The complexity of Domination Games

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Domination Games

Variant of graph searching introduced in FominKratschMüller'03, where it was shown that:

- Domination Games generalise cops and robber games (path-width and tree-width)
- Domination Games are not monotone
- close resemblence to Domination Target Number
- give lower bounds for the domination search number on various classes of graphs
  (such as k-dimensional cubes, asteroidal-triple free graphs, claw-free graphs and graphs with certain types of spanning trees and caterpillars)

Domination Games

Here we will show that Domination Games

- generalise Robber and Marshall Game (hypertree-width)
- have arbitrary monotonicity costs
- are PSPACE-complete
- are not FPT, in fact most variants of Domination Games are not even in XP

Graph Searching Games

Motivation

Two player games played on graph-like structures

- originally motivated by the problem of finding an explorer in a complicated system of dark caves
- provide intuitive definition for many interesting decomposition parameters on graphs (tree-width), hyper-graphs (hyper-treewidth) and directed graphs (directed tree-width)
- close resemblence to several important graph-invariants, e.g. vertex separation number (VLSI-design), black and white pebble games (sequential computation), domination target number

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Introduction

played by two players on the vertices of a graph, i.e the cop- and the robber-player

- the cop player can use an arbitrary amount of tokens to capture the robber, who is hiding in the vertices of the graph
- the robber tries to avoid capture by moving along paths in the graph that are not occupied by a cop
- variants of this game are defined by adjusting the abilities for both players
- the number of cops needed to catch the robber defines the variant we are interested in

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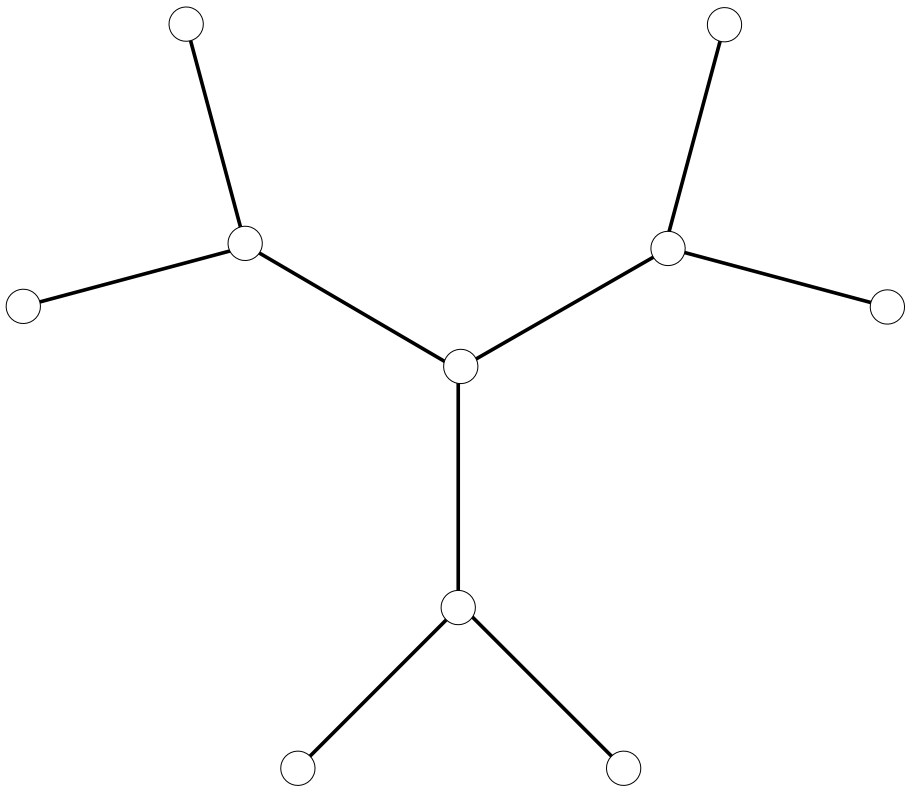
Example



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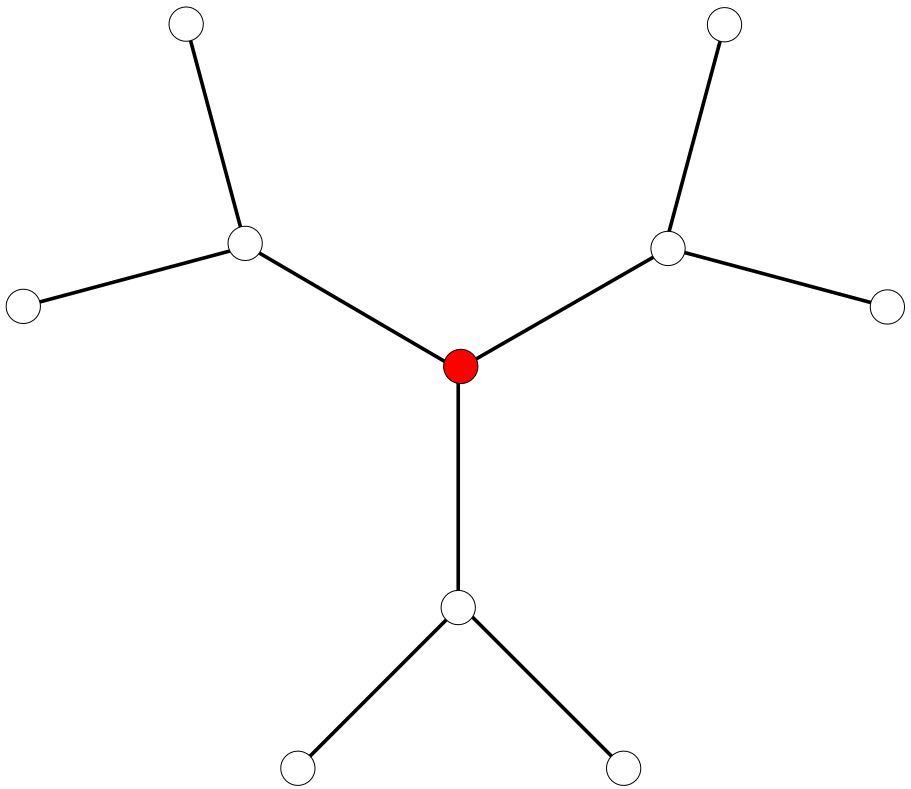

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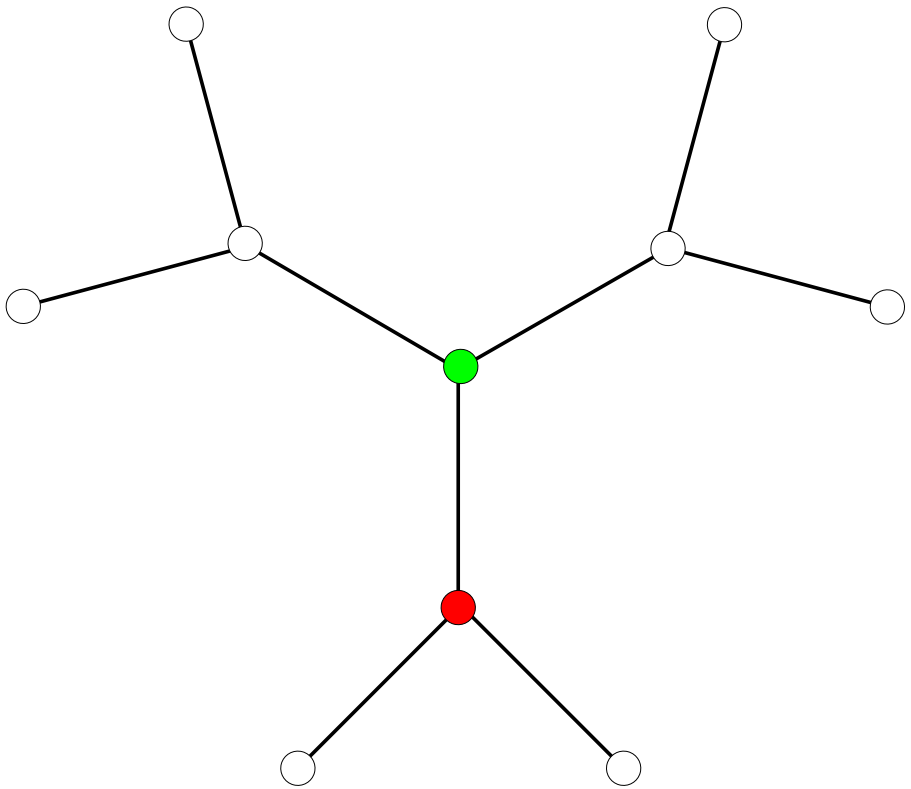

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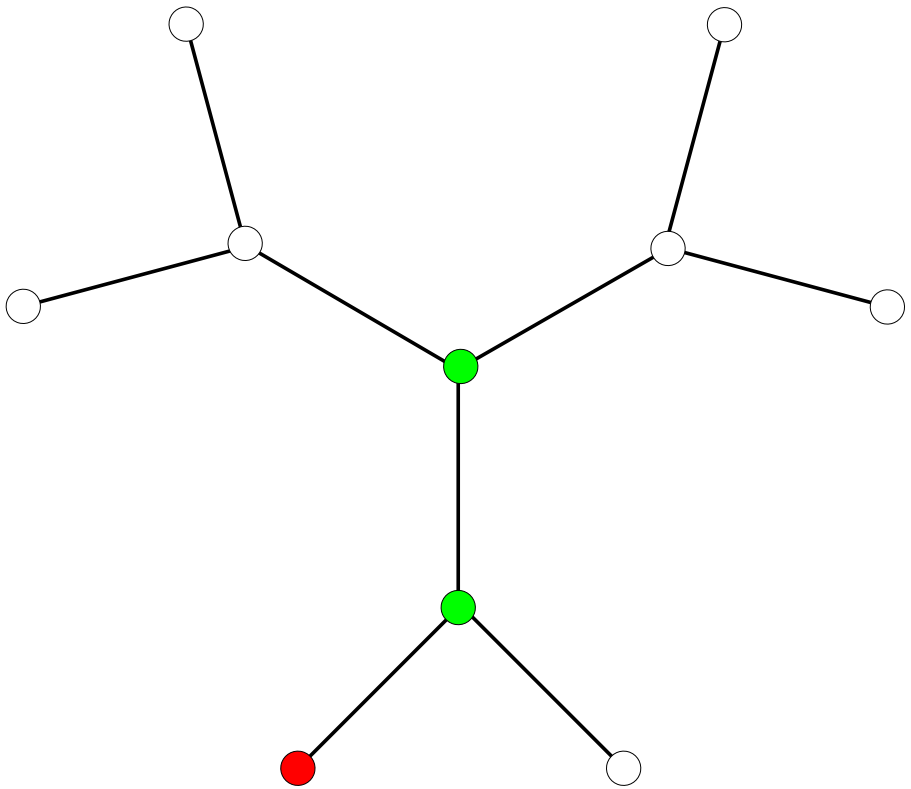

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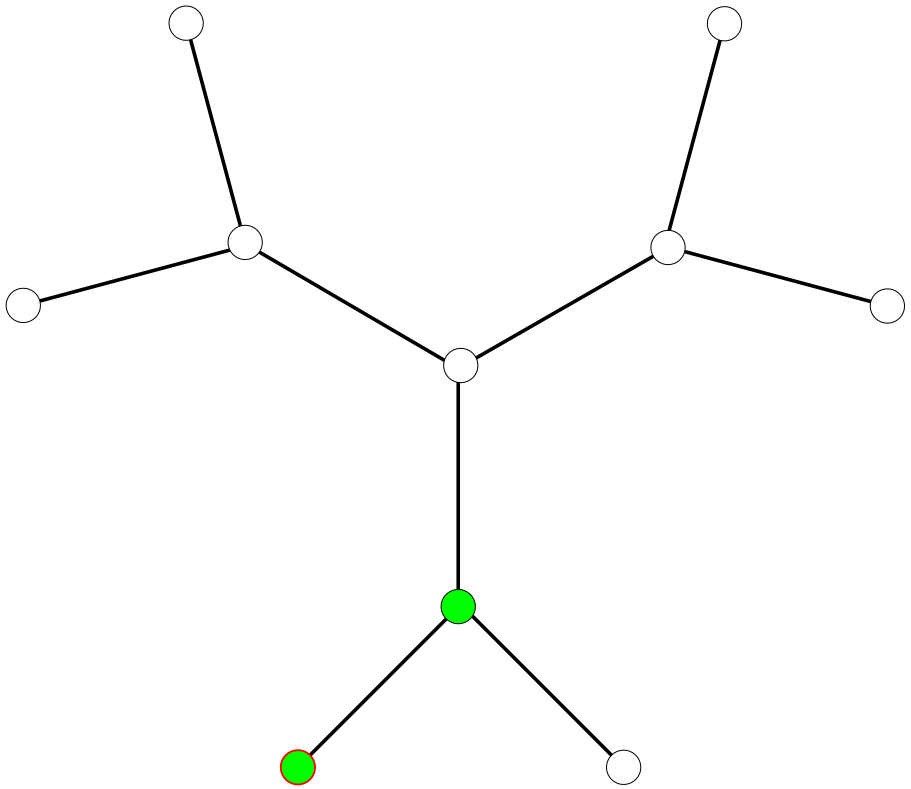

Two cops suffice.

Monotonicity

In general strategies (for the cops) can be arbitrary awkward and long. Monotonicity is a way of defining nice and short strategies.

Definition

A strategy for the cop is monotone, if it does not allow the robber to revisit vertices from which he has previously been expelled.

Definition

A game variant is monotone, if for every winning non-monotone strategy for the cop there exists a monotone winning strategy for the cop that uses the same number of cops.

Variants -revisited

Here we are interested in the following two variants:

invisible robber (iv)

Characterises path-width

visible robber (v)

Characterises tree-width

These variants are NP-complete, monotone and fixed parameter tractable.

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Domination Games are a variant of cops and robber games, where the cop player not only guards the vertices he occupies but also all their neighbours.

It is known that, DG: FKM03

- generalise cops and robber games, and thus are at least NP-hard
- are not monotone (invisible case, difference one)

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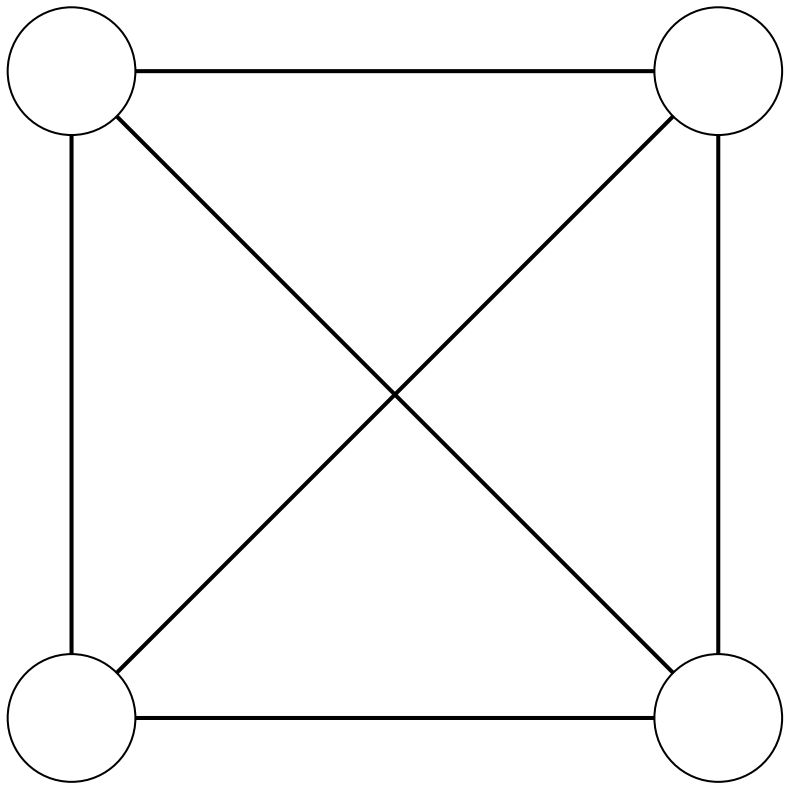

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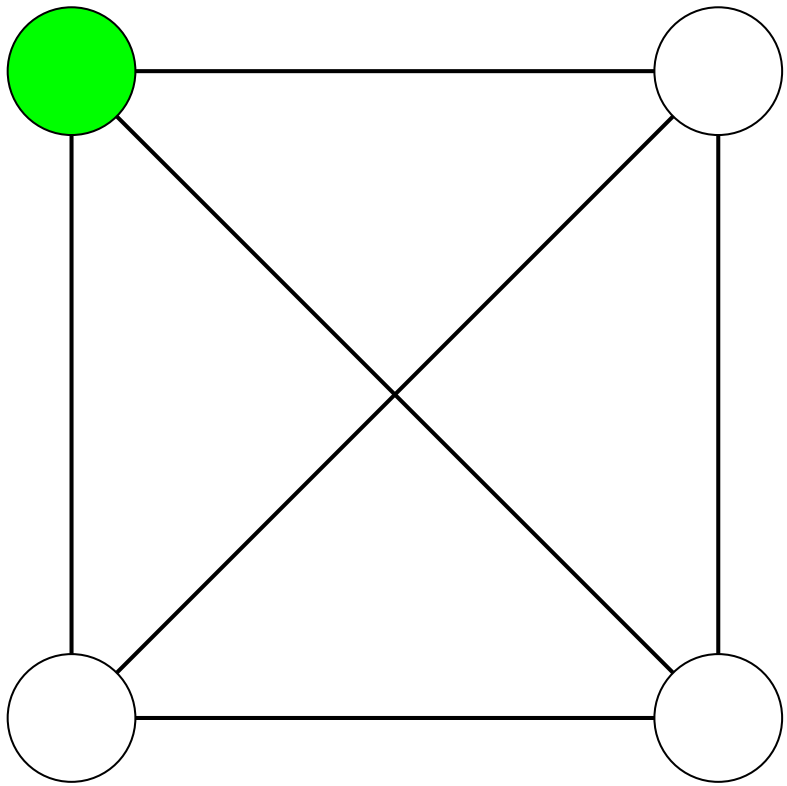

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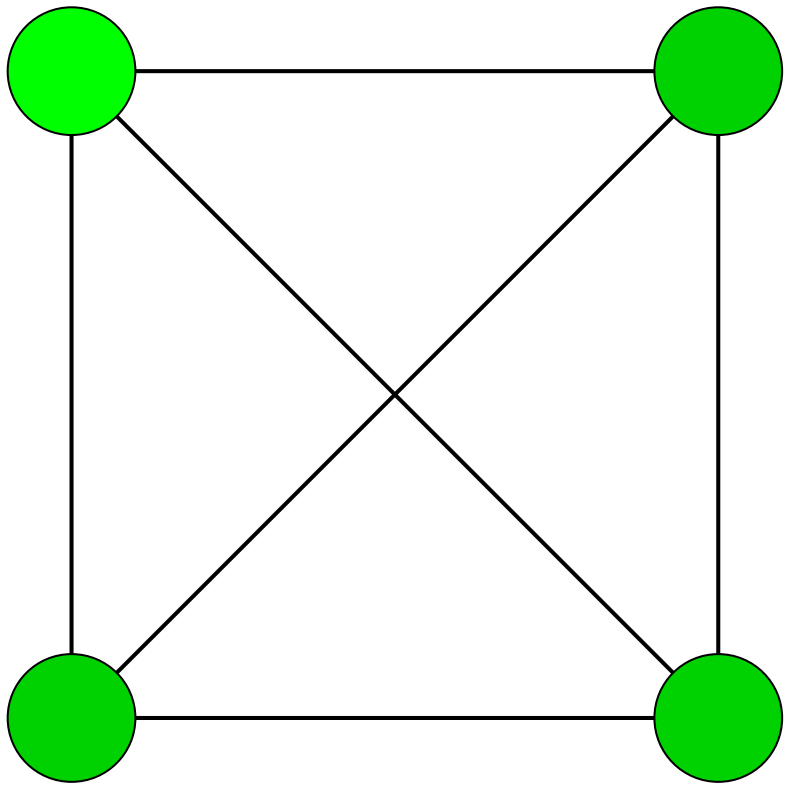

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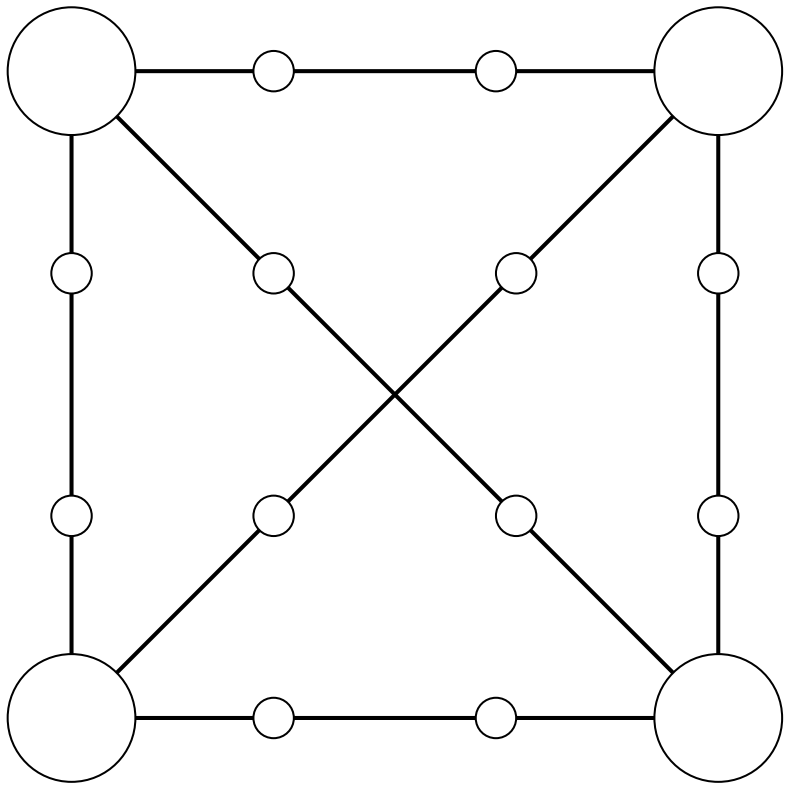


$CR(G)=DS(G')$.

Domination Games - complexity

Theorem: Kreutzer, O. 08

The following problem is $W[2]$-hard:

DOMINATION SEARCH	
Input:	Graph G, integer k.
Parameter:	k.
Problem:	Decide whether k cops have a (in-)visible (non-)monotone domination search strategy on G.

Domination Games - complexity

Theorem: Kreutzer, O. 08

The following problem is NP-complete:

COP-MONOTONE INVISIBLE DOMINATION SEARCH	
Input:	Graph G
Problem:	Decide whether 2 cops have an invisible cop-monotone domination search strategy

Domination Games - complexity

Theorem: Kreutzer, O. 08

The following problem is fixed parameter tractable:

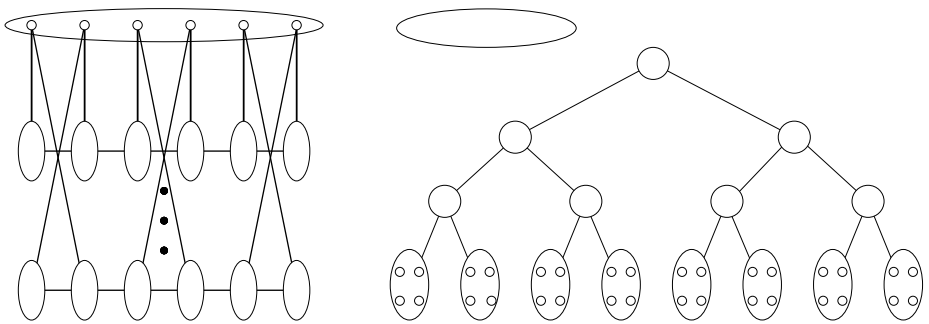
DOMINATION SEARCH	
Input:	Graph G of maximum degree d, integer k.
Parameter:	$k + d$.
Problem:	Decide whether k cops have a (in-)visible cop-monotone domination search strategy on G.

Domination Games - monotonicity

Theorem: Kreutzer, O. 08

For every $k \geq 1$ there exists a graph G_k, such that

- 2 cops have a non-monotone strategy on G_p.
- k cops are needed to search the graph with a monotone strategy.



Monotonicity - revisited

Even the length of non-monotone strategies is bounded by the size of the graph.

$\rightsquigarrow$ Non-monotonicity does not tell us much about the complexity of the game.

The reason for non-monotonicity is that the cops accidentally dominate parts of the graph that are not needed to search the graph.

$\rightsquigarrow$ Define a new variant Selective Domination Search of the game in which the cops can choose which vertices to dominate.

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Monotonicity - revisited

In the selective domination search all previous examples are monotone, but:

Theorem: Kreutzer, O. 09

There exists a graph, such that:

- 2 cops have a non-monotone strategy.
- 3 cops are needed to search the graph with a selective monotone strategy.

Open Problem:

- How large are the cost of monotonicity in the selective game?
- Is there a bound on the length of selective non-monotone strategies?

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Robber and Marshall Games

Robber and Marshall Games

Robber and Marshall Games are a variant of Graph Searching that define hypertree-width.

- Are played on hypergraphs instead of graphs.
- The cops (Marshalls) are occupying edges instead of vertices.
- The robber can move along hyperedges that are not currently occupied by a Marshall.

Robber and Marshall Games

Theorem: Kreutzer, O. 09

For every hypergraph H there exists a graph $D(H)$, such that $RM(H) = DS(D(H))$.

Robber and Marshall Games

As selective monotone domination search generalises generalised hypertree-width, it follows that:

Theorem: Kreutzer, O. 09

The following problem is NP-complete:

SELECTIVE MONOTONE (IN-)VISIBLE DOMINATION SEARCH	
Input:	Graph G
Problem:	Decide whether 3 cops have an (in-)visible selective monotone domination search strategy on G

Conclusion and Open Problems

Domination Search Games generalise cops and robber and Robber and Marshall Games, but behave very differently, i.e. DG:

- have no bound on the monotonicity cost.
- DG are not fixed parameter tractable.
- most variants are not even in XP.
- Are PSPACE-complete

Open Problems:

- What is the complexity of the visible variant of this game?

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