Kritchman & Raz – exposition

Notices AMS 2010
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BERRY:

the minimal natural number that cannot be defined
by less than thousand English words

CHAITIN: For every formal theory T there exist some C s.t. no statement of the form "$K(\ldots) > c$" is provable

program: find (first) x that provably $\boxed{K(x) \geq 10^{10^{10}}}$ does not have a program shorter than $10^{10^{10}}$ producing x

where do we cheat?

Program: Enumerate all proofs (in ZFC ...) of statements of the form "$K(x) \geq n$" and wait until "$\boxed{K(x) \geq 10^{10^{10}}}$" for some x

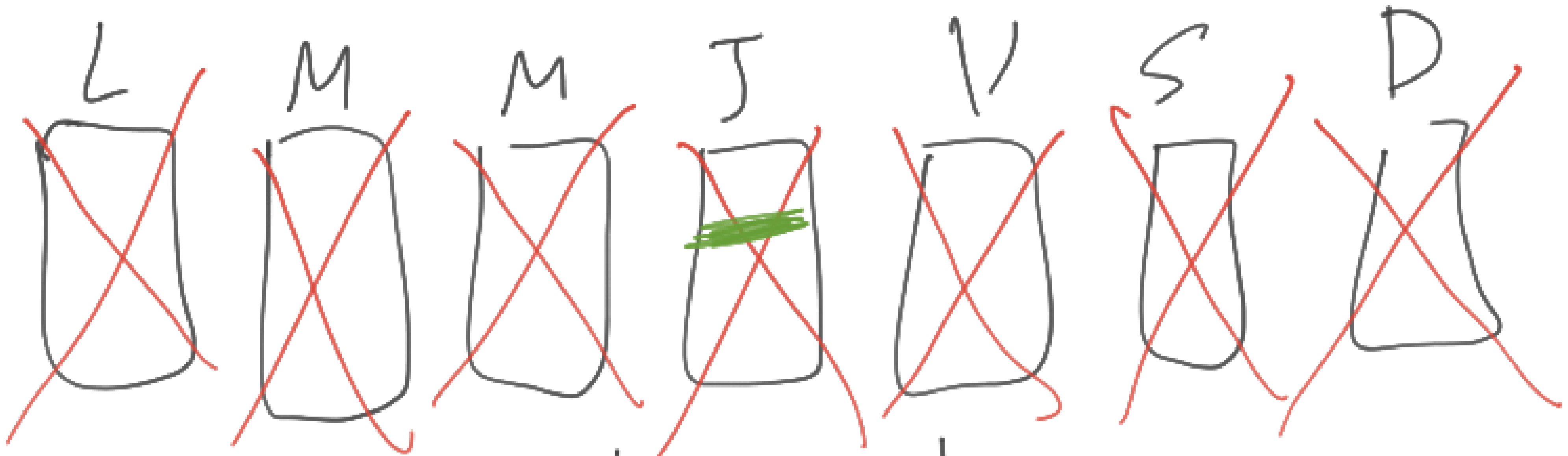
Then output "this x"

Either ZFC proves false things or it never proves

Godel: some true staments cannot be proven

Chaitin:

Unexpected security drill



next week

Real paradox?

Kritchman – Raz paradox

incompressible strings exist for every n

$|x| = n$ is incompressible if $K(x) \geq n$

2^n

shorter program $1 + 2 + \ldots + 2^{n-1} < 2^n$

Large n : no proof of $K(x_0) \geq n$

Unexpected drill ← the number of incompressible strings of length n

\# of day before the end

Assume only one string is incompressible

Prove: $\exists \geq 1$ incompressible

$\exists \geq 2$ incomp. .

...

$\exists \geq 2^n$ incompressible

a specific string is provably incompr.

Start looking for shorter programs for all other strings, wait until they are found.

All other strings are compressible and provably compressible

(N — large for Chaitin

→ "There are at least 1 / 2 / 3 incompr. strings of length N"

Assume $T \vdash \underbrace{\text{"there is at least } k \text{ incompr. strings"}}_{\varphi_k}$

~~Const +~~ $T \vdash$ "there is at least $k+1$ incompr. strings"

T'

Assume not. Then it has exactly k.
They are provably compressible
With φ_k we know that all other are provably incompressible
And no one should be (Chaitin, n large)
Contradiction

$T \vdash \varphi_k$

$\Downarrow$

$\mathrm{Con}\,T + T \vdash \varphi_{k+1}$

true

$\varphi_1 \rightsquigarrow \varphi_2$

provable in T

$\varphi_2 \rightsquigarrow \varphi_3$

false

$\rightsquigarrow \varphi_{2^n}$

false

$T + \mathrm{Con}\,T$, $T + \mathrm{Con}(T + \mathrm{Con}\,T)$, ←