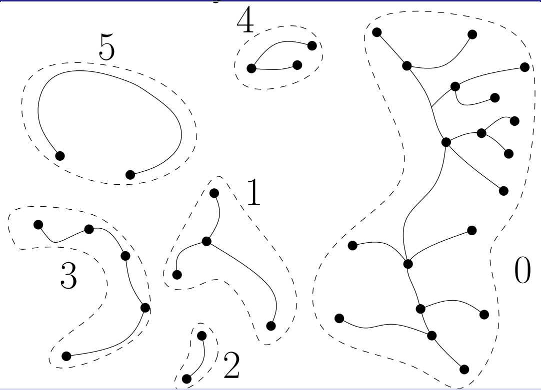

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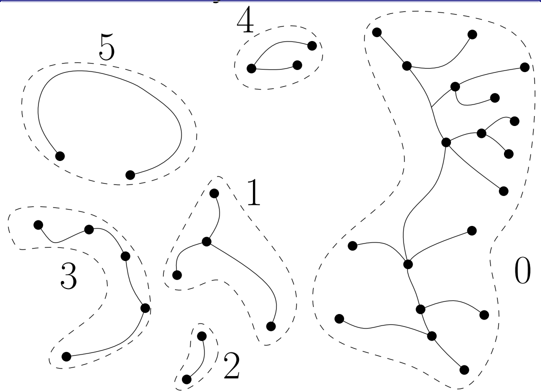

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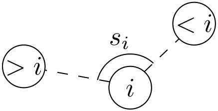

Switch s_i, $i = 1, \ldots, k-1$

- s_i connects V_i to V_j with $j > i$
- unused vertex of s_i is in V_ℓ with $\ell < i$

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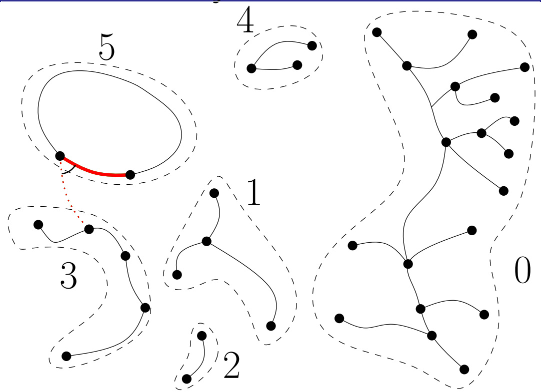

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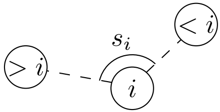

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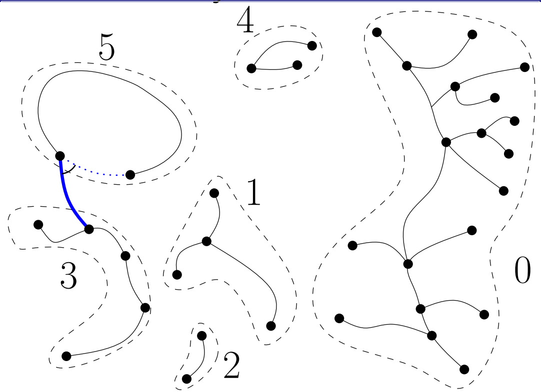

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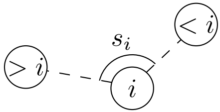

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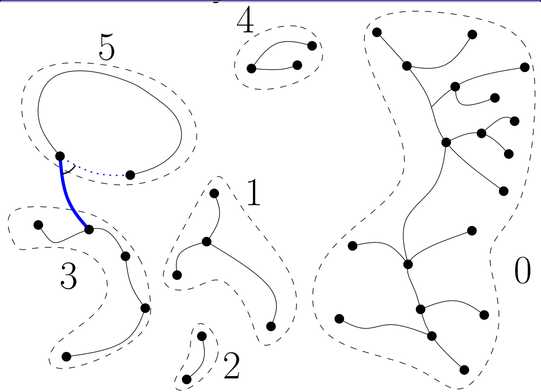

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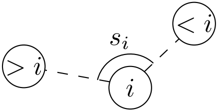

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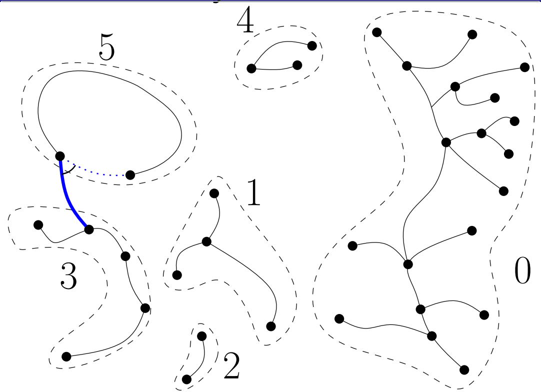

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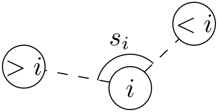

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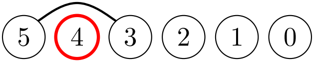

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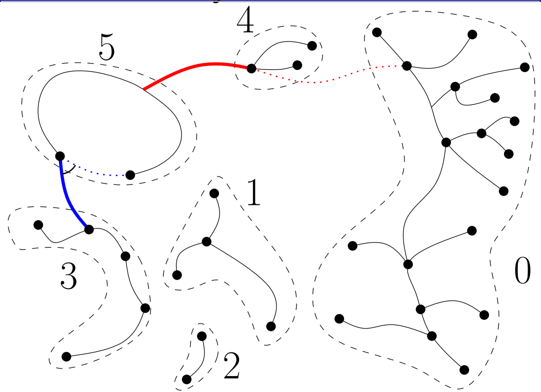

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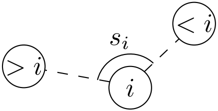

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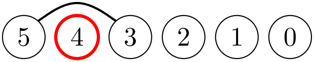

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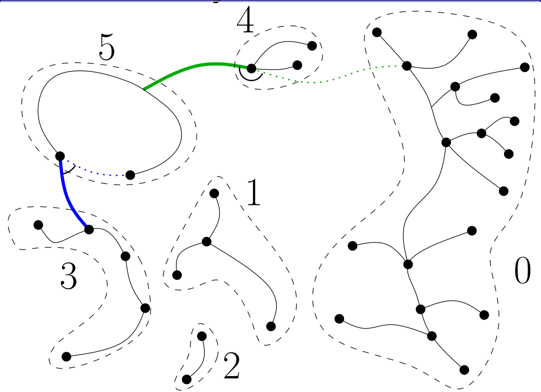

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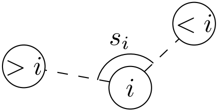

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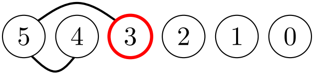

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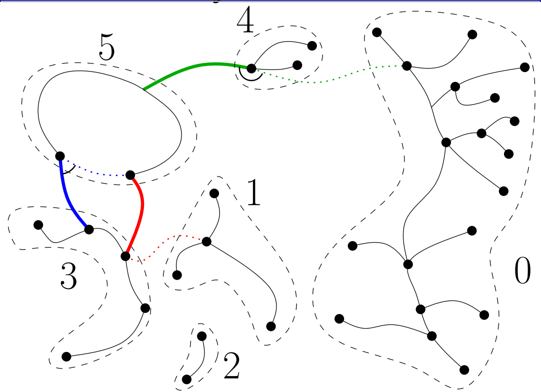

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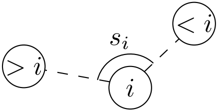

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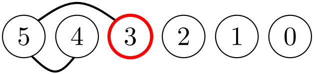

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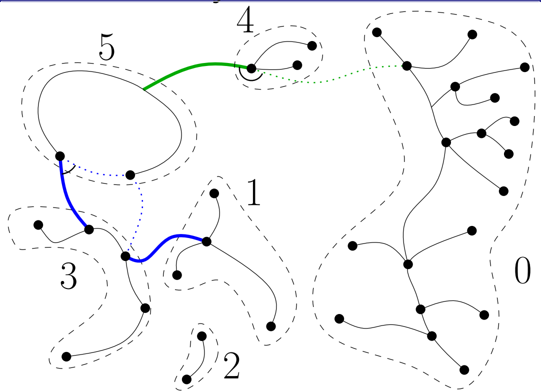

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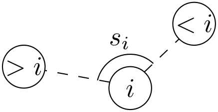

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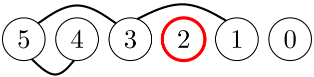

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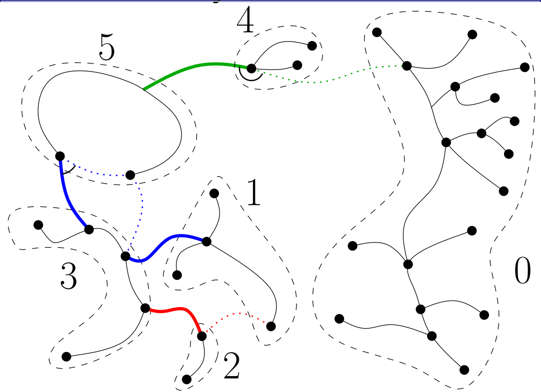

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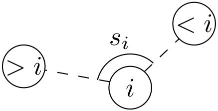

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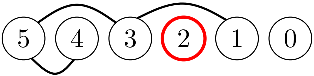

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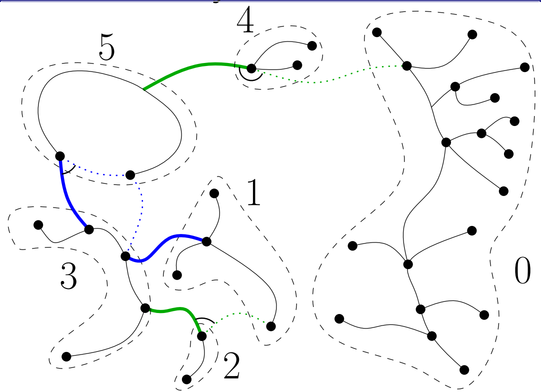

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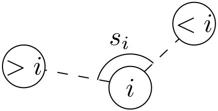

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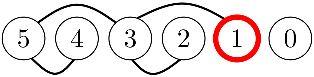

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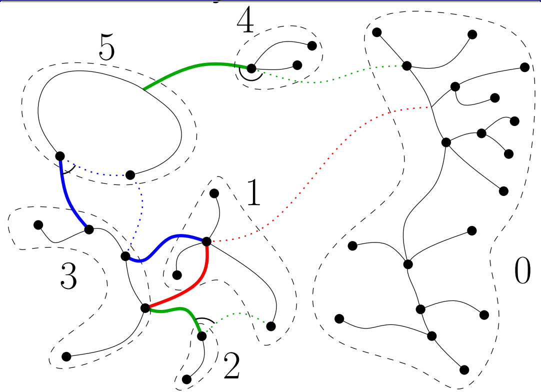

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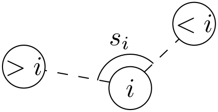

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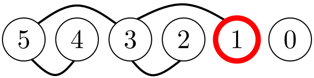

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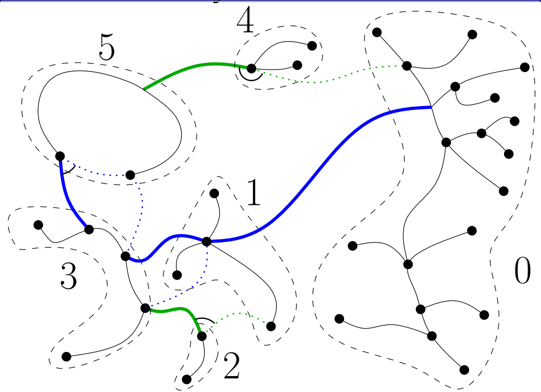

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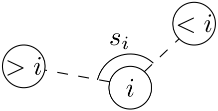

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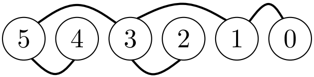

Testing independence

Lemma (Independence Test)

Given independent set of switches S' with acyclic configuration and an additional switch s we can compute in $O(kn)$ time either

- $S'' \subseteq S' \cup \{s\}$ *with* $|S''| >= |V(S'')|$ *or*
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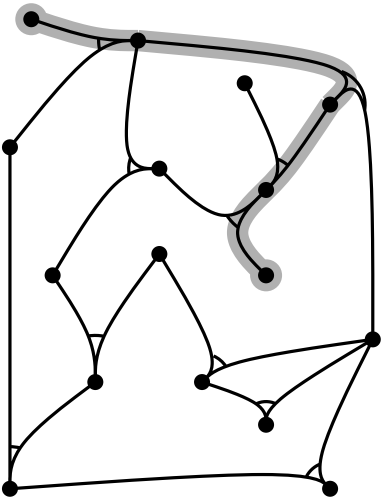
Running time can be improved to $O(km + kn^2)$.

Local Connectivity

Definition (a–b-Connectivity)

Given a switch graph $G = (V, S)$ and $a, b \in V$.
Is there a configuration c such that a, b are connected in $G_c(S)$?

a-b-Connectivity (examples)



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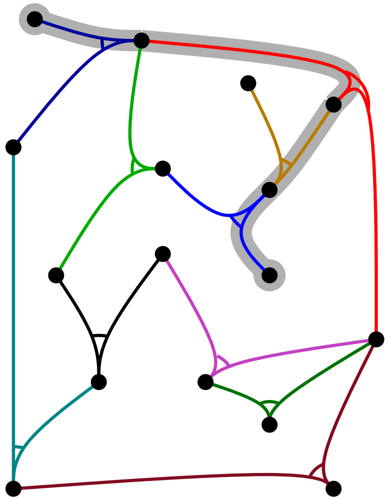

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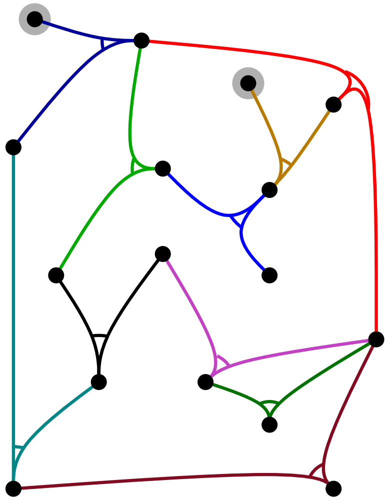

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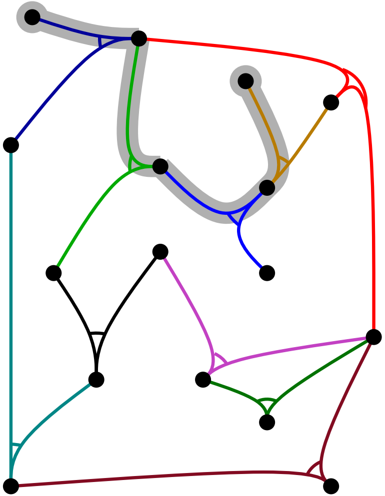

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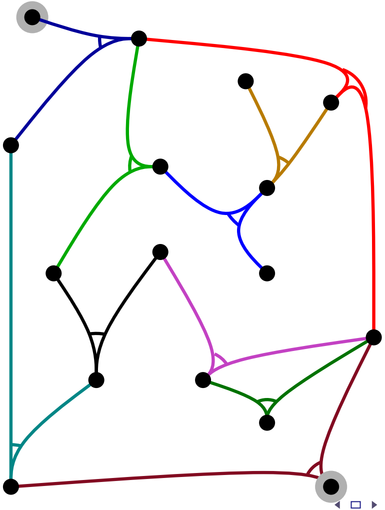

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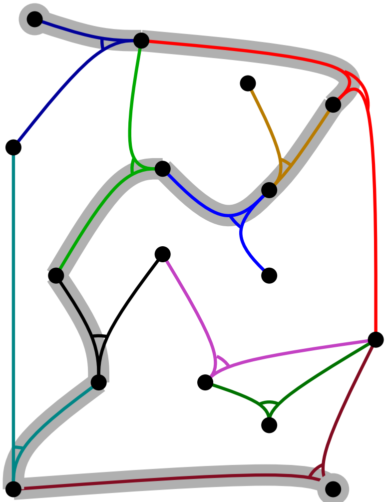

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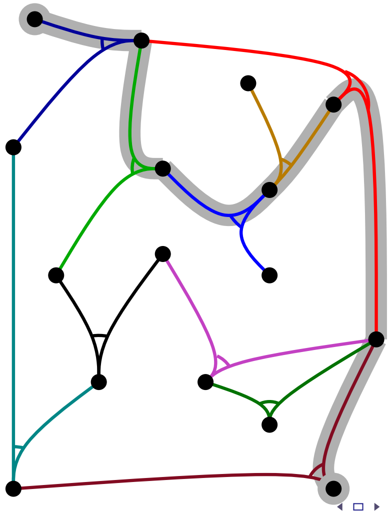

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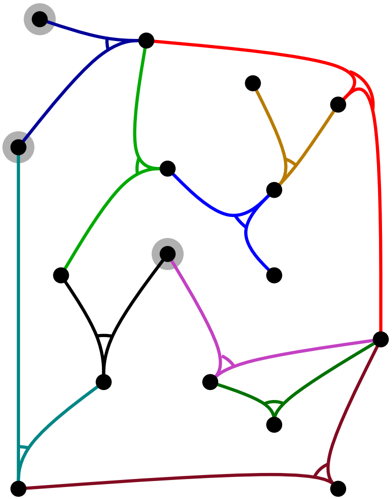
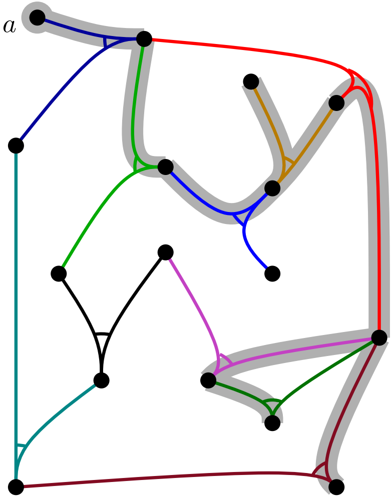


Searching along a tree

- Build search tree with root *a*
  - Edges used from target to pivot block the switch.
  - Edges from pivot to target may all be explored.



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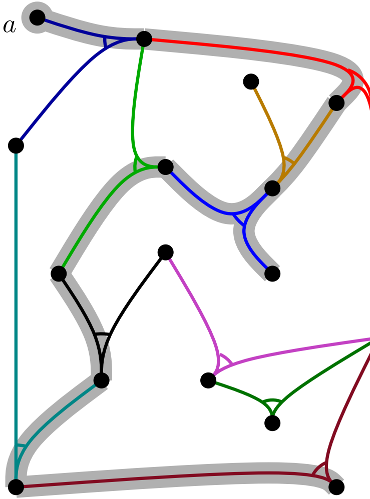

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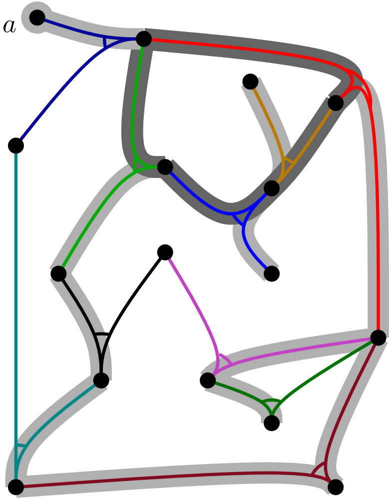
  - Direction of traversal is important for cycles of switches.
  - Idea: contract cycles.



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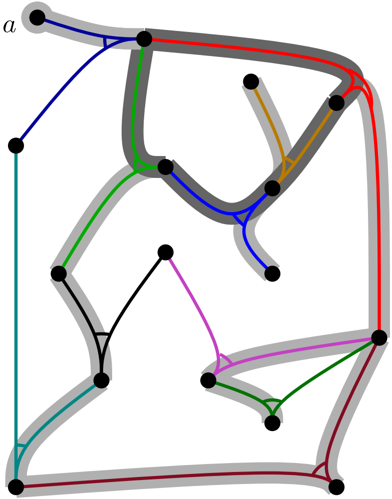
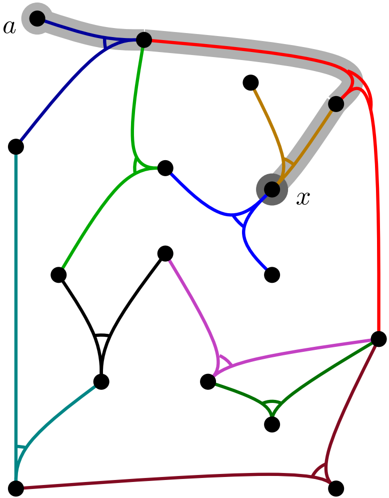
- Correctness? Running time?



A contraction lemma

Lemma

Let $x \in V$ with an x–a path P s.t. all switches of P are used “forward”. Contracting switches with pivot x that are not on P does not change the vertices reachable from a.



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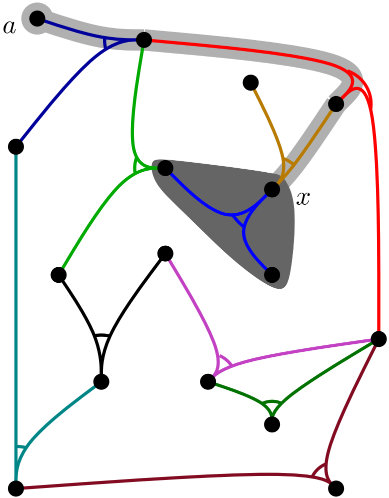

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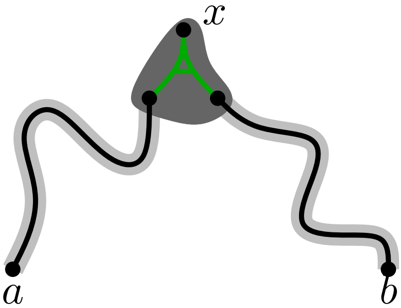

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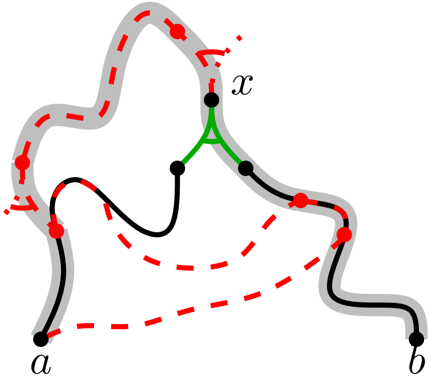

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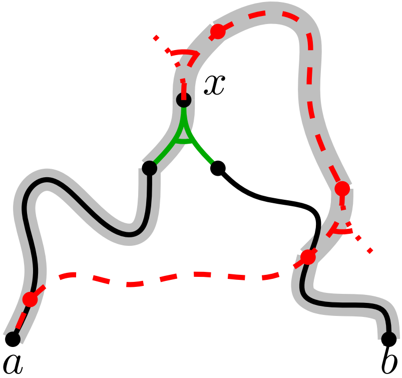

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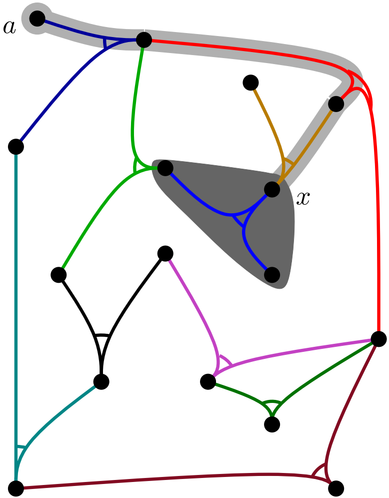

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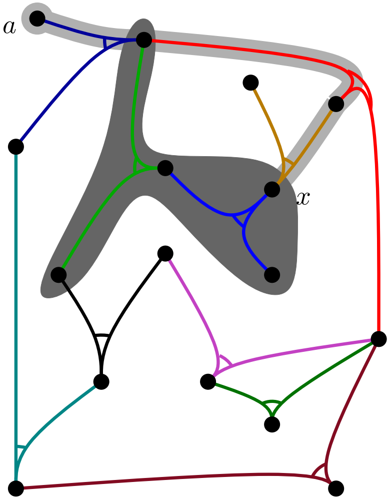

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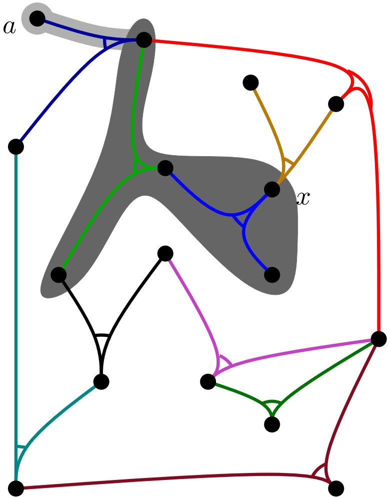

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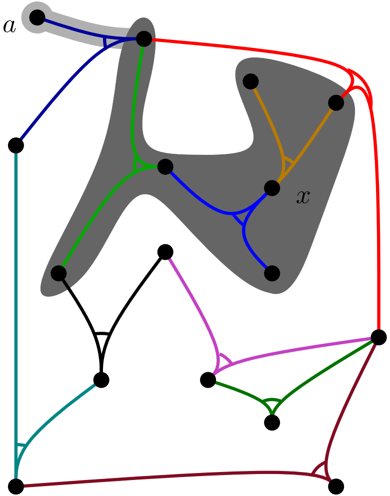

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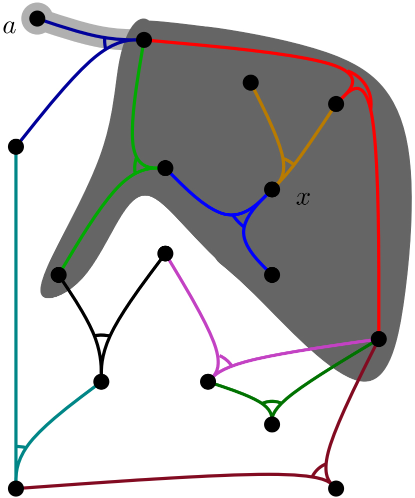

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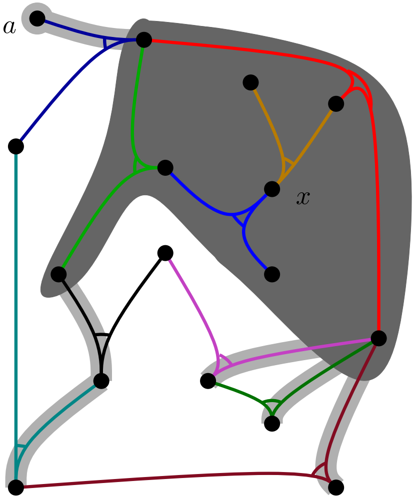
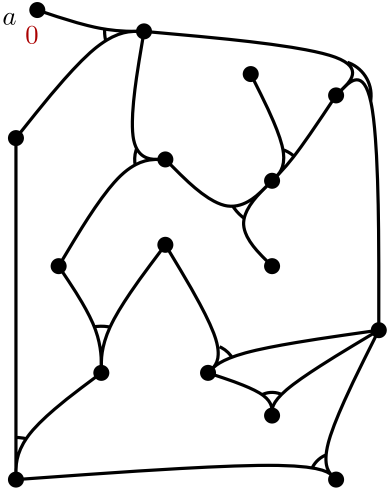
- preserves reachability from a
- contracts cycles of switches
- b reachable from a iff b has forward path from b to a
- $O(n^2 + nmk)$ algorithm!



Near linear time: search tree

- Pivot receives label:
  - Mark tree edge blue
  - Mark blocked edges red



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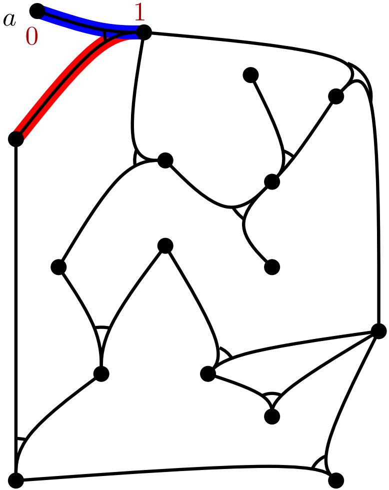

Near linear time: search tree

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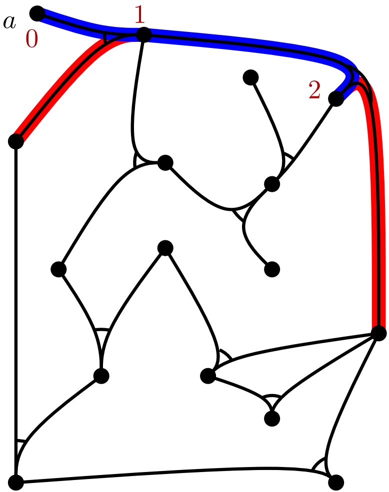

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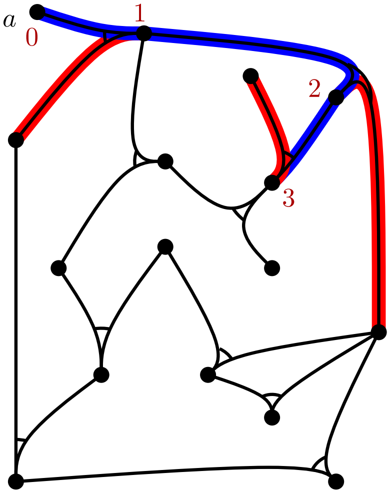
- Target receives label:
  - Contract edge



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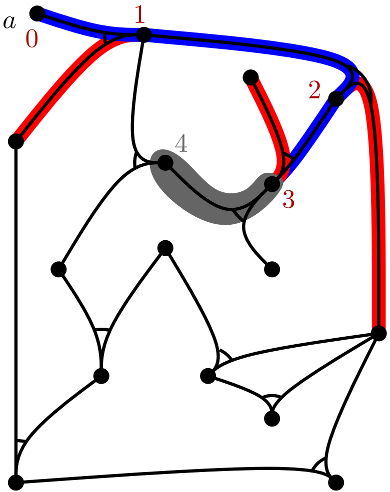

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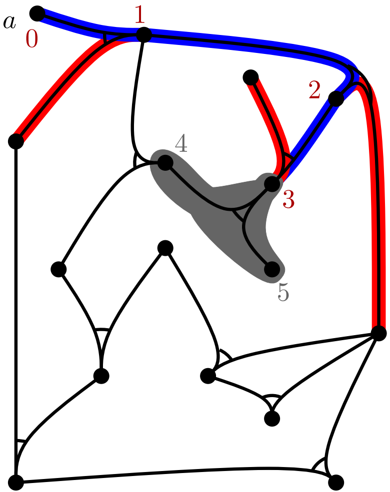

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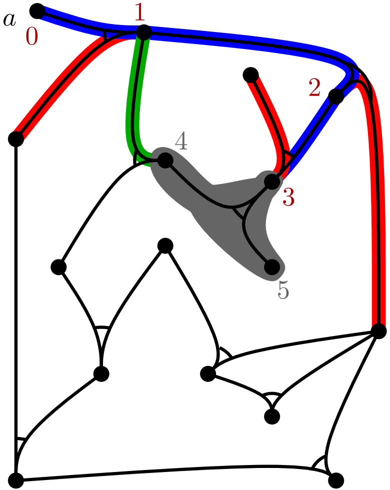

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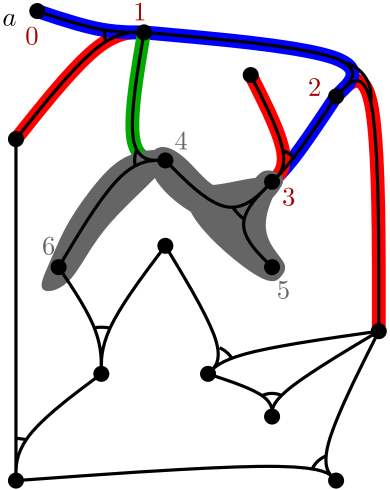

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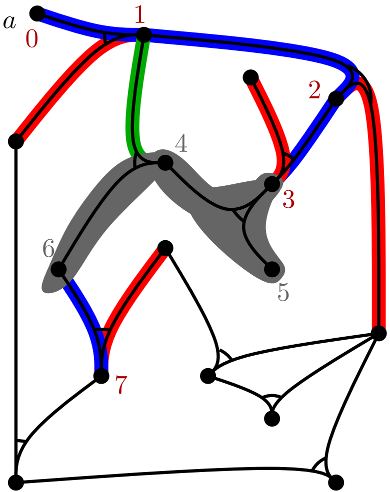

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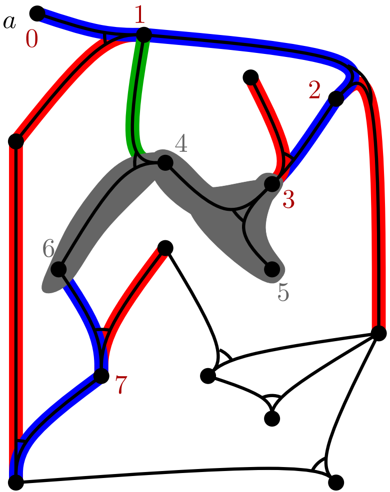

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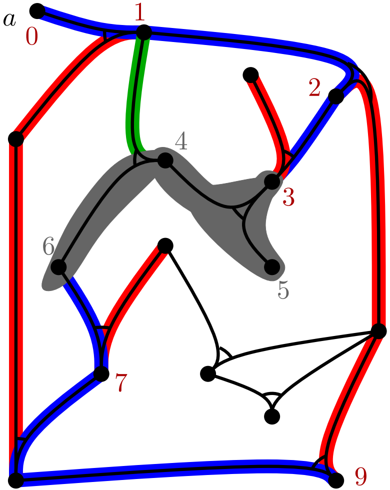

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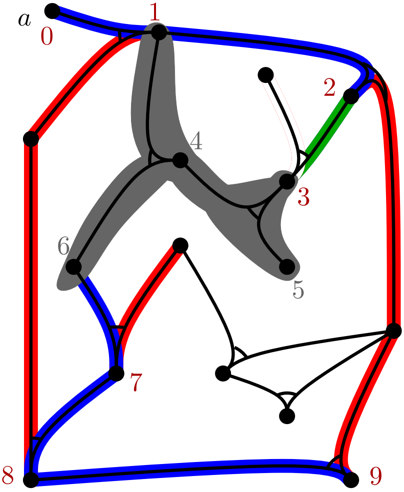

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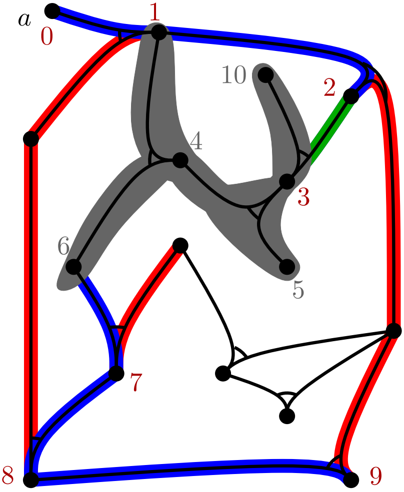

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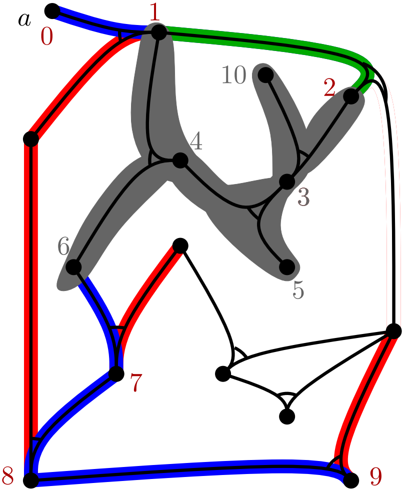

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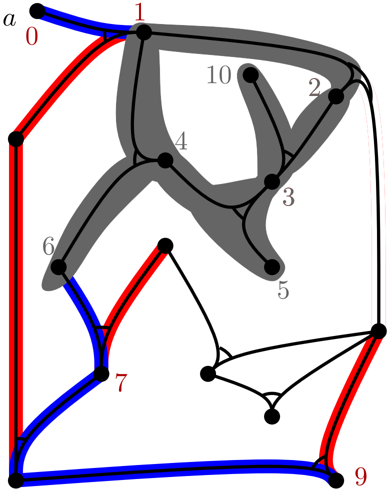

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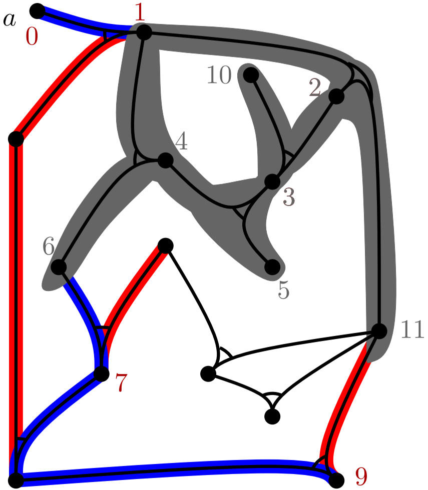

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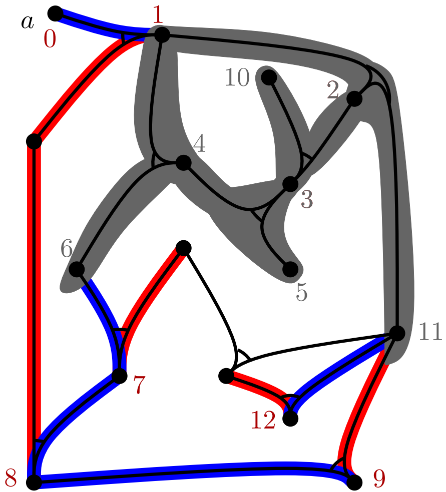

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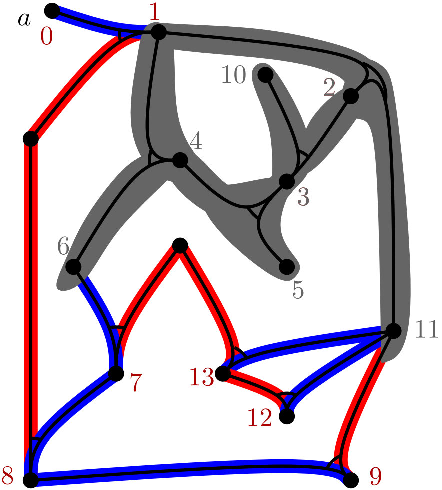

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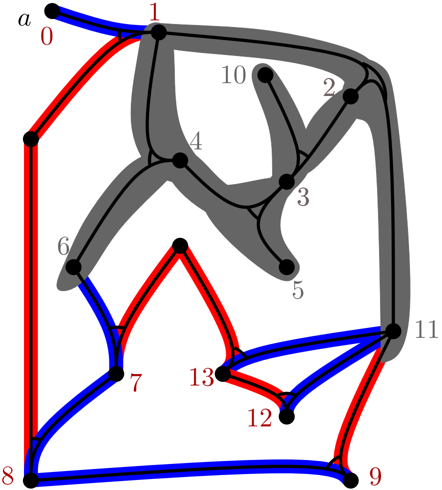
- Running time: $O(km + n\alpha(n))$



Conclusion and Open Problems

Known: Finding configurations with the following properties is NP-hard: Bipartite, triangle-free, Eulerian, biconnected, strongly connected, planar, minimum number of directed cycles

Problem (SwitchConnect-T)

Input: Given a switch graph $G = (V, S)$ and a set $T \subseteq V$.
Question: Is there a configuration c such that in G_c all vertices in T are in the same connected component.

Results:

- Efficient algorithms for $|T| = 2$ and $T = V$.
- NP-hard for arbitrary T.

Open: What about $|T| = 3$? FPT with parameter $|T|$?