# Fully Dynamic Representations of Interval Graphs

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# Plan

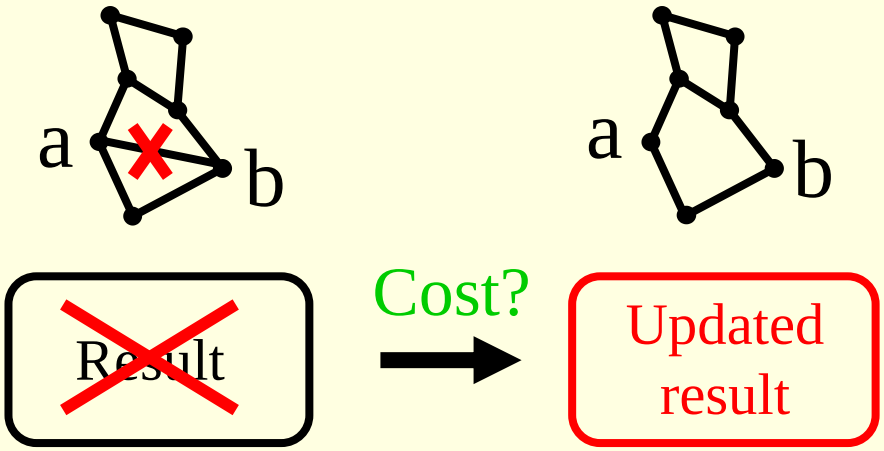


1. Problem and difficulty
2. Representations of Interval Graphs
3. Vertex insertion

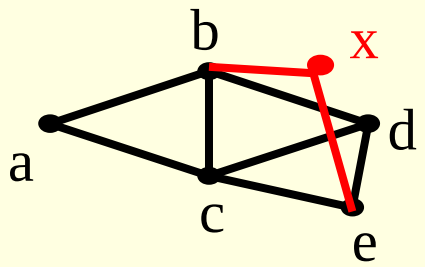
# Dynamic Recognition Algorithms

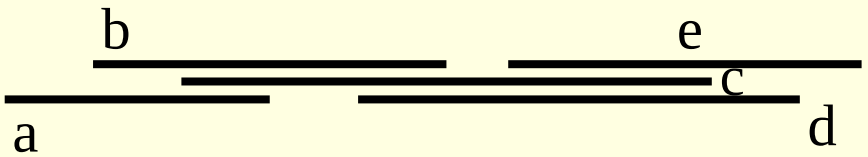


Dynamic Algorithms




Interval graphs



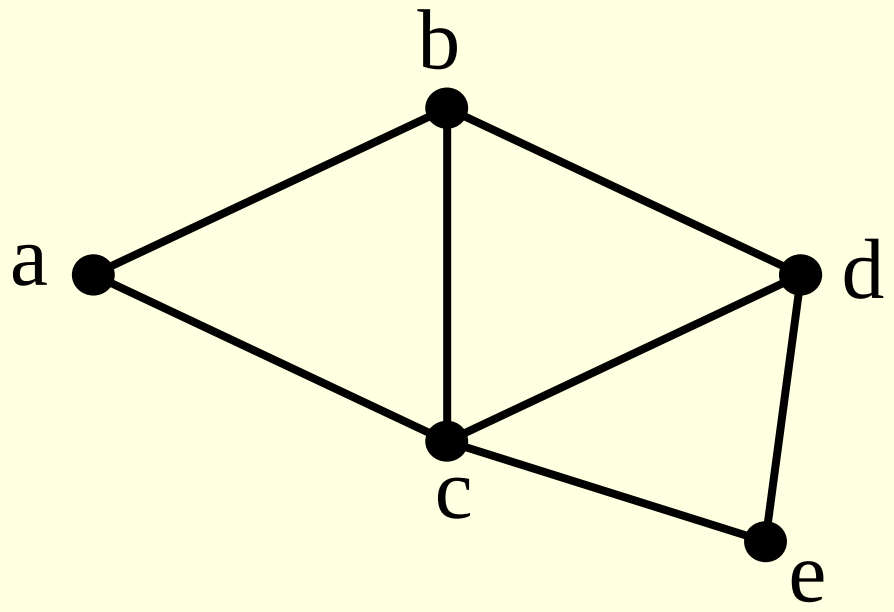



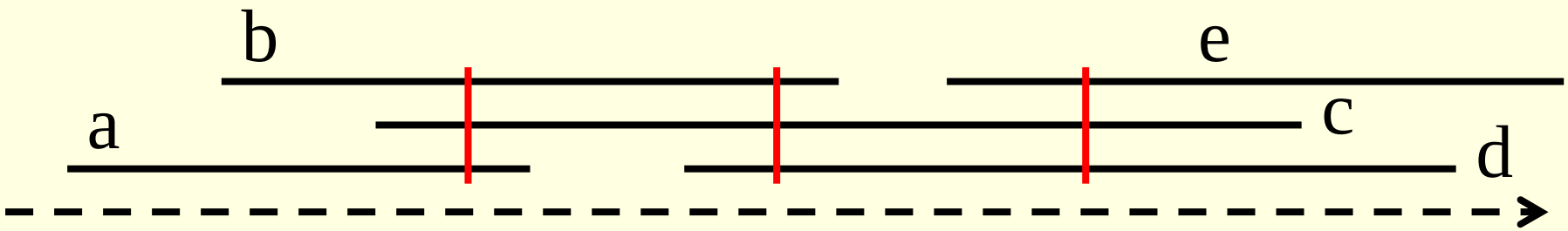

Interval graph?

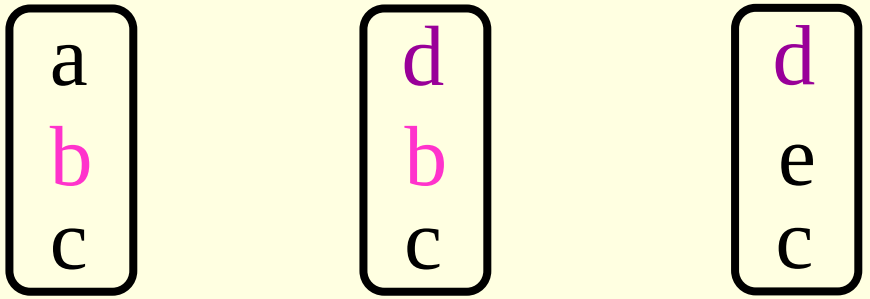
Updated representation?

# Interval graphs



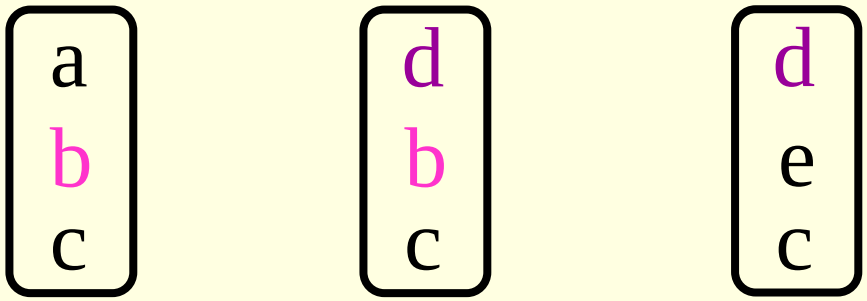


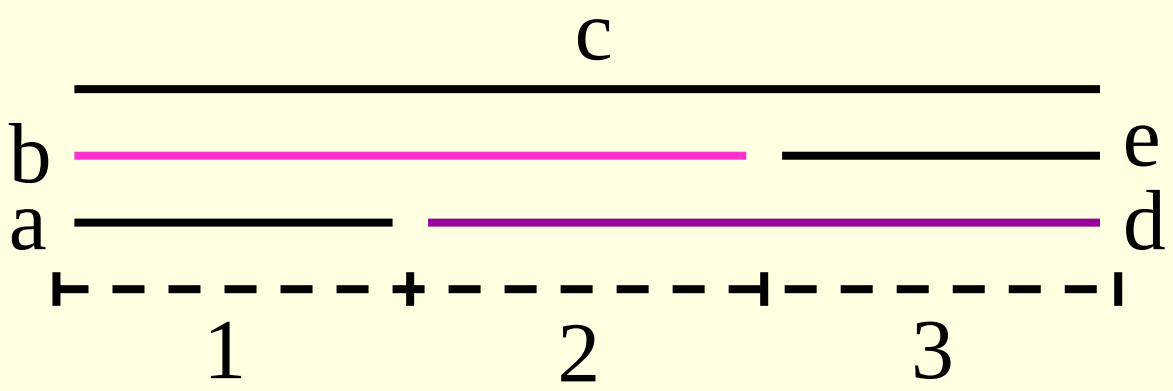



# Minimal models






a (1,1)

b (1,2)

d (2,2)

a (1,1)

d (2,2)

O(n) space

Adjacency in O(1) time

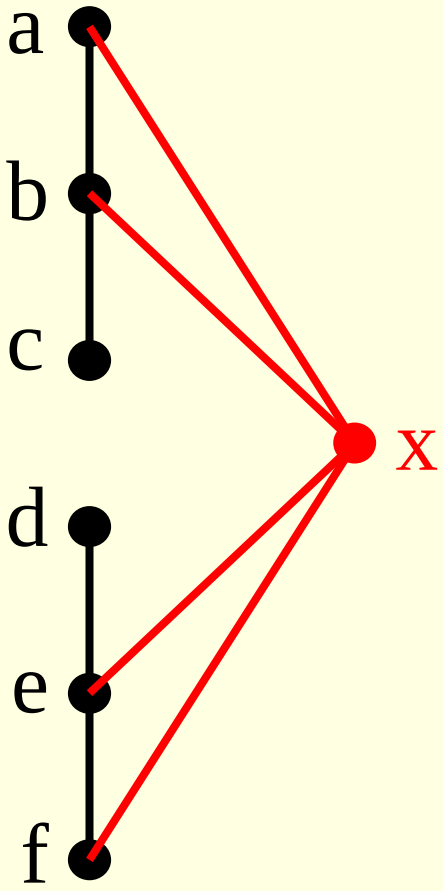
# Interval graph recognition

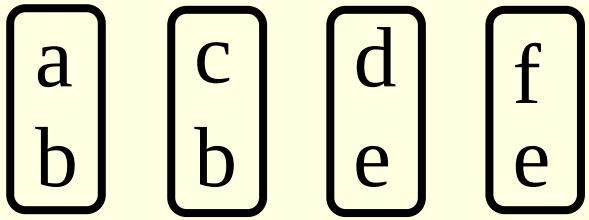


- Booth and Lueker 1976 : static recognition and computation of a model in O(n+m) time.
- Korte and Möhring 1989 : Incremental Algorithm, with static precomputation, in O(d) time.
- Hsu 1996 : vertex-incremental recognition algorithm, in O(d log n) amortized time.
- Ibarra 2001 : Edge-fully-dynamic recognition algorithm, in O(n log n) time.
- Fully dynamic algorithm, both on vertices and edges, in O(n) time.

# Different models





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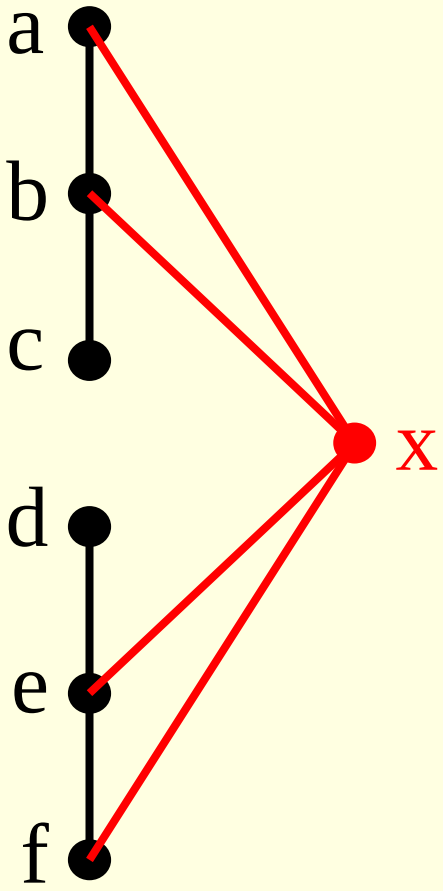


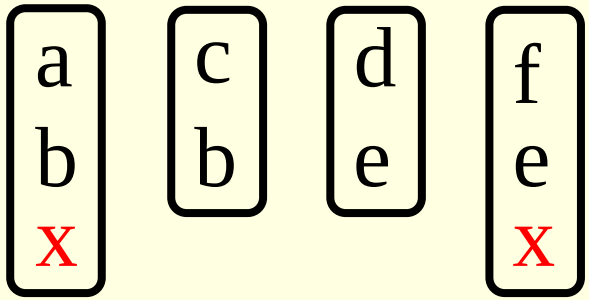


IMPOSSIBLE

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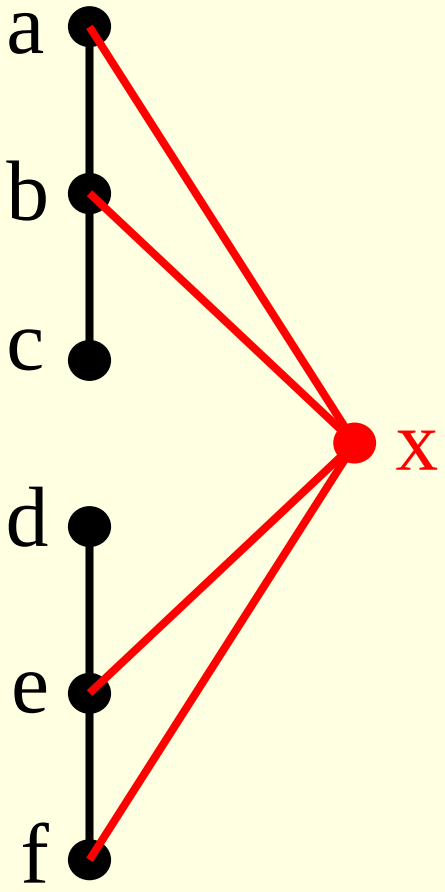


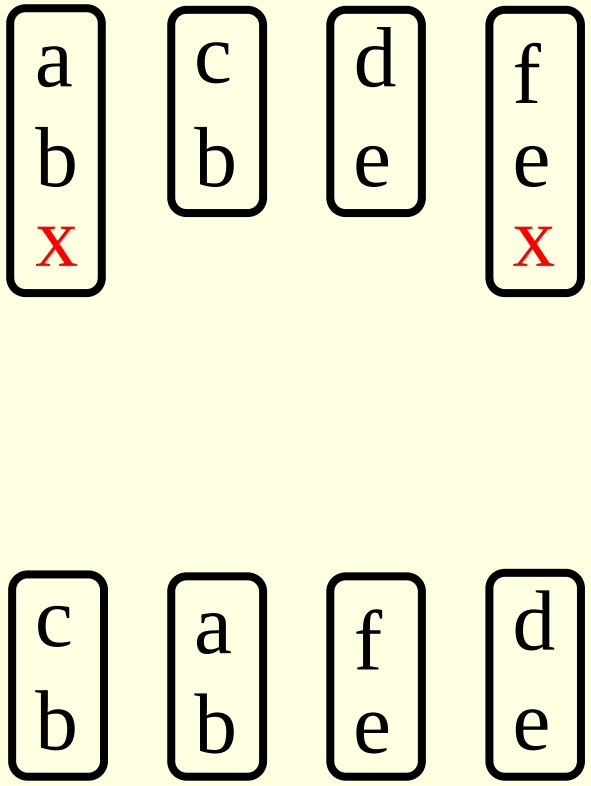


IMPOSSIBLE

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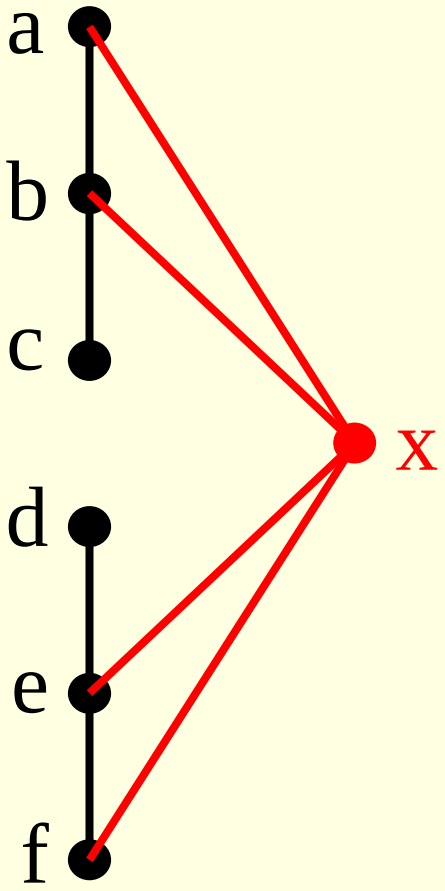
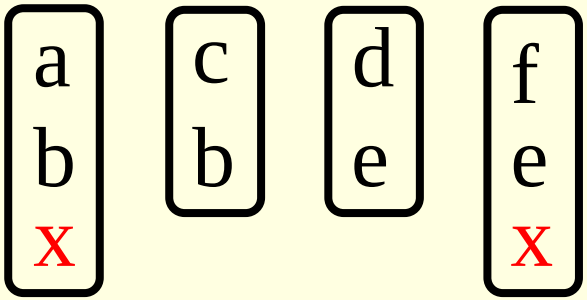
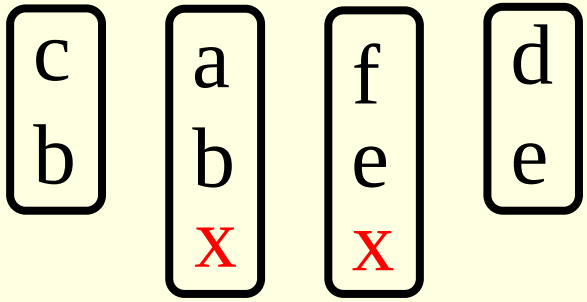


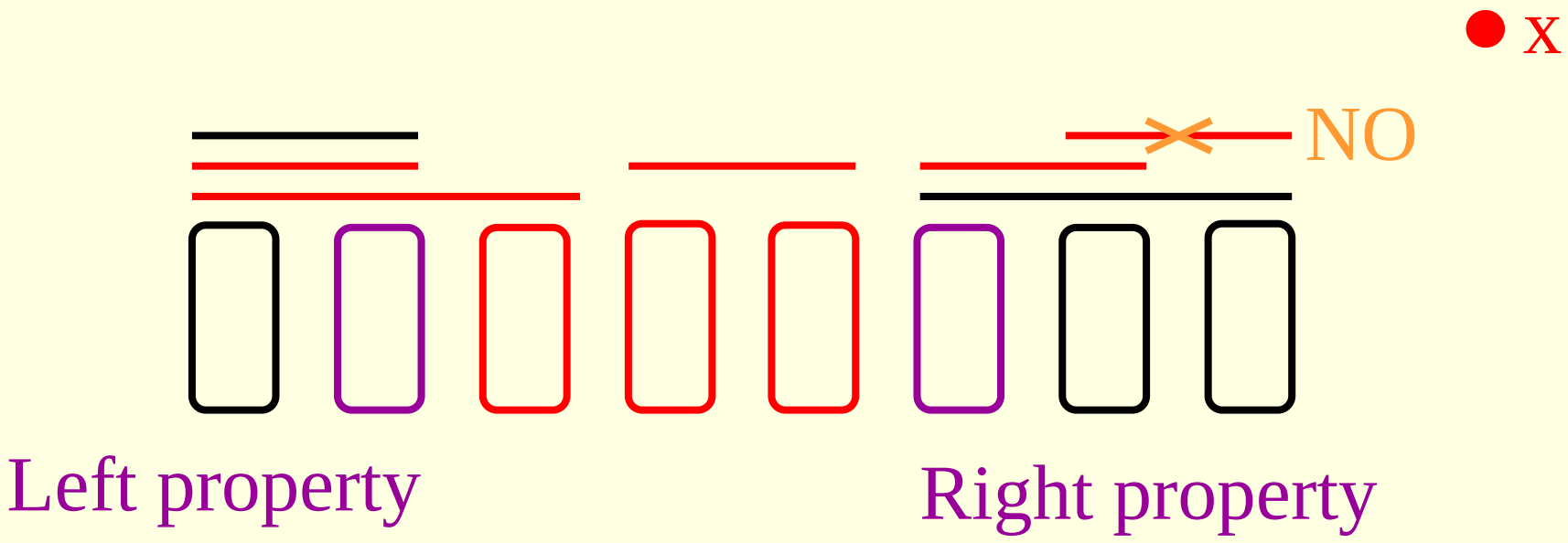

IMPOSSIBLE

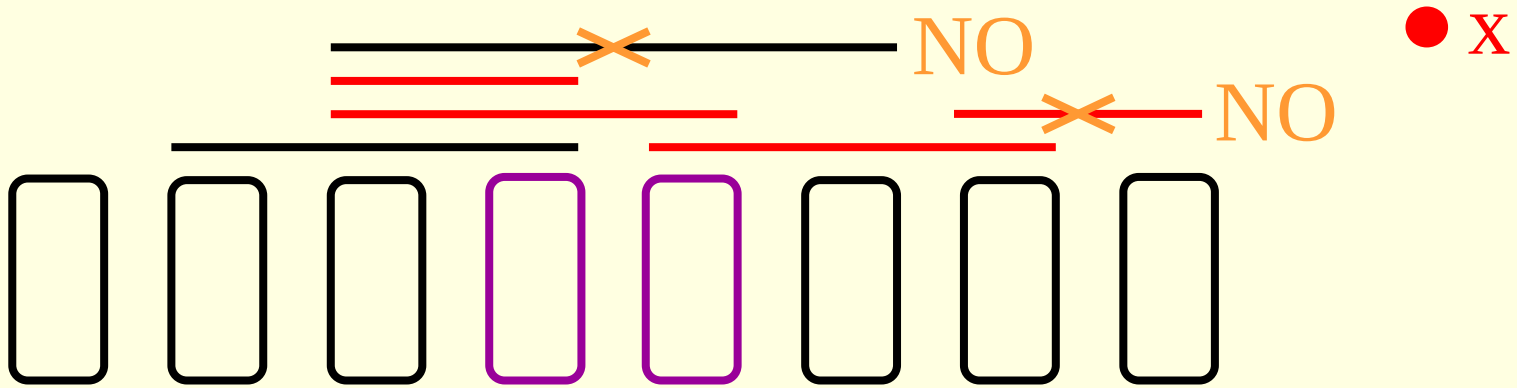



INSERTION OK

# Inserting x in a given model





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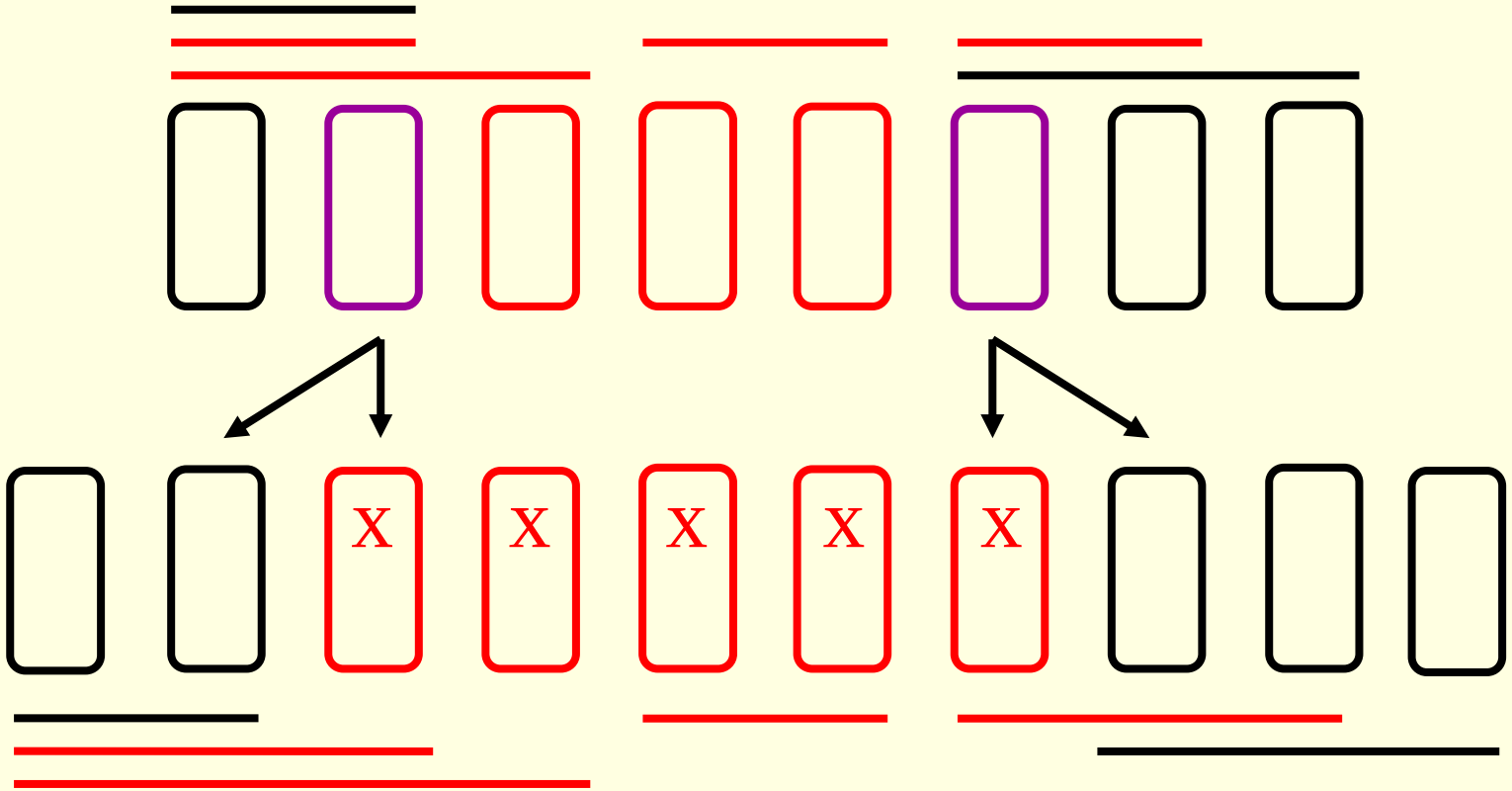

# Plan

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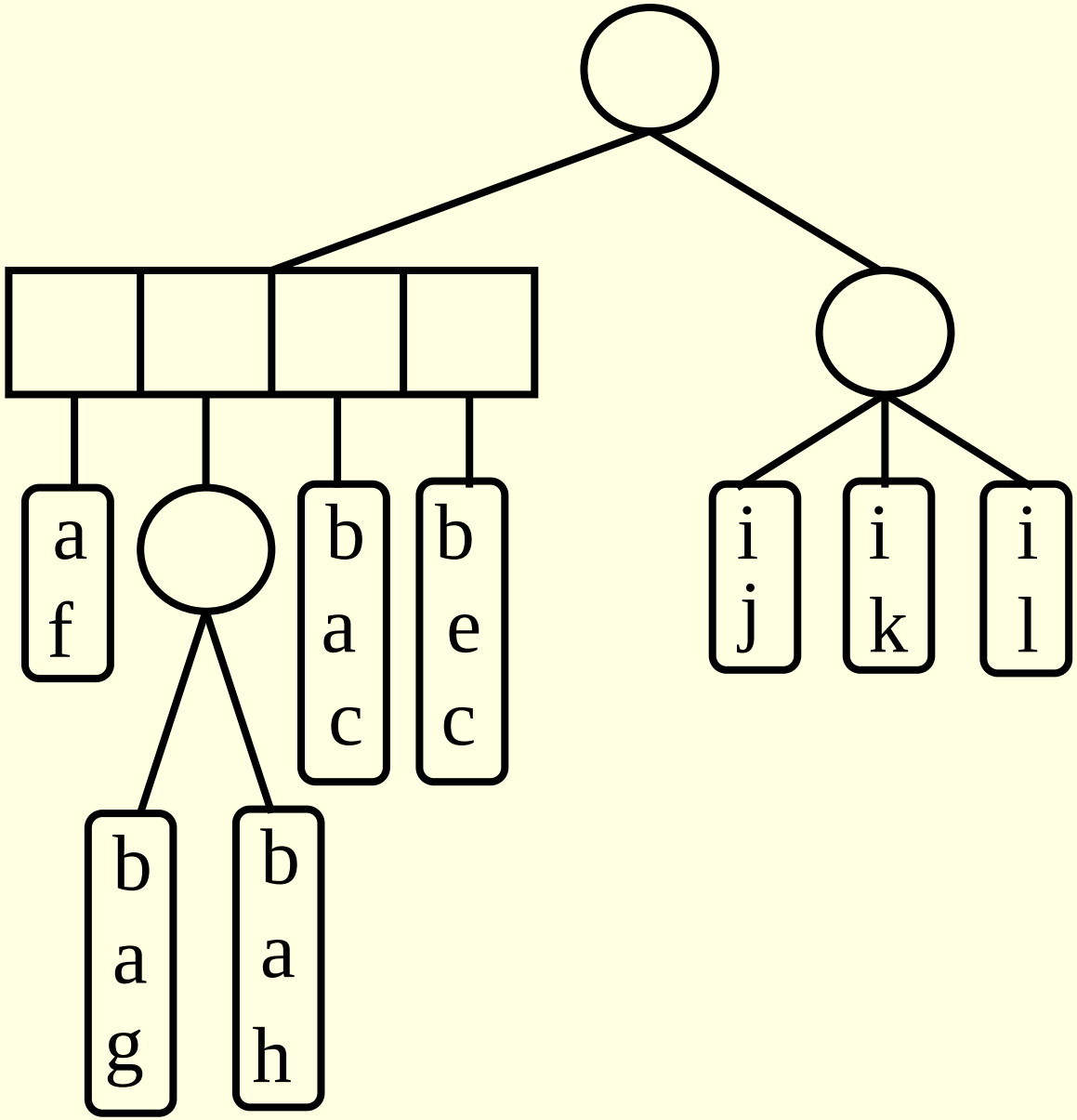
# PQ-tree



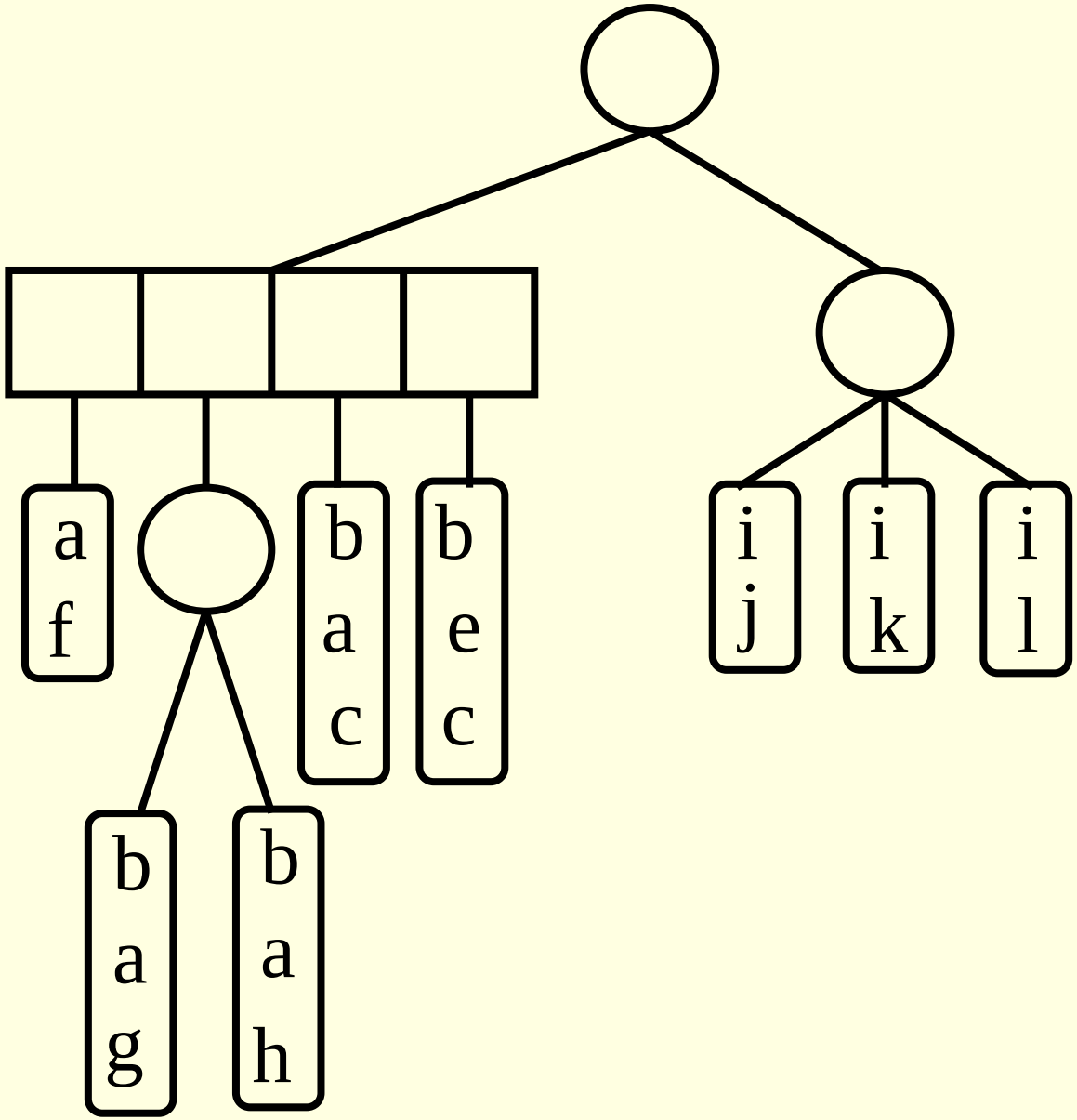
## Booth and Lueker 1976



# PQ-representation



Korte and Möhring 1985



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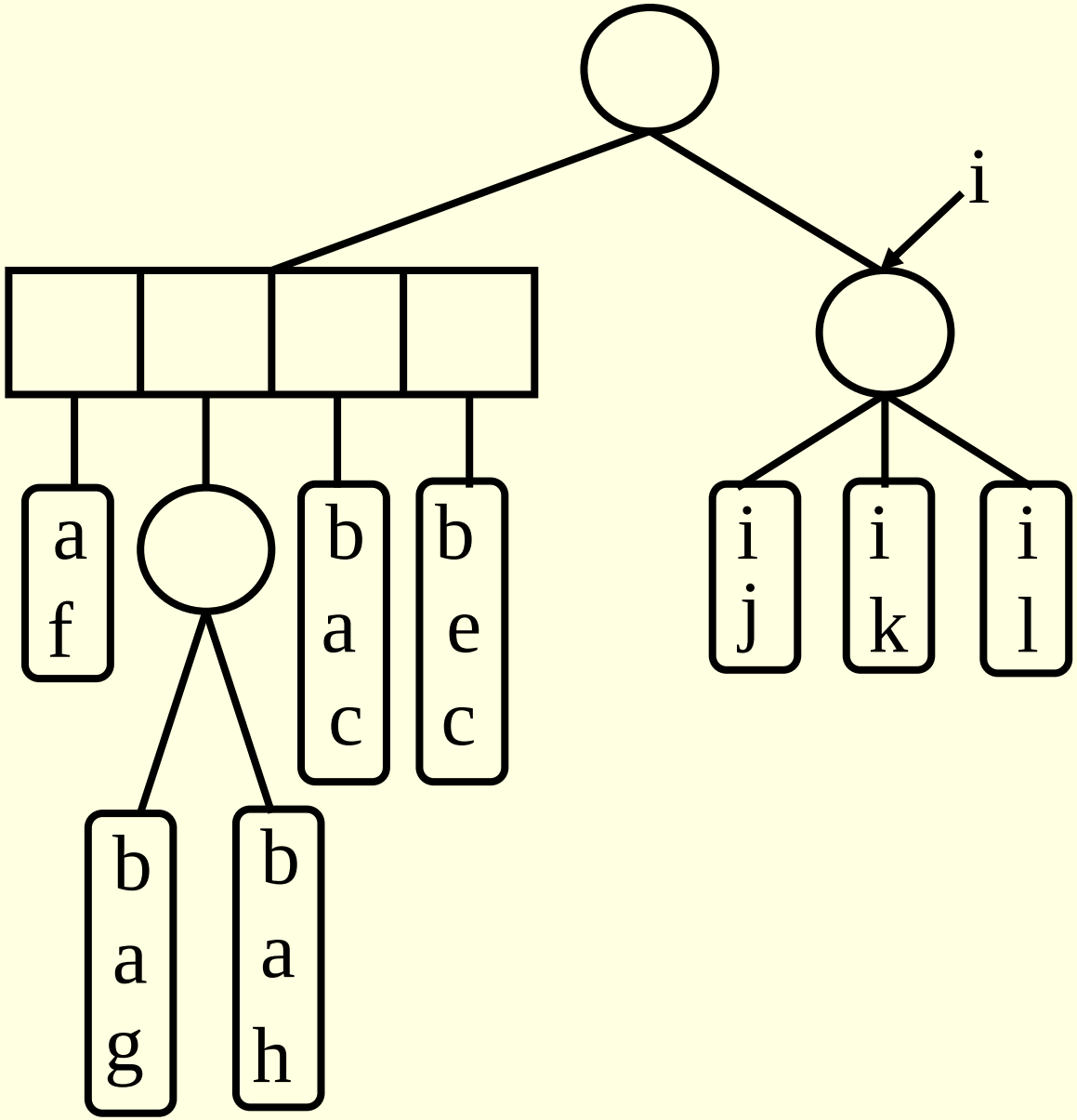

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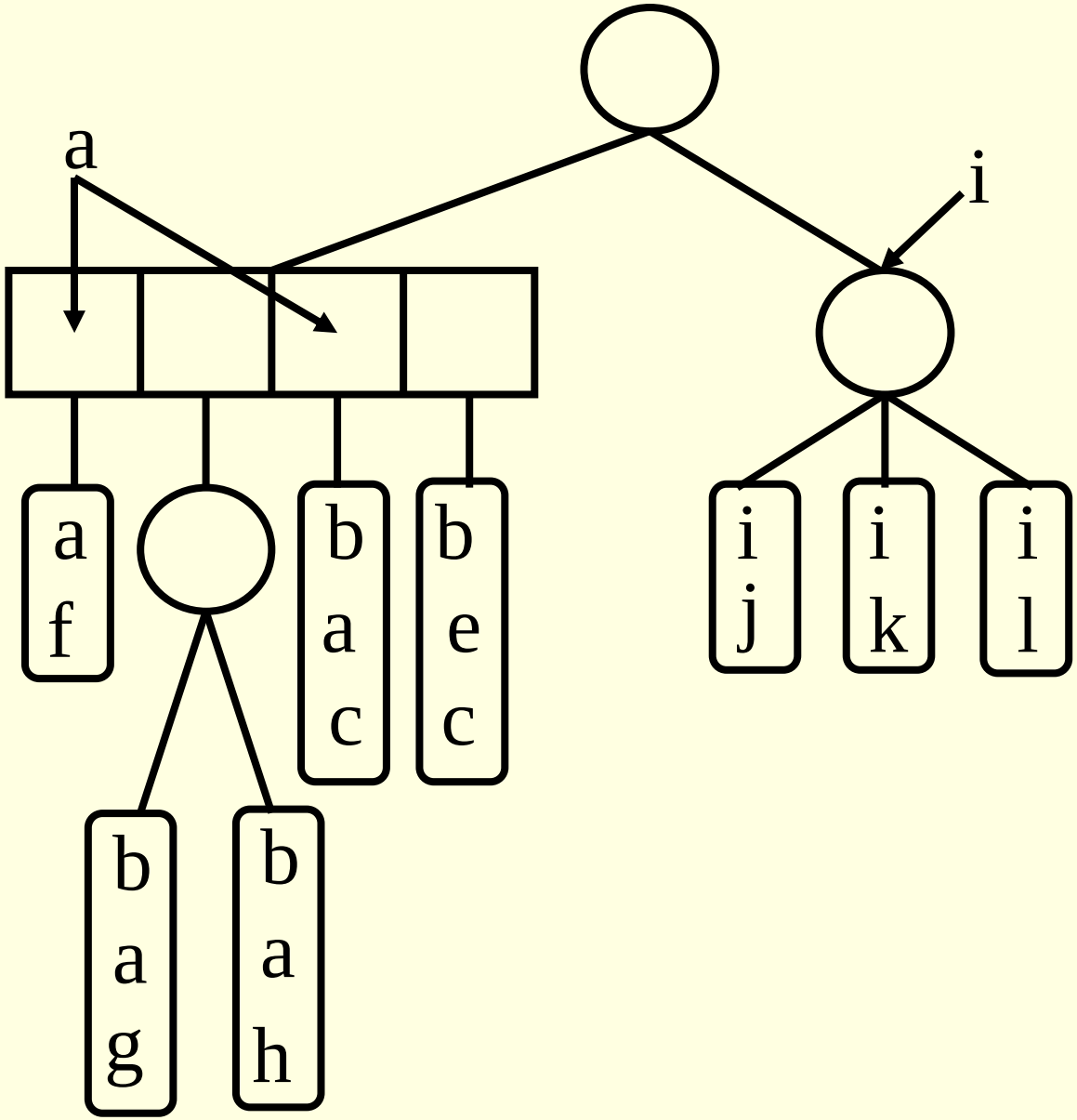

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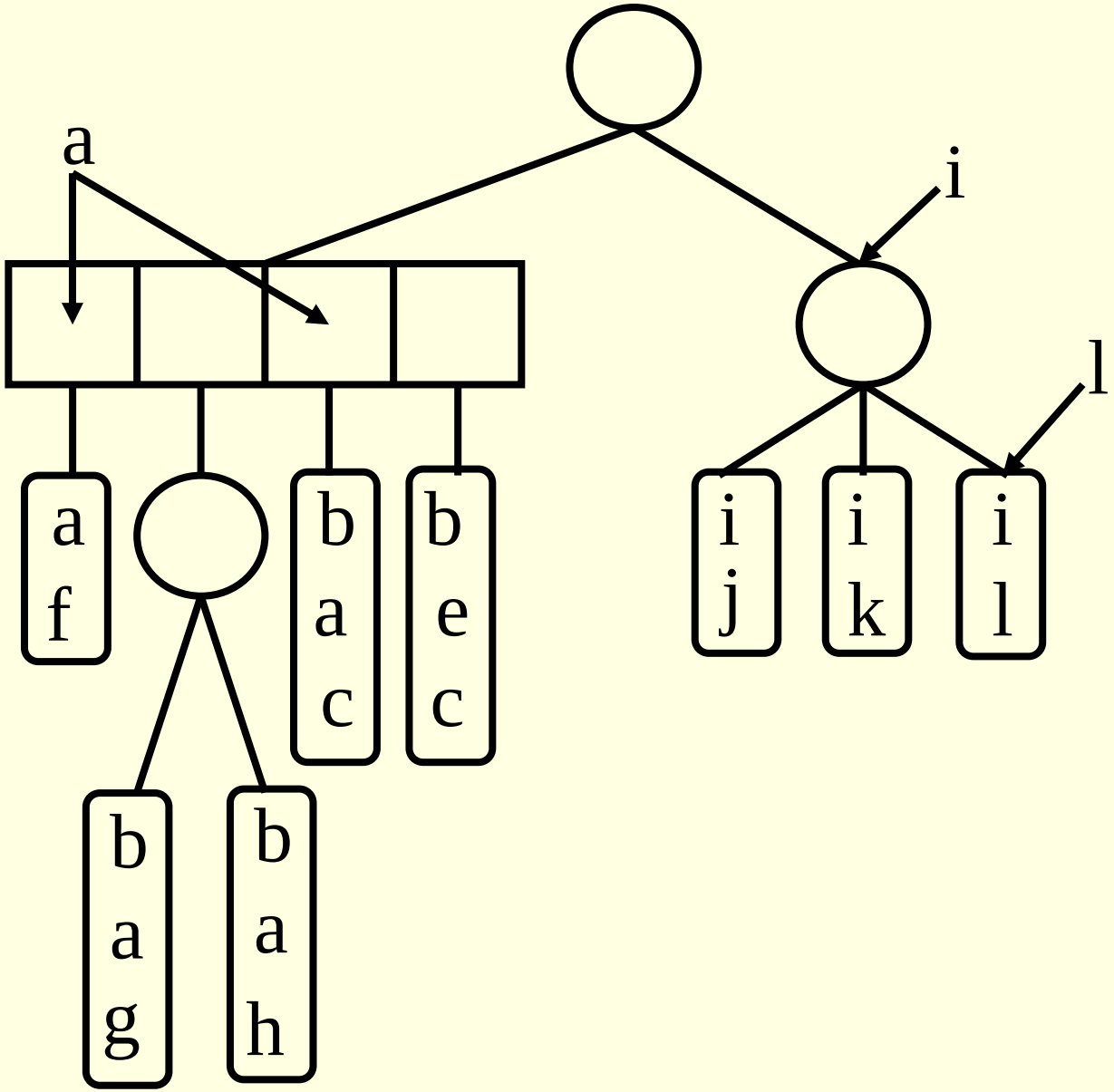

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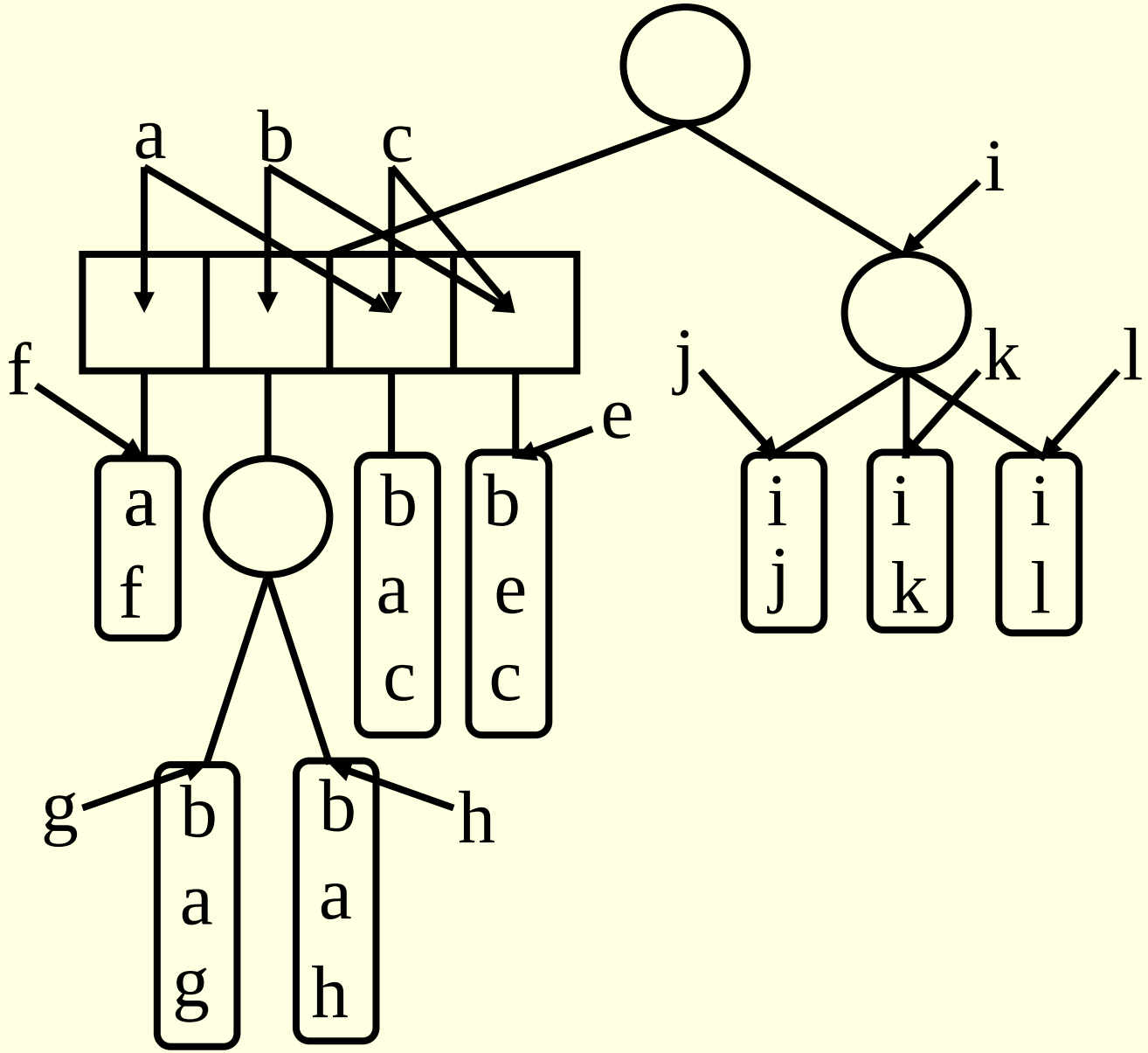

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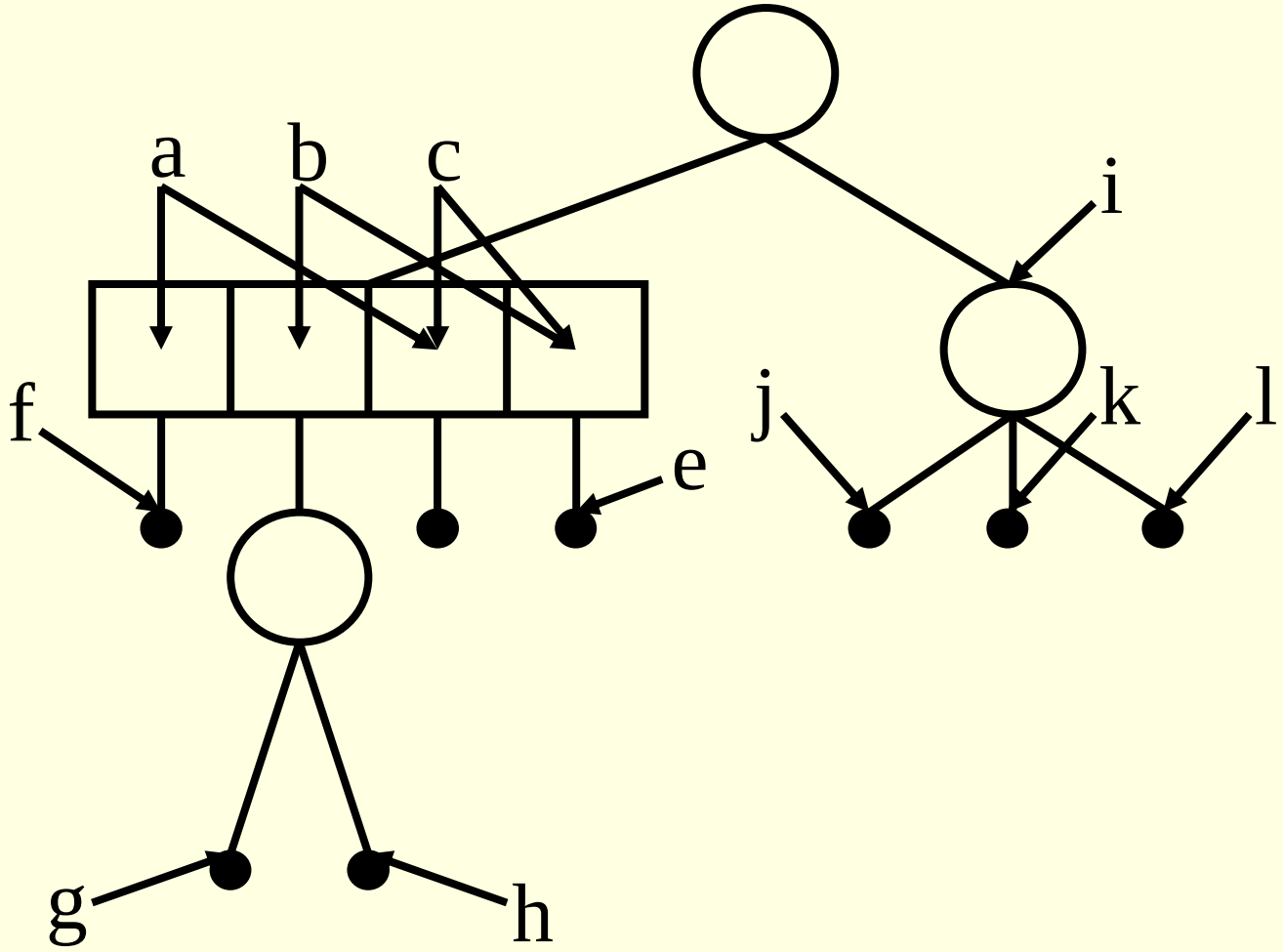
Korte and Möhring 1985



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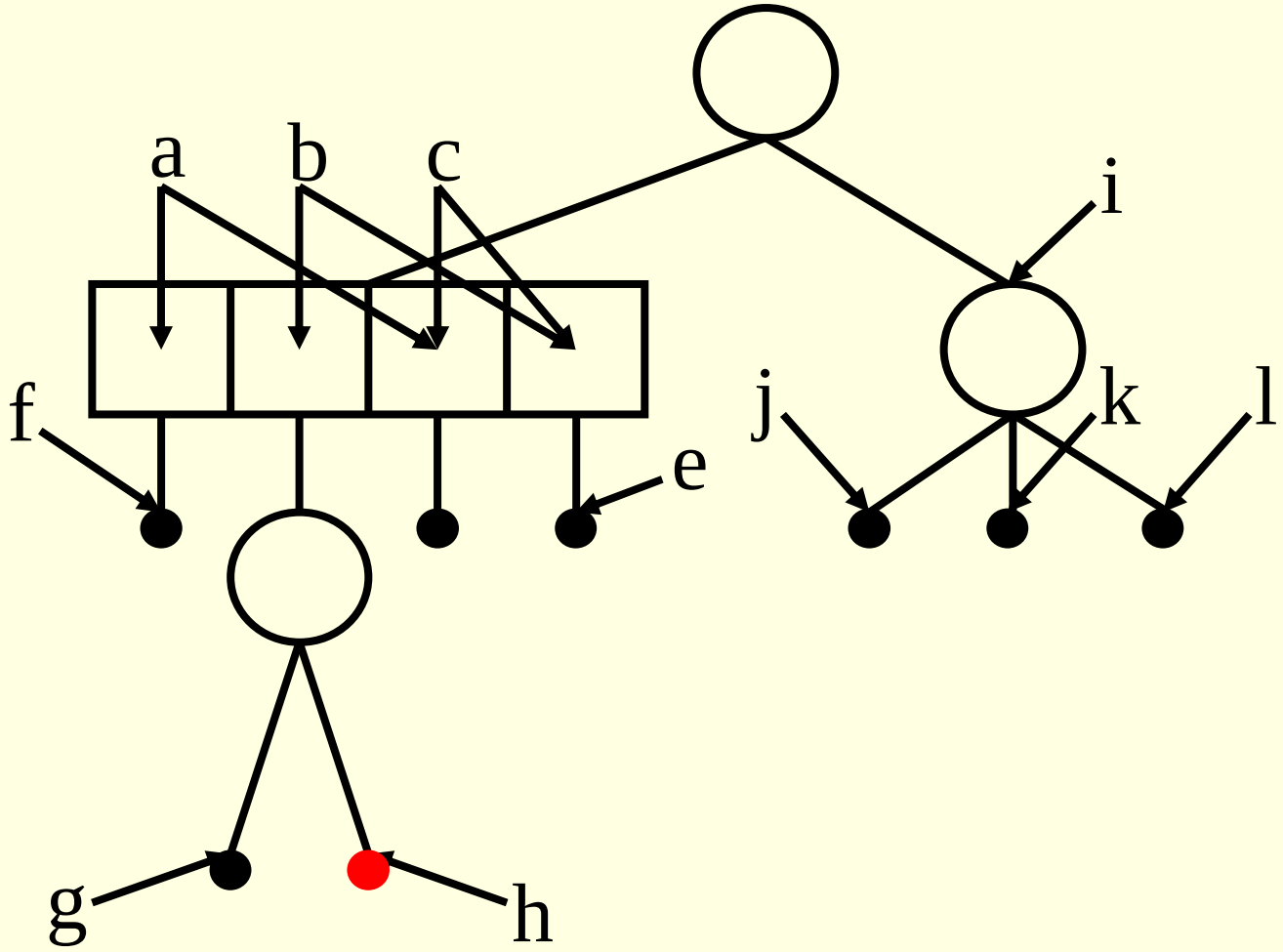
Korte and Möhring 1985



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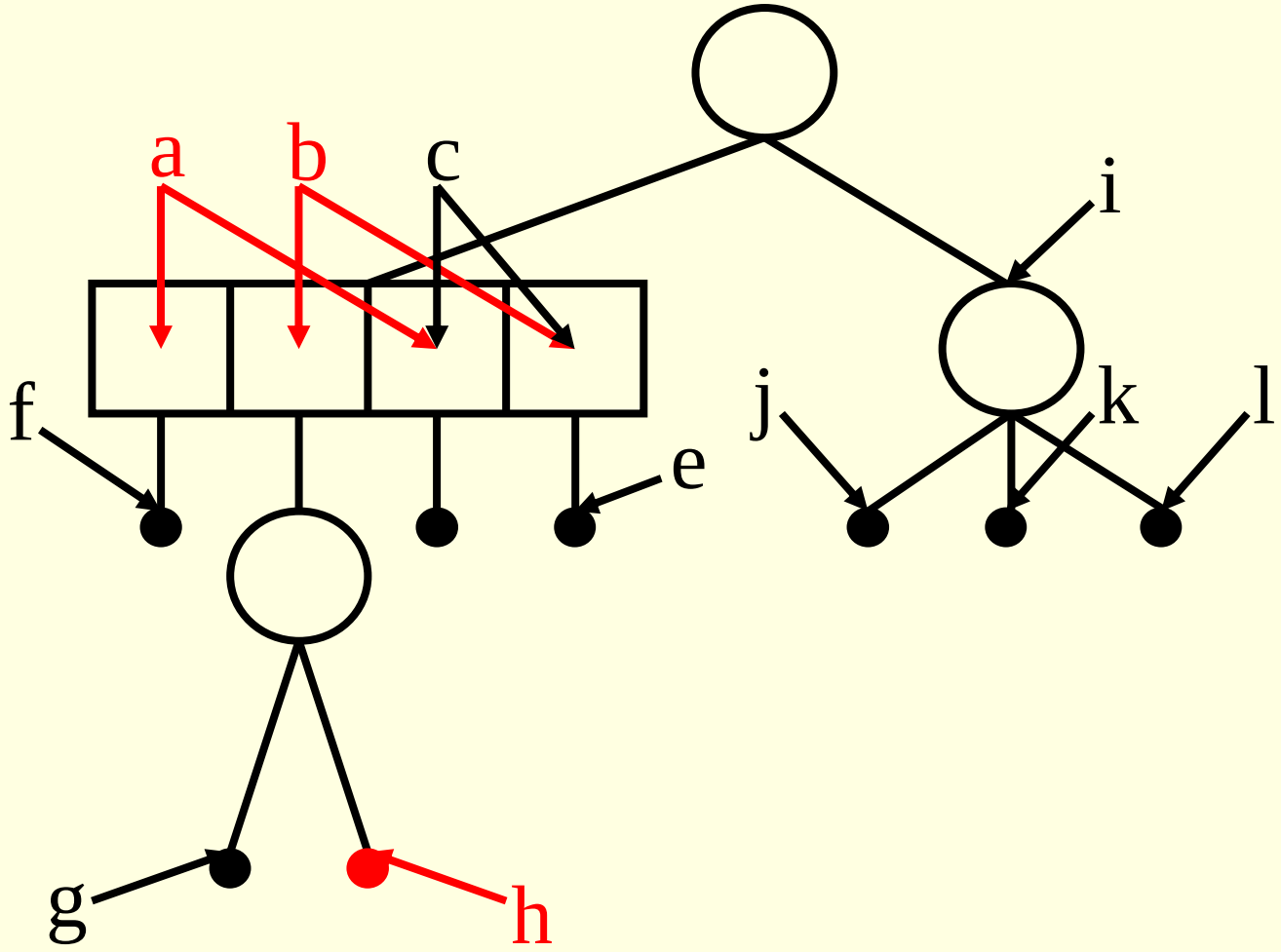
Korte and Möhring 1985



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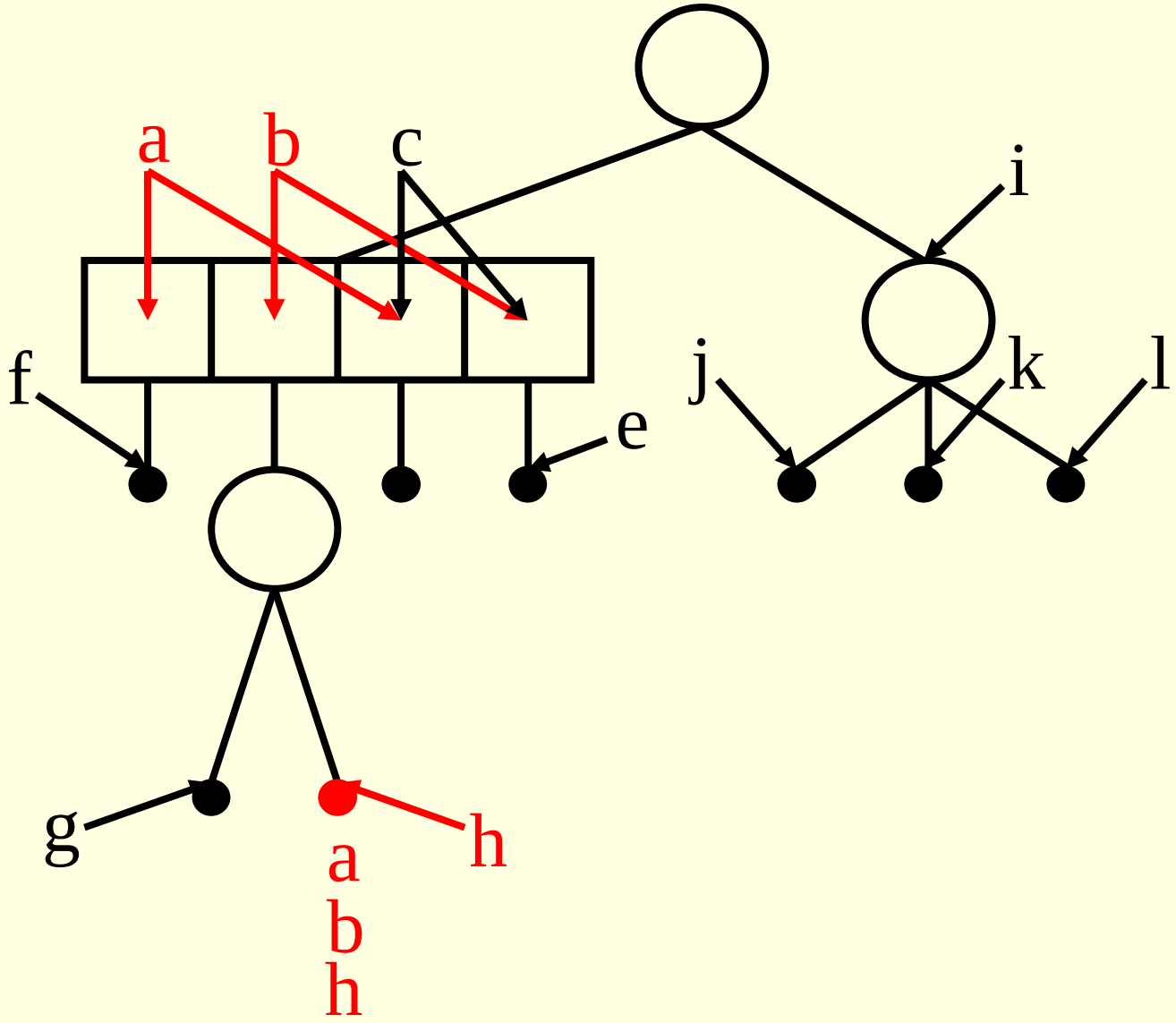
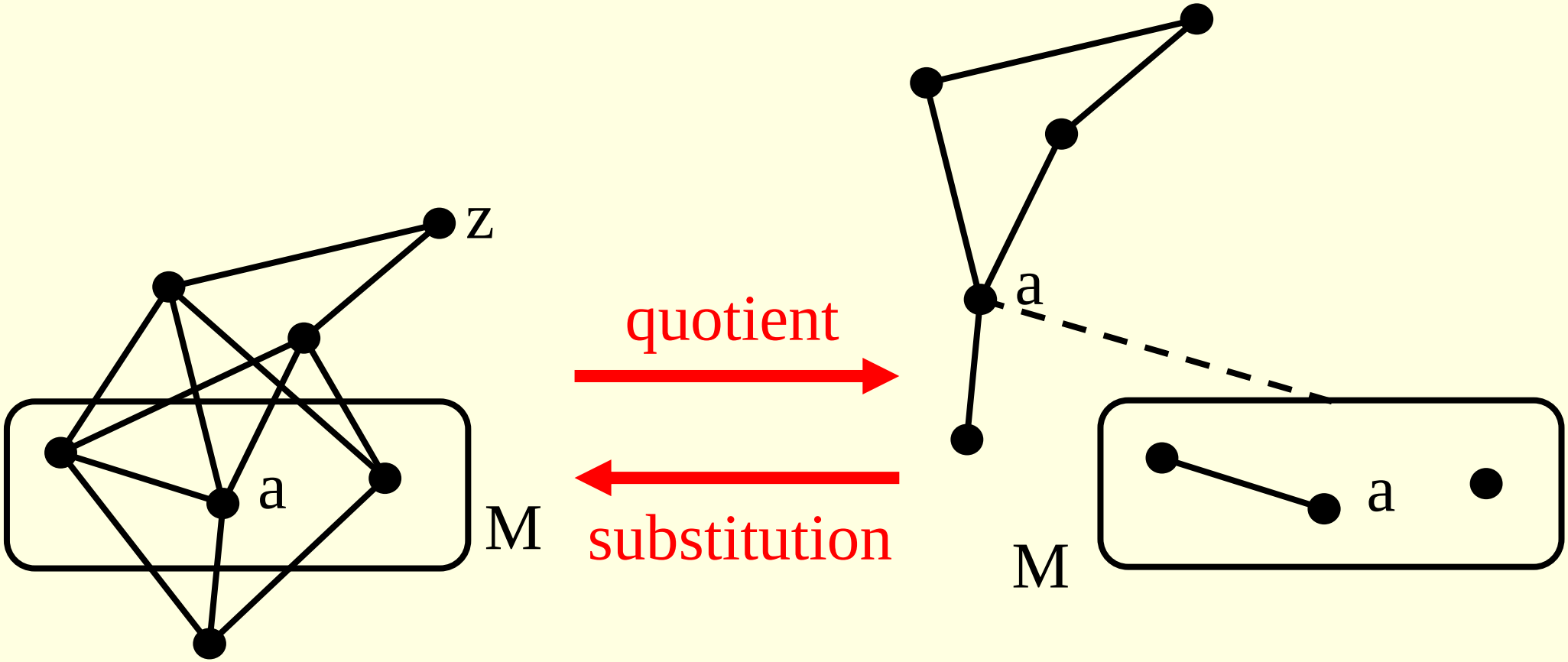


# Equivalence of

# PQ-tree and modular decomposition

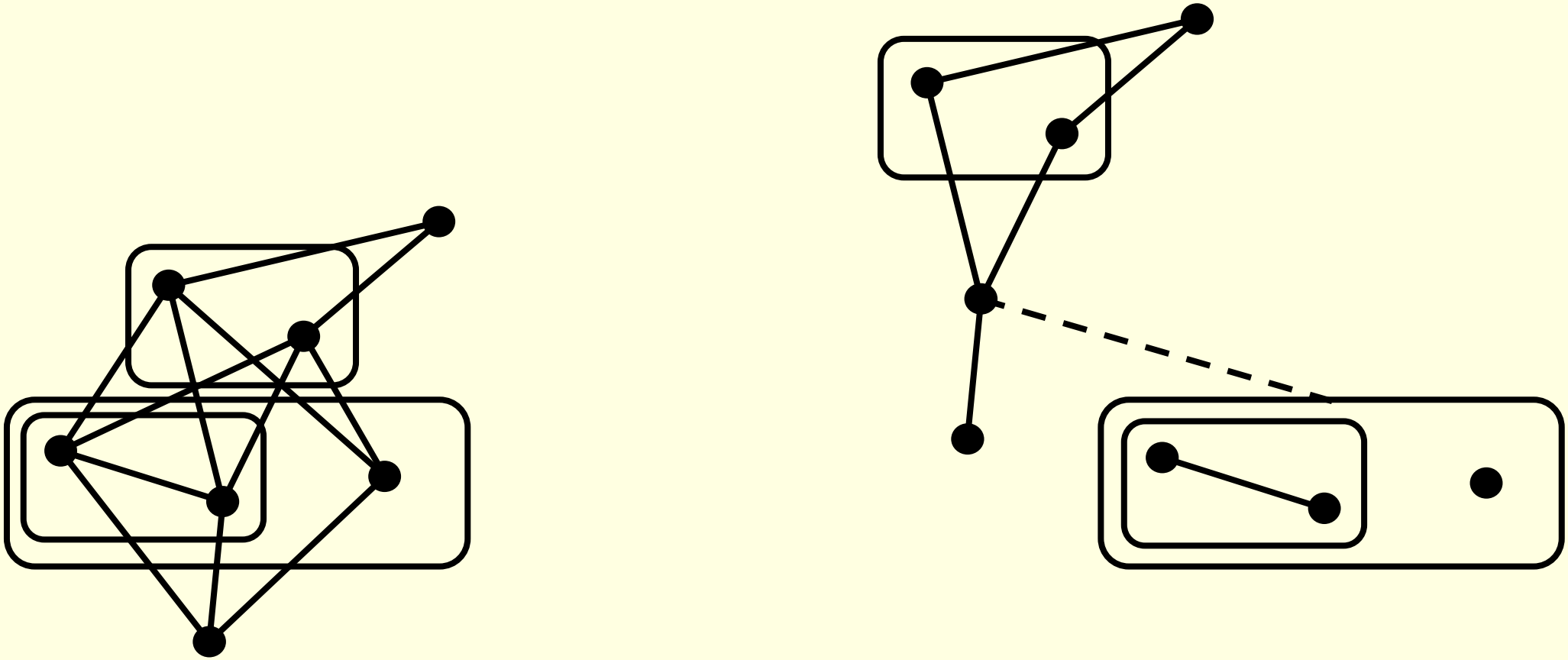
# Modular decomposition





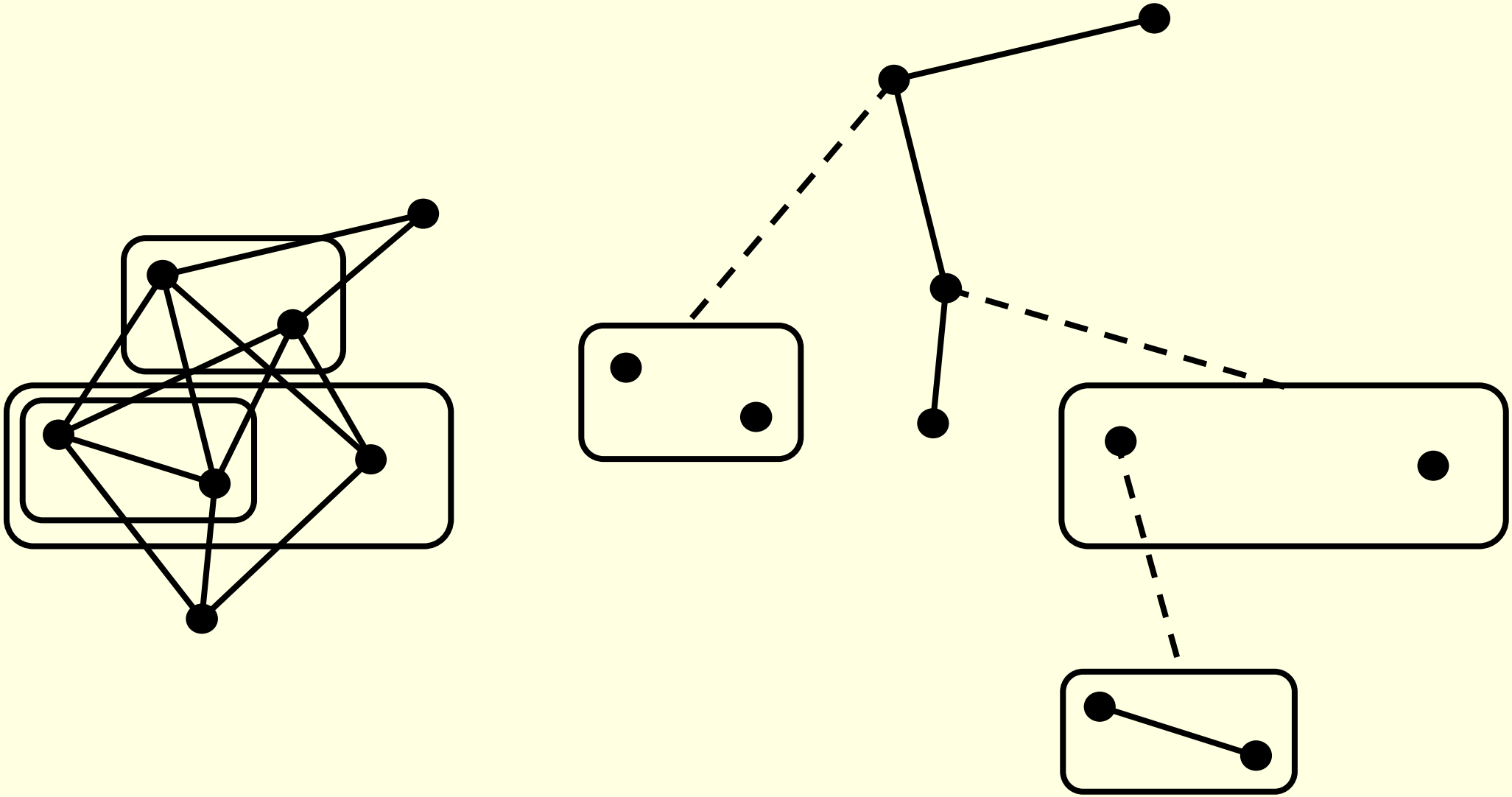
# Modular decomposition





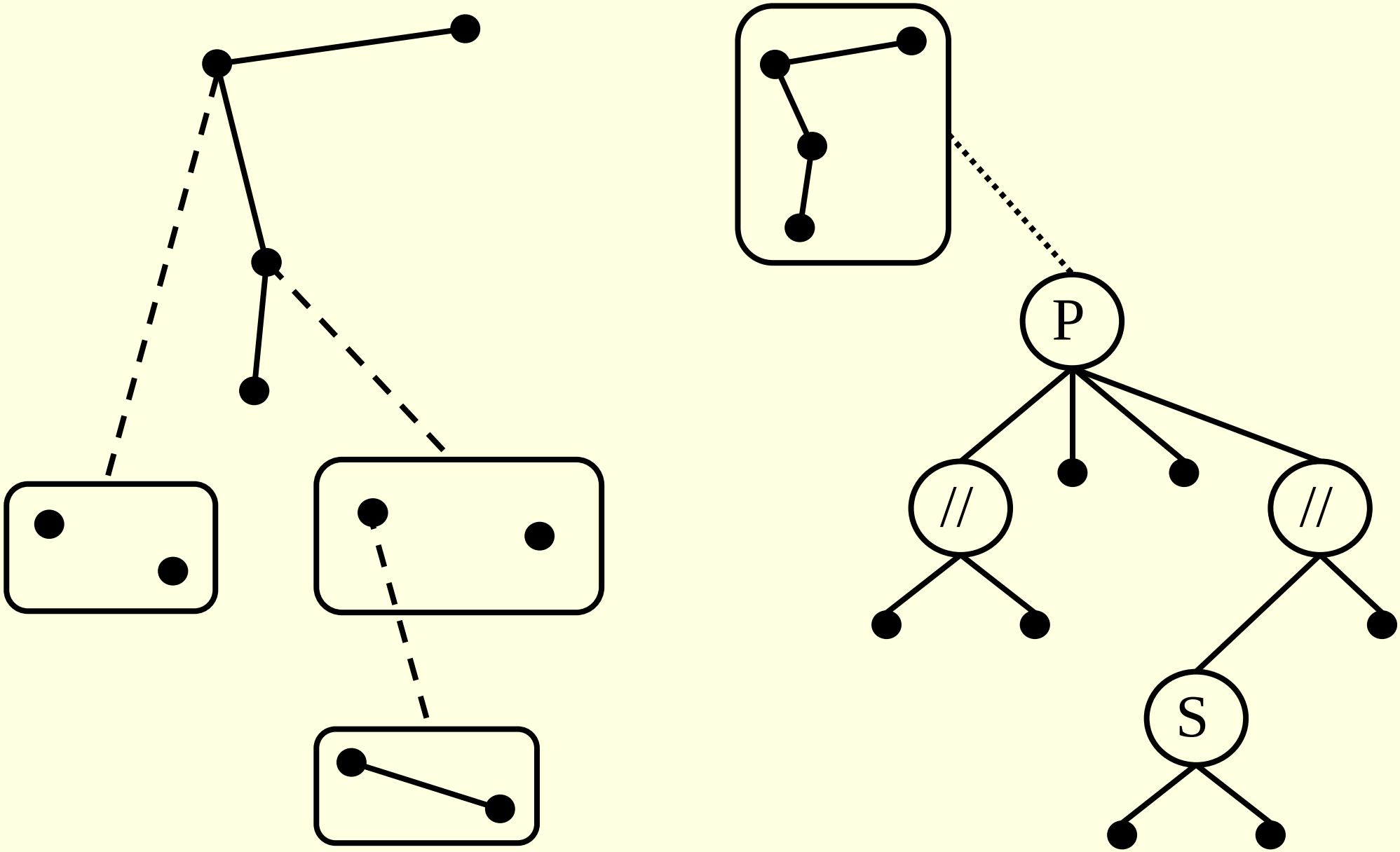
# Modular decomposition





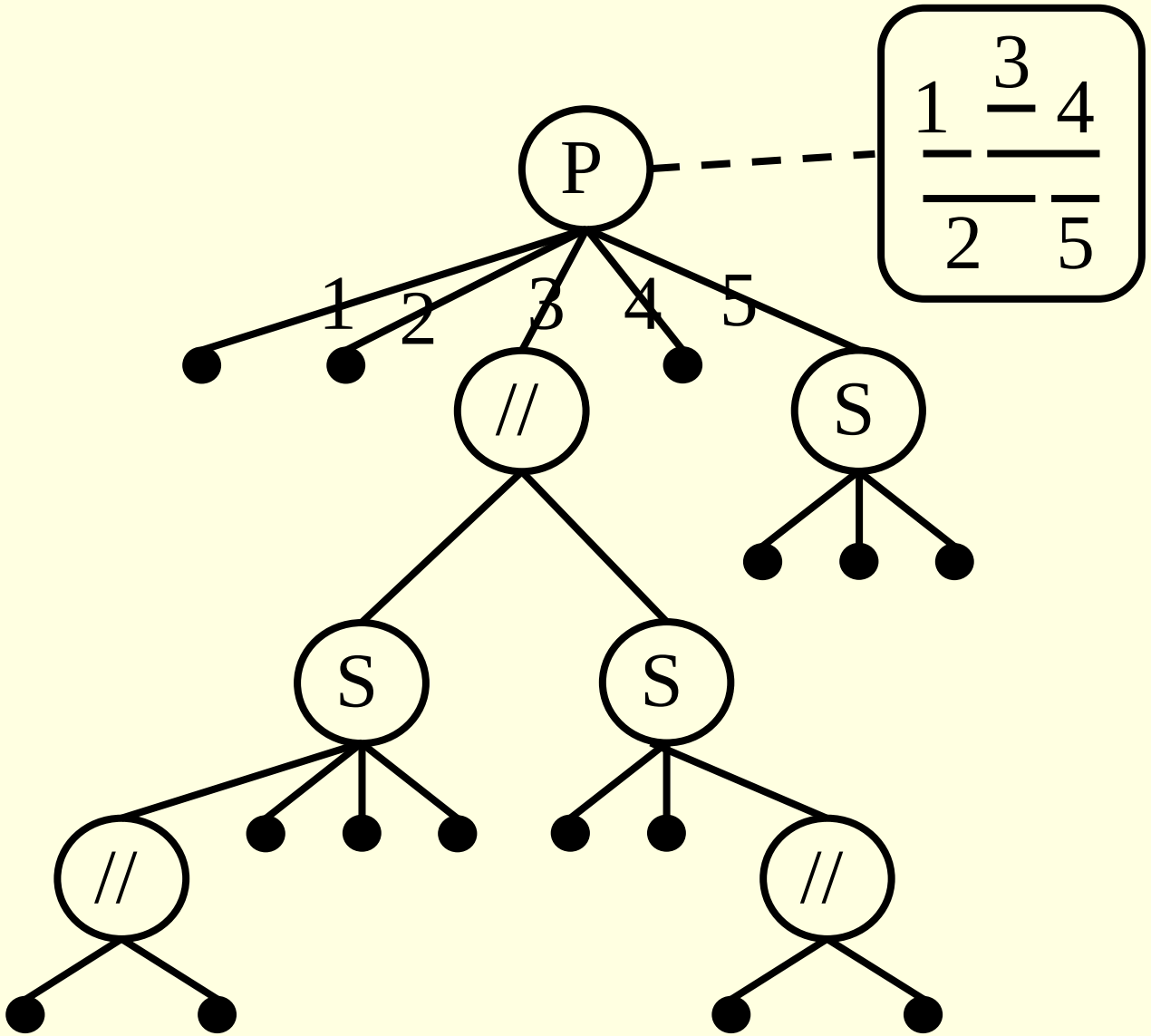
# Modular decomposition





# Modular decomposition of interval graphs

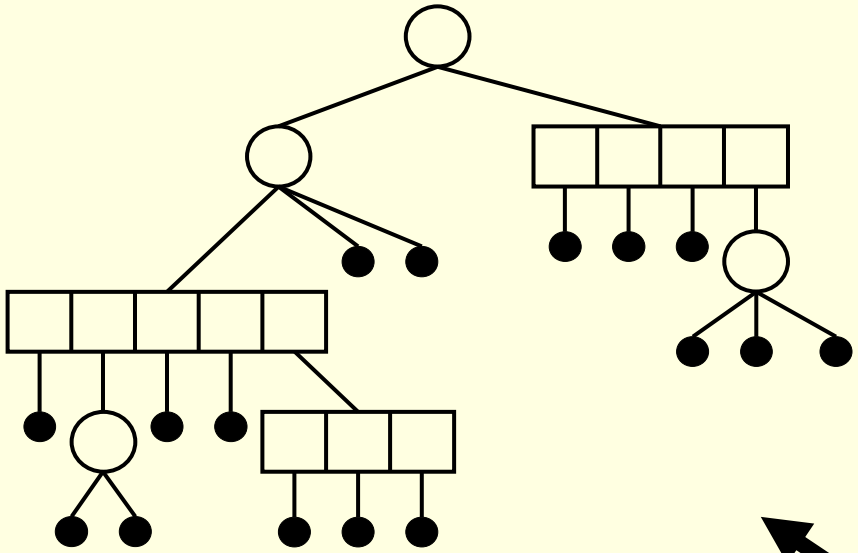




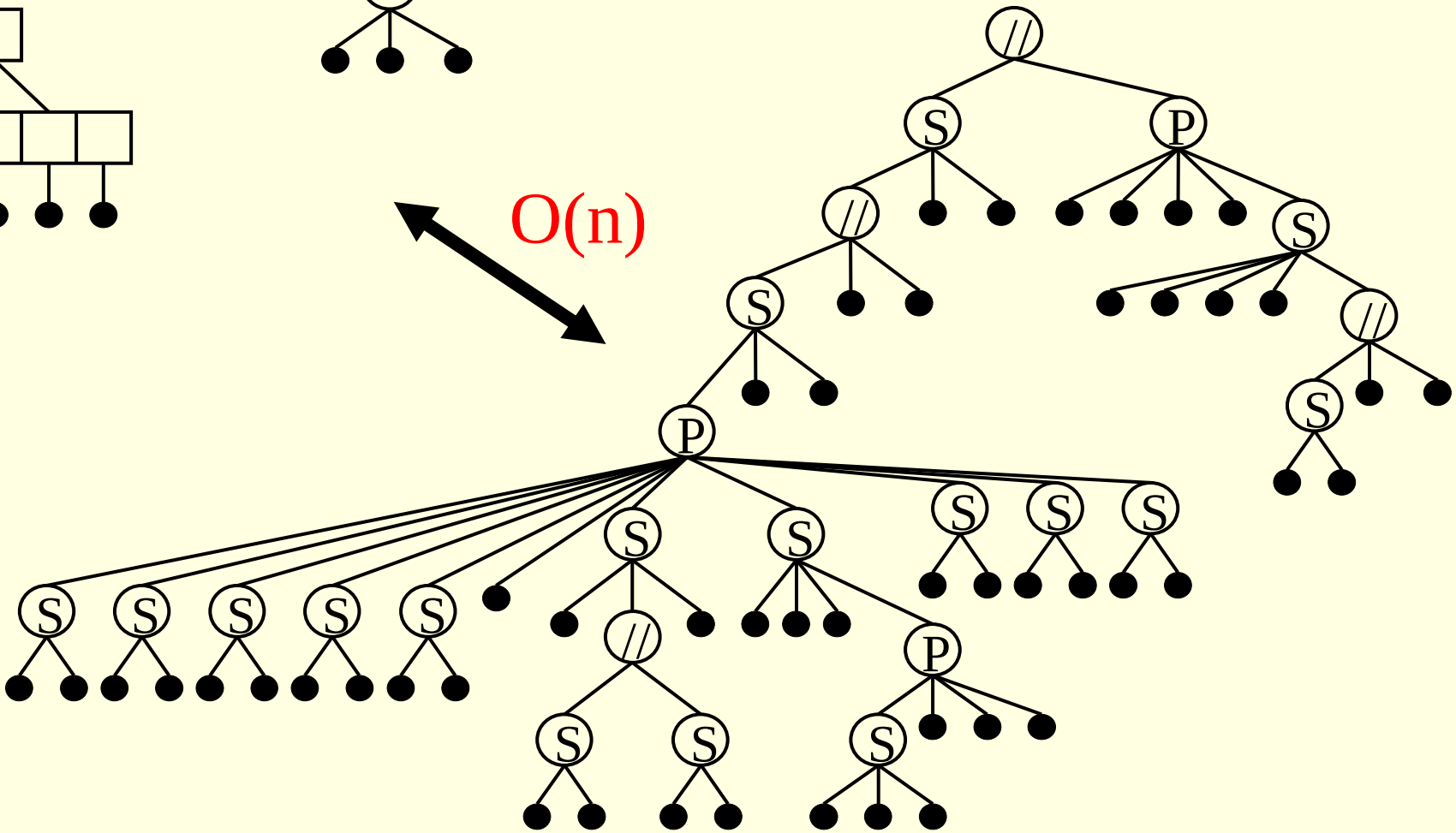
# PQ-tree and modular decomposition



PQ-tree



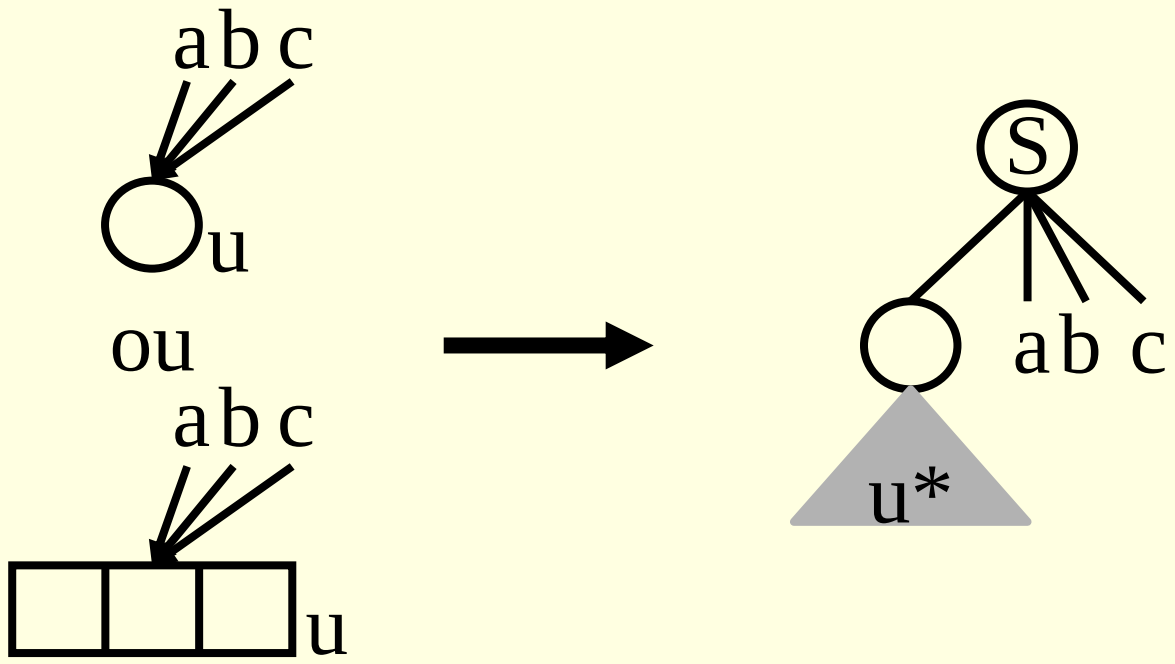
Modular decomposition



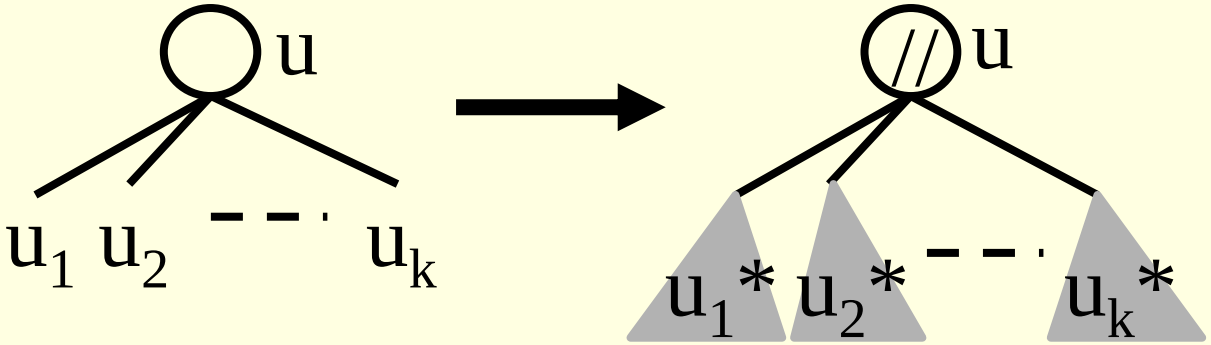
# Degenerate nodes



Series



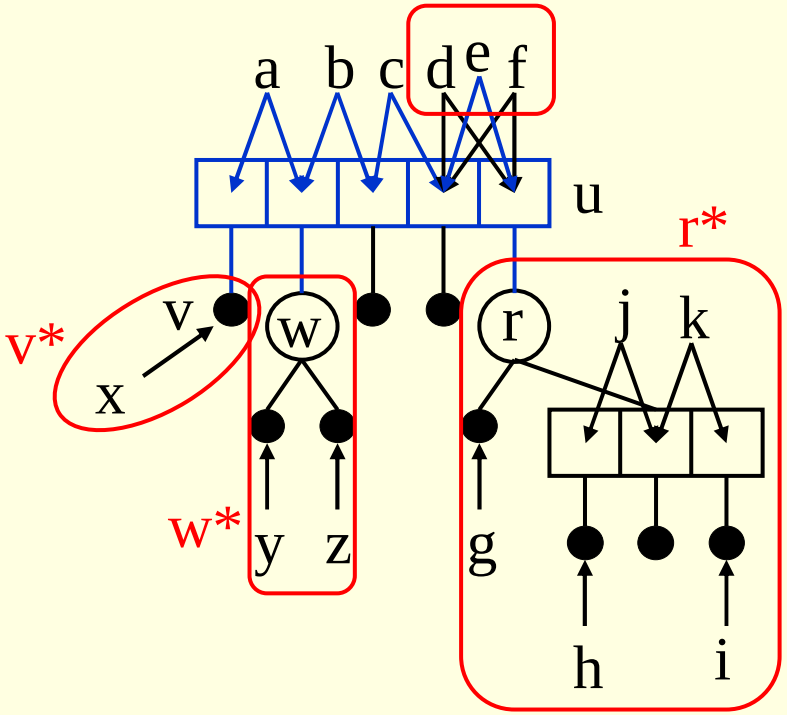

Parallel

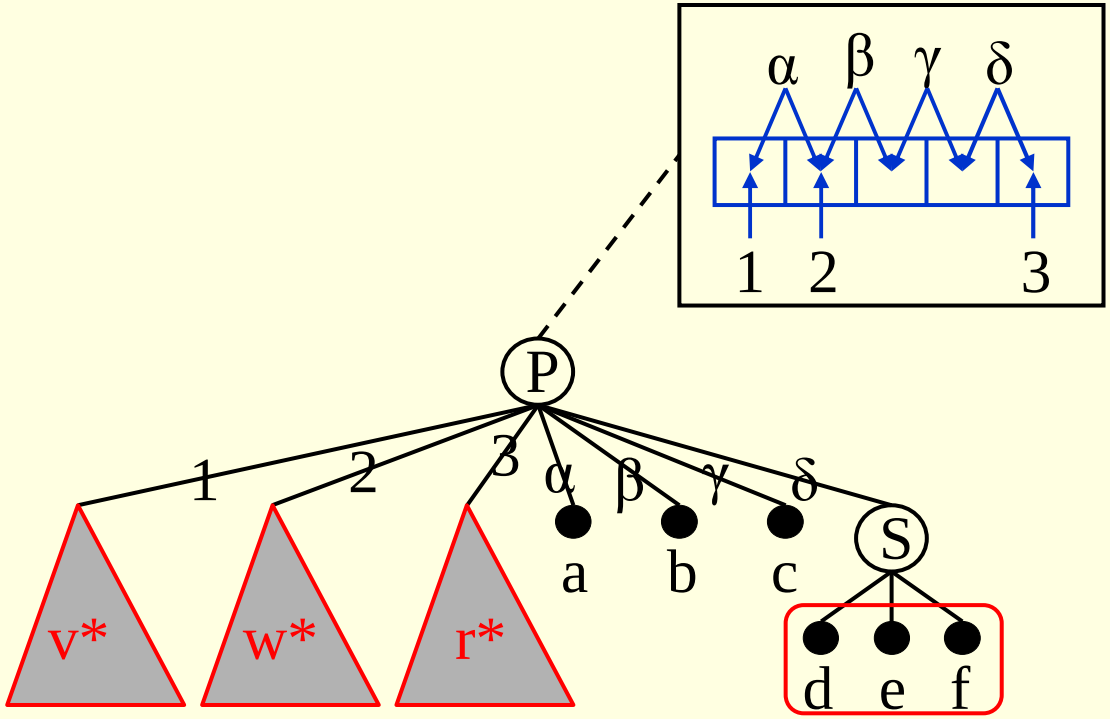


# Prime nodes



## Maximal strong modules

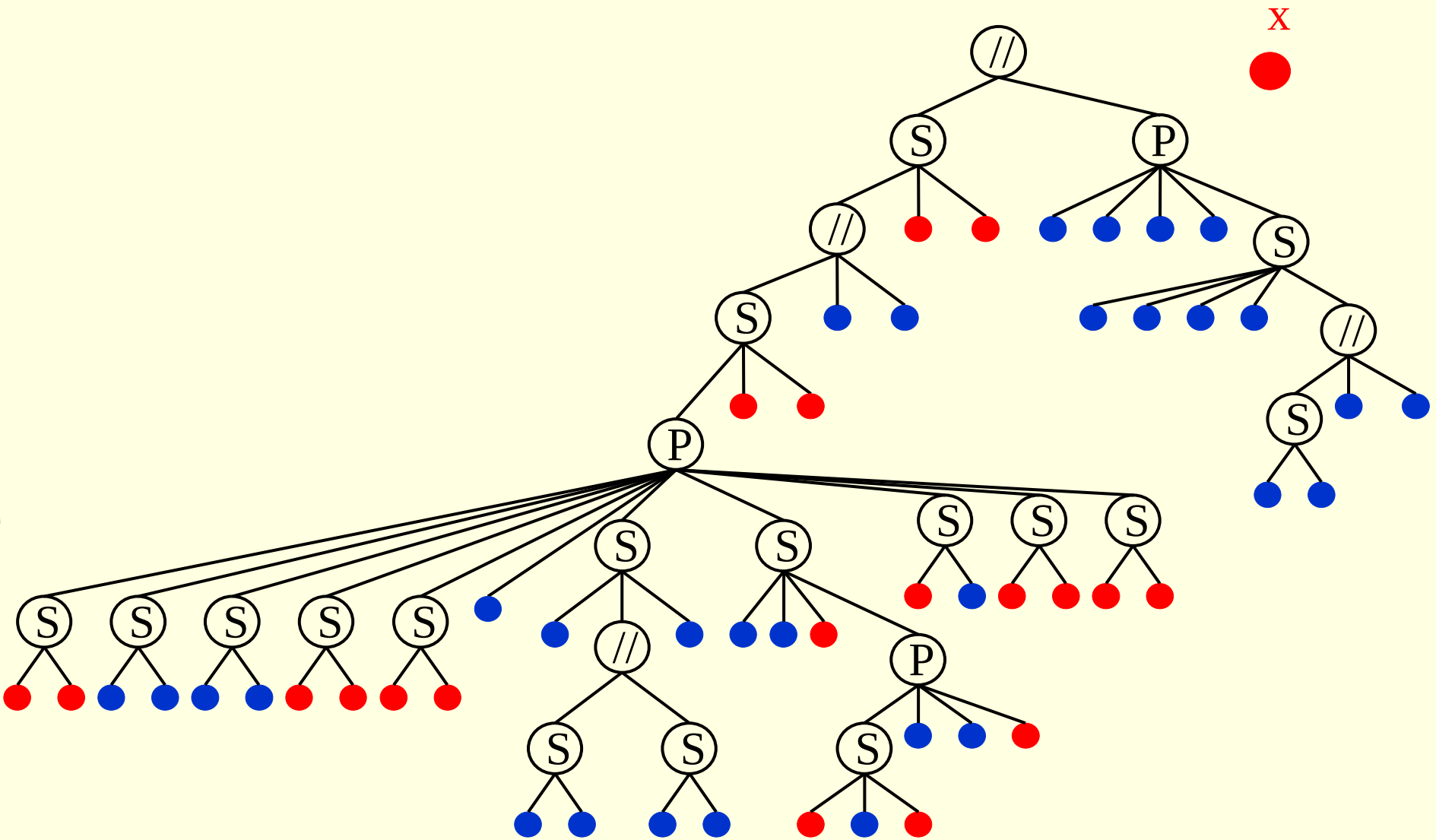


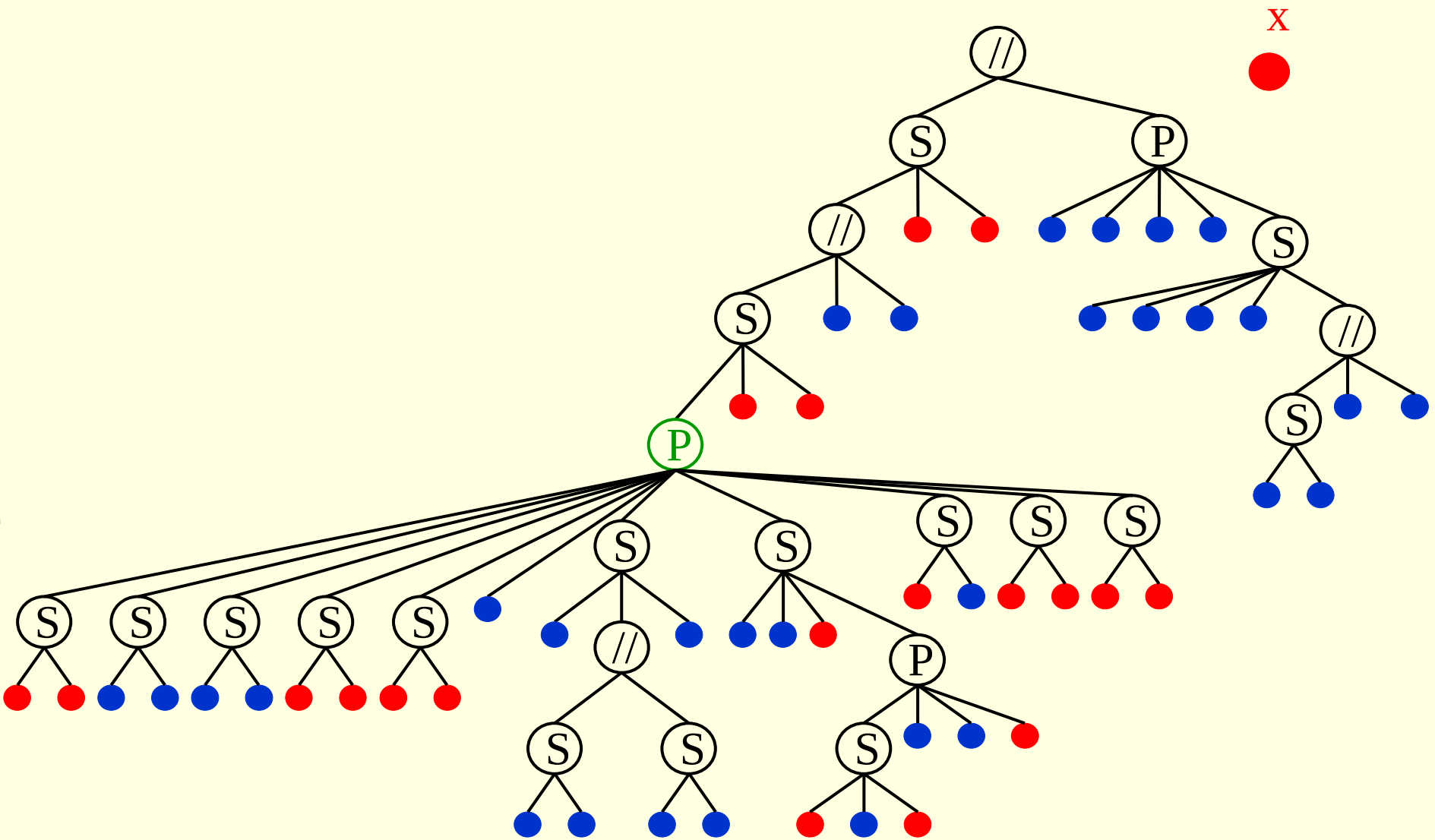



# Plan

1. Problem and difficulty
2. Representations of Interval Graphs
3. Vertex insertion

# Muller and Spinrad 1989
# Crespelle and Paul 2005

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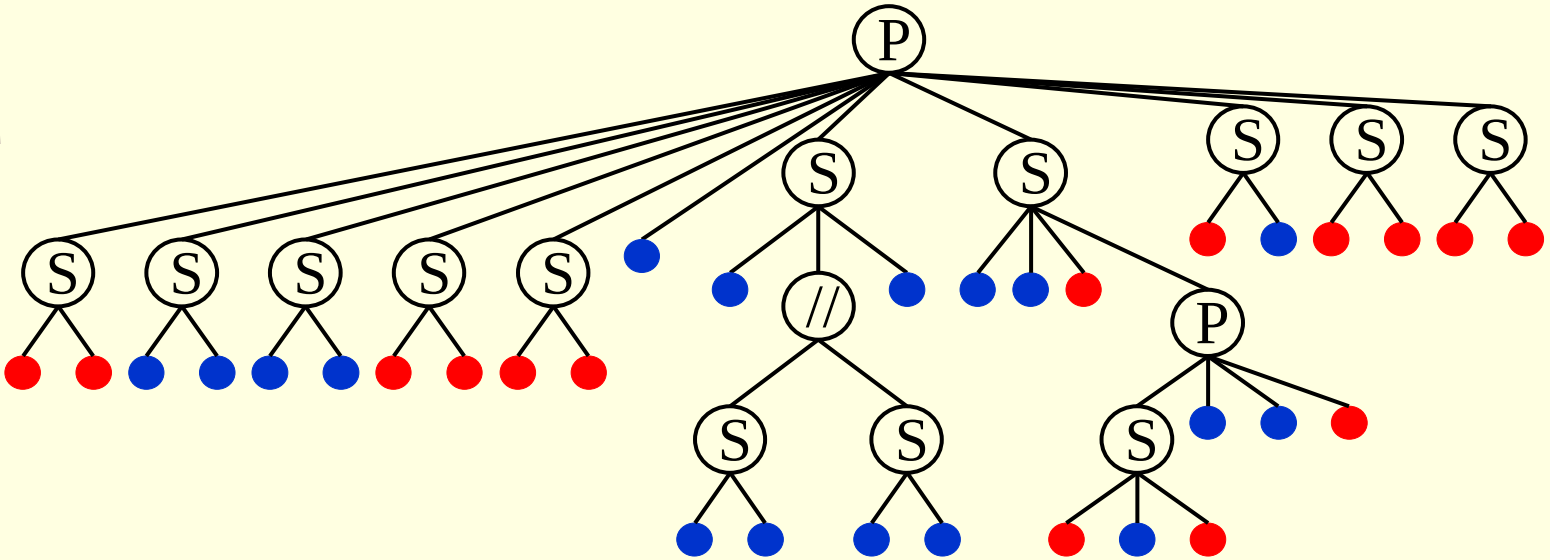
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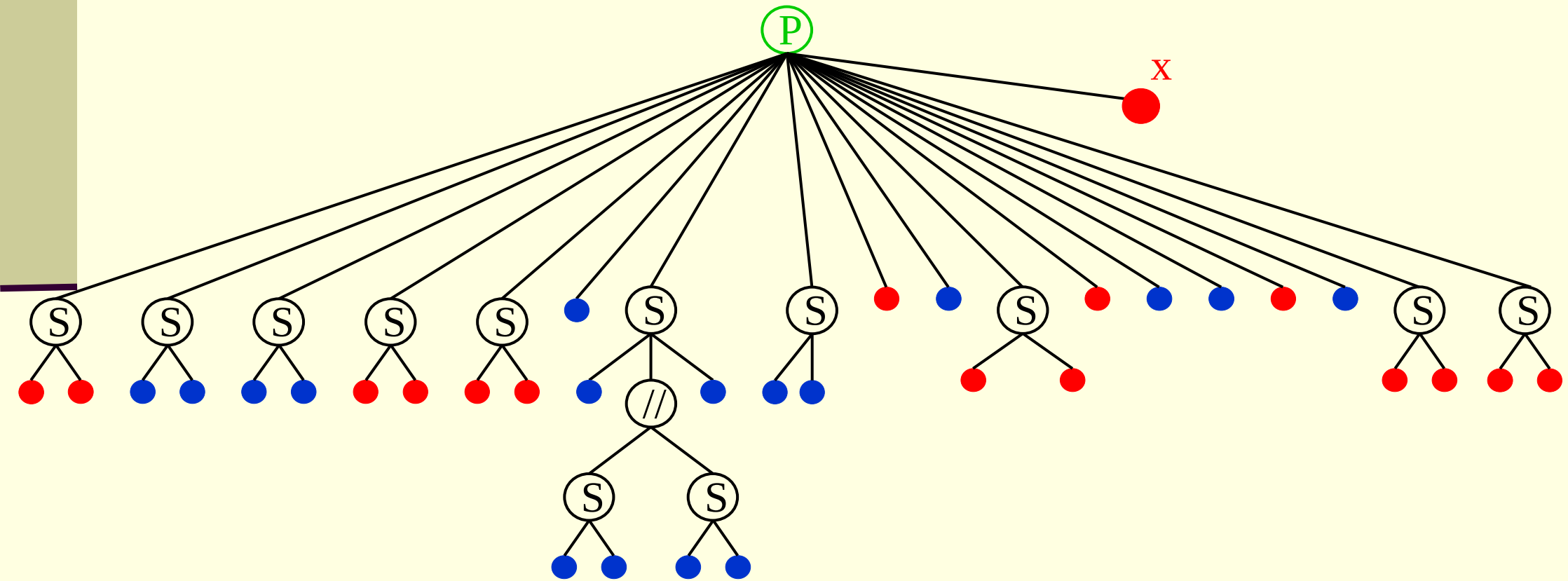
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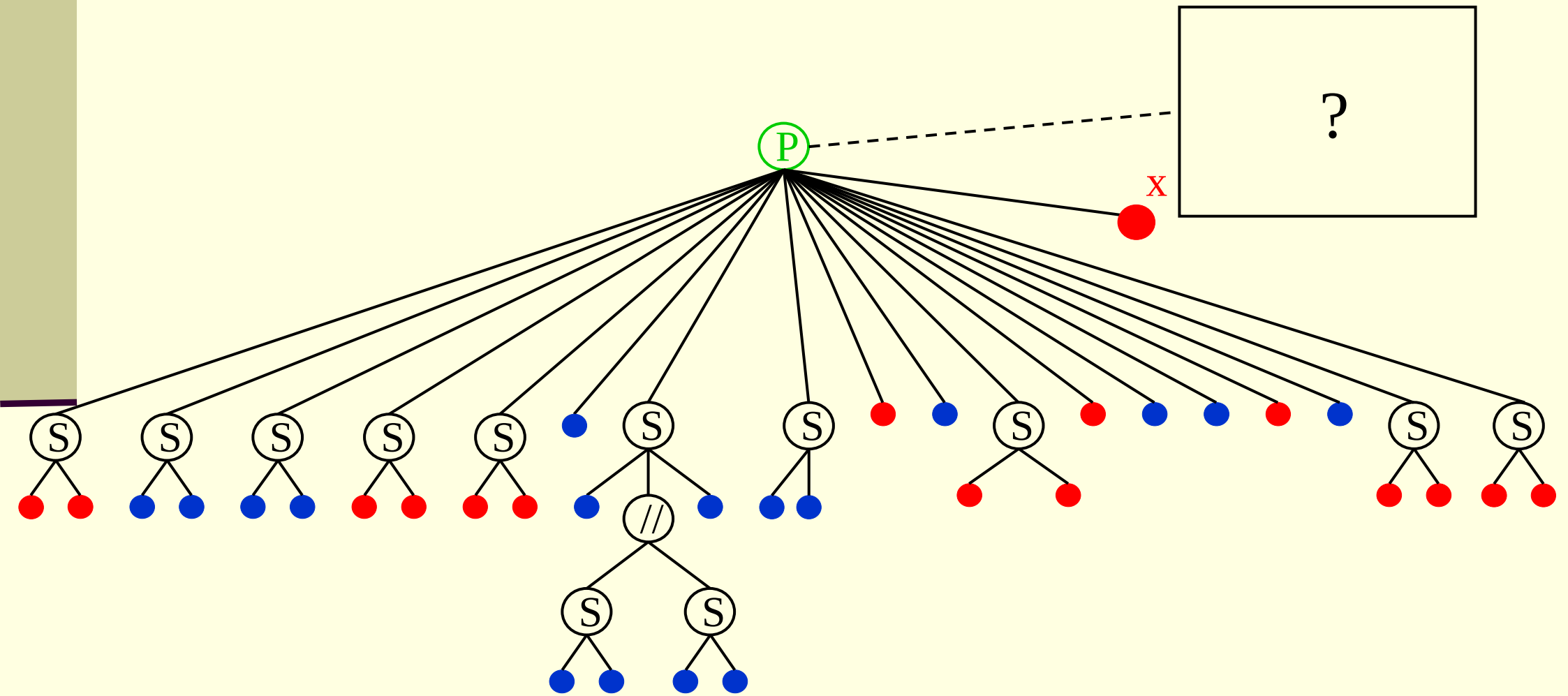
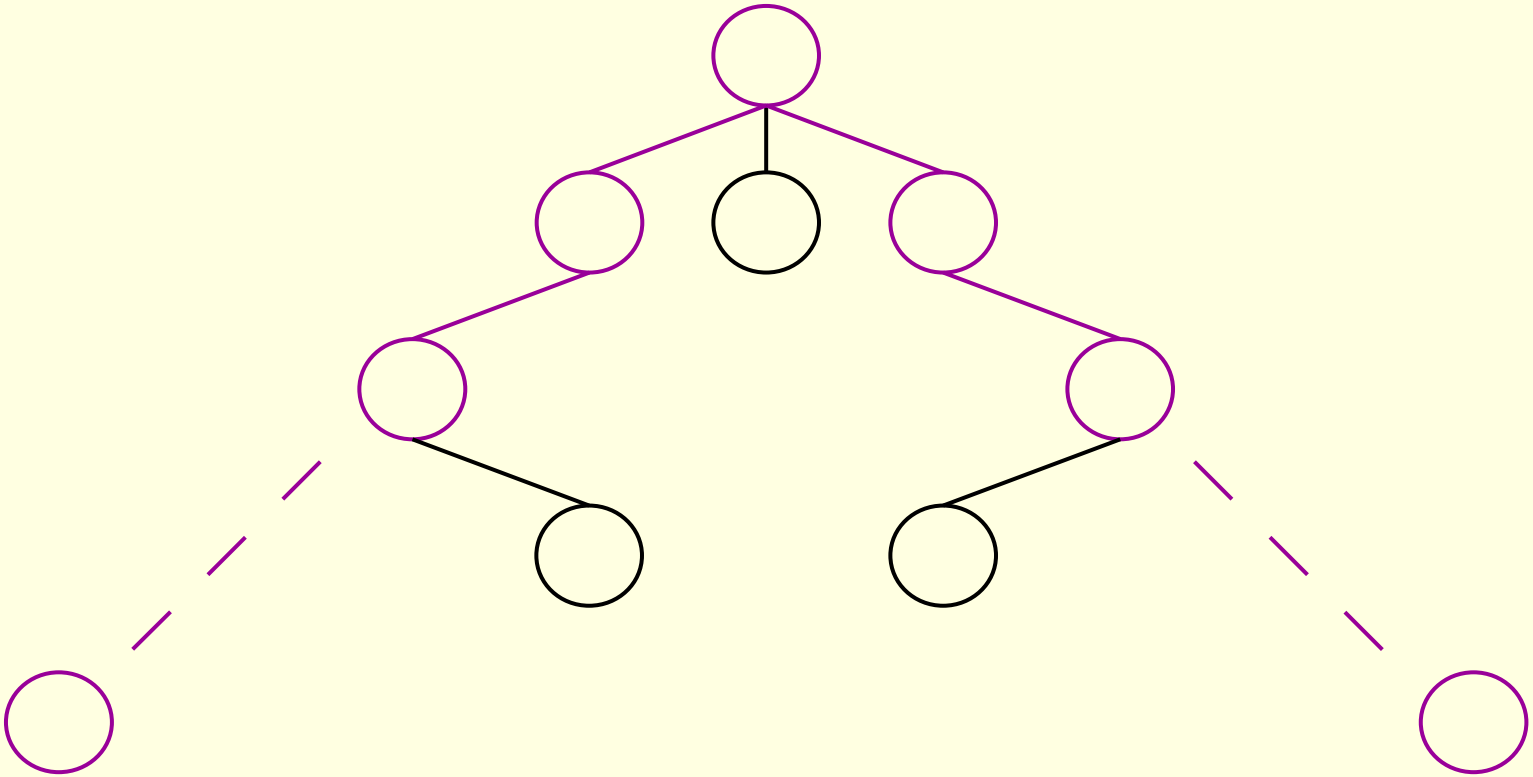
# Is G+x an interval graph?



# Conditions for a successful insertion



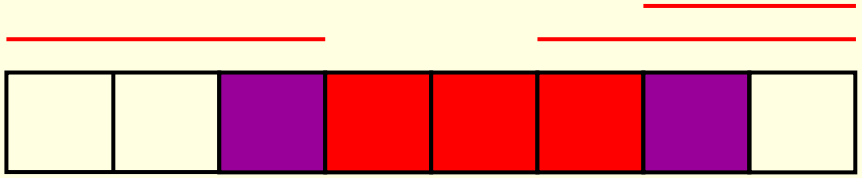
The insertion node w has at most two mixed children and any node u of $T_w \backslash \{w\}$ has at most one mixed child.



# Conditions for a successful insertion



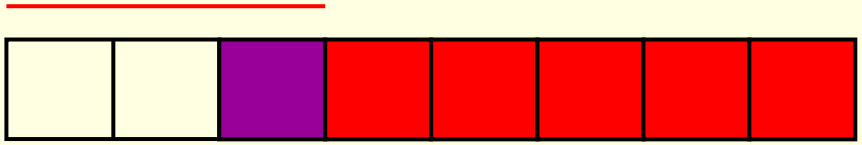
Conditions on w.



# Conditions for a successful insertion

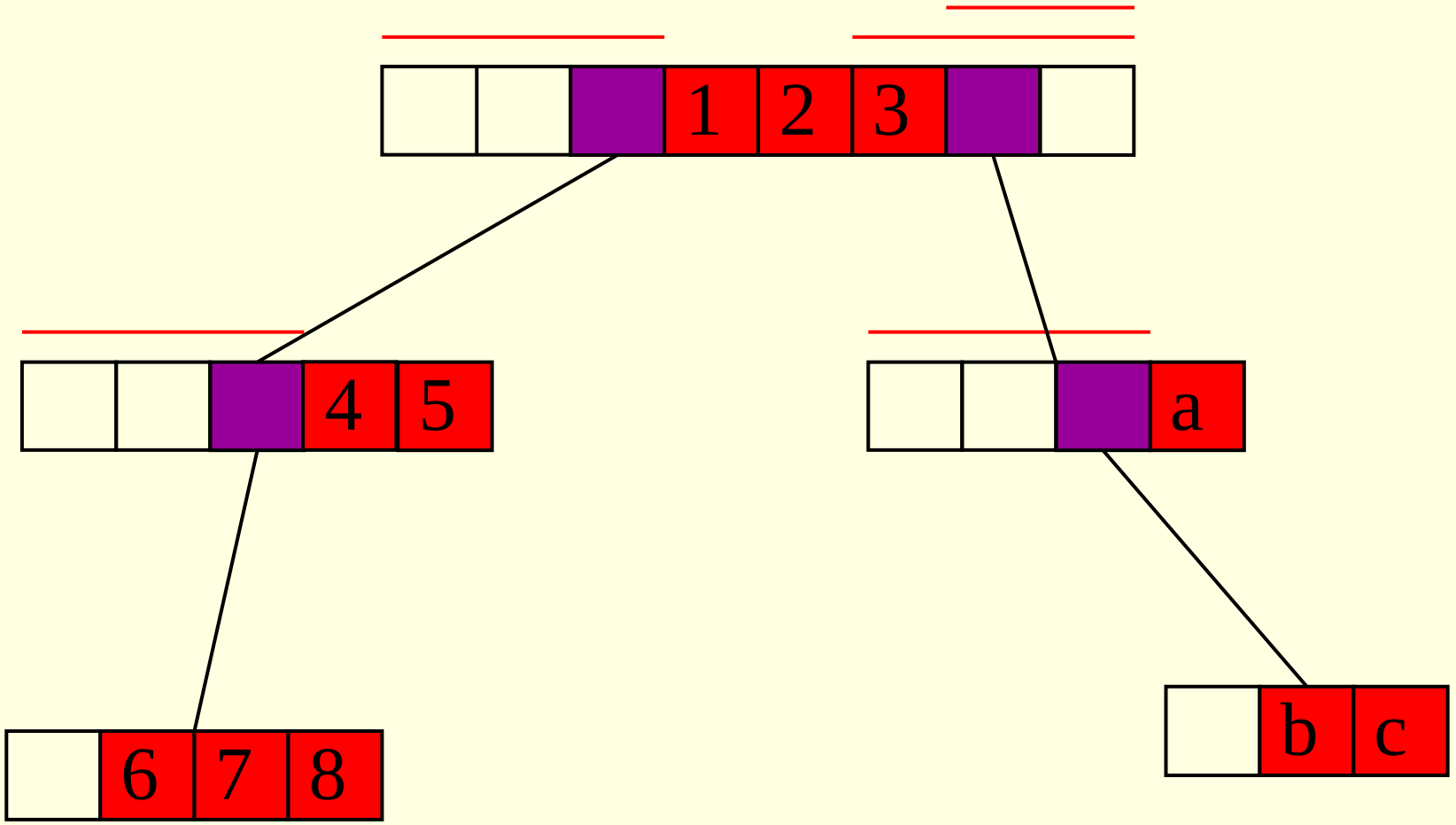


Conditions on a node u of $T_w \backslash \{w\}$.



# Updating the PQ-representation





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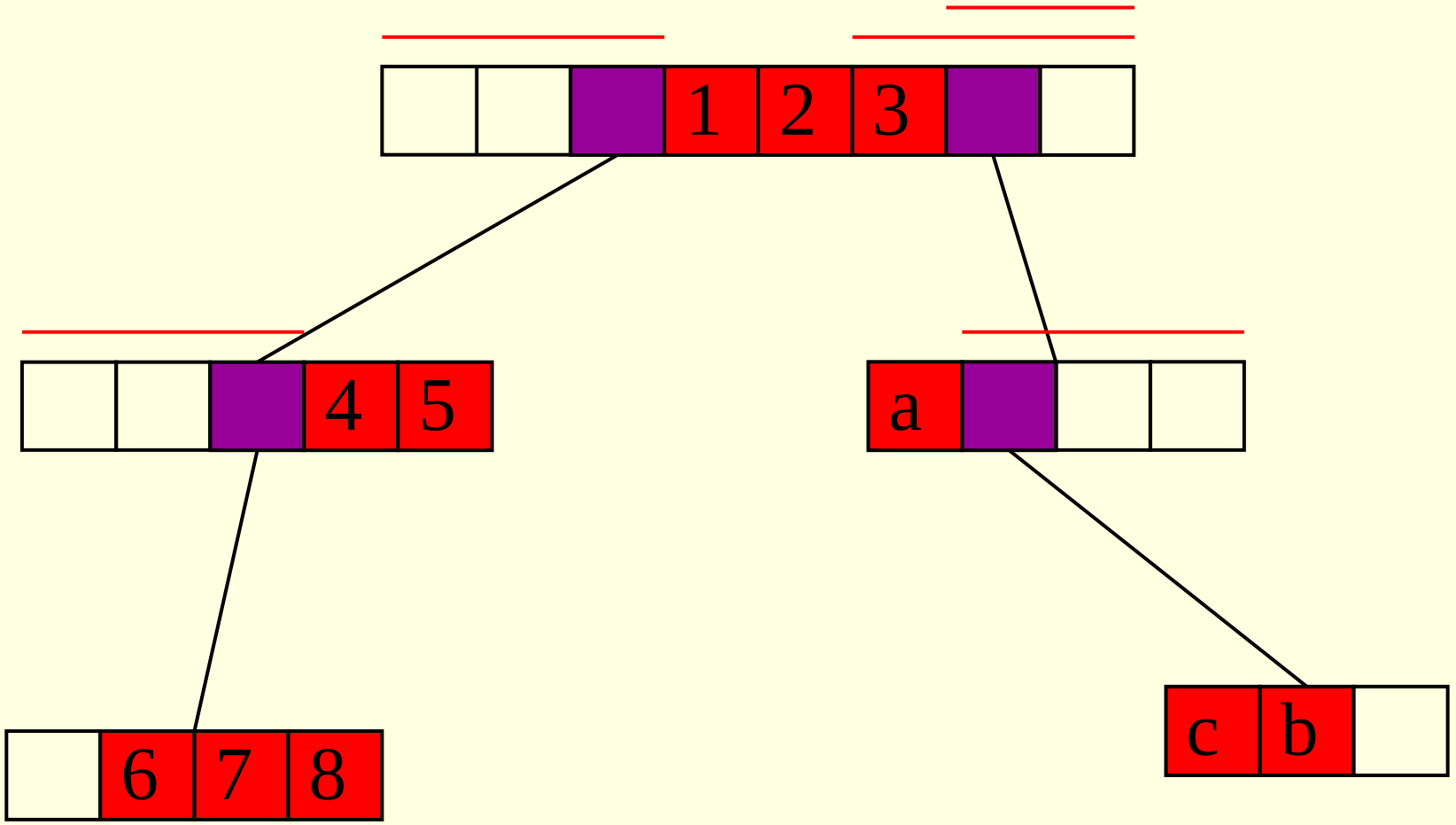

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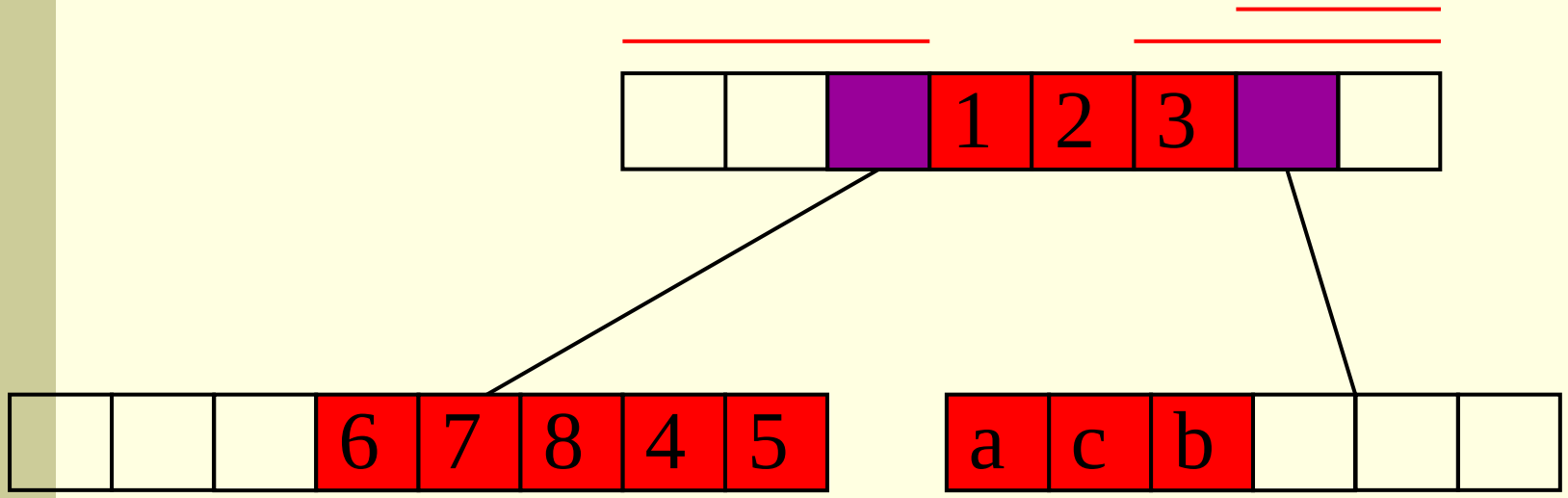

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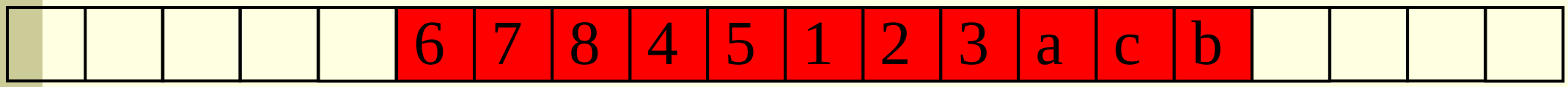

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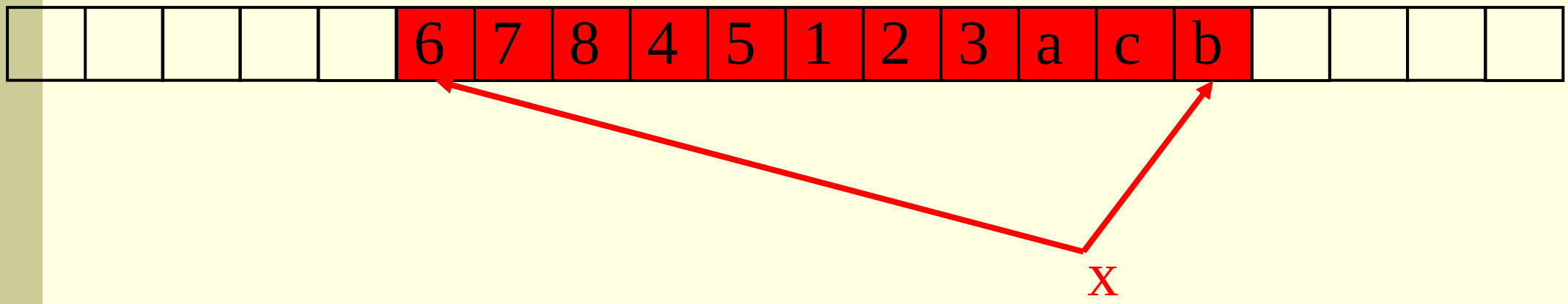

# Complexity



- Vertex insertion : O(n)

  Every node is treated in time proportional to its degree and the number of vertices pointing toward it.

# Complexity



- Vertex deletion : O(n)

  The difficult case is managed thanks to the algorithm of McConnell and de Montgolfier 2005

- Insertion / Deletion of edges : O(n)

  Managed by two vertex modifications