Topological arguments and Kolmogorov complexity

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Supported by ANR NAFIT grant

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- not always possible: $C(x)$ should be at least n

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- strangely, for $C(x) \gg n$ this argument does not work (only for $C(x) \leq \mathrm{poly}(n)$)
- so $C(x) \geq n + O(\log n)$ is enough, but two essentially different arguments are needed at both ends

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- discrete intermediate value theorem guarantees that $C(x|y) = n + O(1)$ for some y on the way

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- then both $C(x|y)$ and $C(y|x)$ are $n + O(\log n)$
- $O(1)$ cannot be obtained in this way (since all the arguments about random and independent bits work with $O(\log n)$ precision only)

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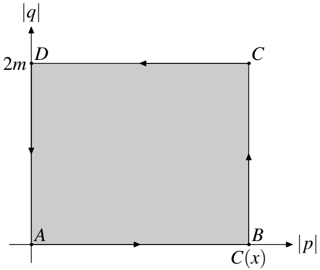

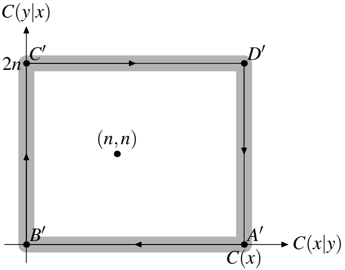

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- Alternative: repeat the proof for discrete case

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- an open problem in the general case

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- and finally show how to win the game

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  - $\leq 2N^3$ hopeless elements; all others in X are ultimately connected to some $y \in Y$ and this connection lasts forever

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- but both complexities are at least n, otherwise refused

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- new partner for x is found if possible (=there is $y \in Y$ with matching experience not tried earlier with x)

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- the first is $< N$; the second a priori is unbounded, but also will be kept $< N$ due to agency strategy
- when (x, y) is terminated, numbers updated
- invariant: in all pairs people have matching experiences (#sent = #received for the other)
- corollary: #refusals received $< N$
- new partner for x is found if possible (=there is $y \in Y$ with matching experience not tried earlier with x)
- otherwise x is declared hopeless

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- there are N^2 experience classes; if class reaches $2N$, it stops growing since y can be always found in the class ($< 2N$ are tried earlier with given x), so $O(N^3)$ hopeless

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- to the audience for following the talk to that point :-)