RaCAF ANR-15-CE40-0016-01: Dépasser les frontières de l'aléatoire et du calculable (Randomness and Computability: Advancing the Frontiers)

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March 22, 2018

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- Combinatorics: randomness extractors

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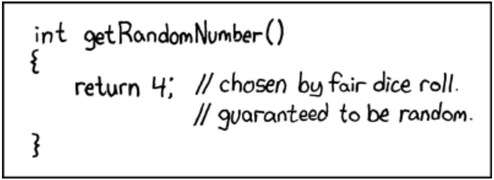

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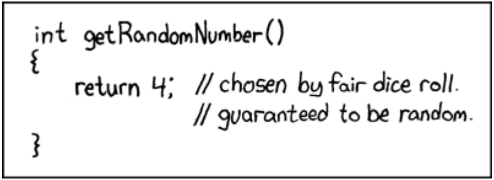

- of course, we usually speak about sequences, not individual digits

Tables of random numbers

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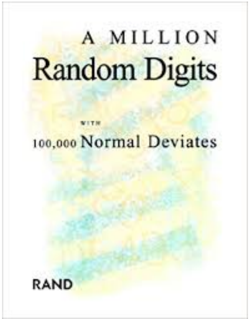

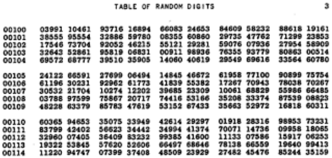

TABLE OF RANDOM DIGITS 3

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00108	03788	97599	75867	20717	74416	53166	35208	33374	87539	08823
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($600 hardcover, $41 paperback, $9 digital)

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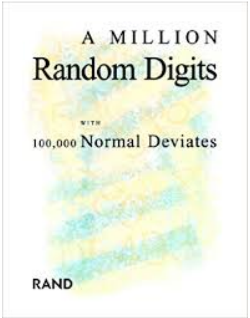

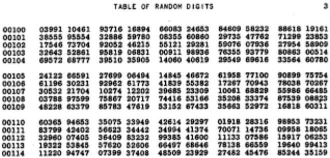
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can we complain to amazon if “non-random”?

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- (our main field of expertise)

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- why better than $x_{n+1} = x_n + 1 \bmod 2^{32}$?

Statistical tests

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- compressors as random tests: compression by k bits corresponds to p-value below 2^{-k}

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- conditional existence (factoring is hard, the existence of one-way functions)

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- solution attempt: “extracting randomness from weak randomness”

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- see below (and also pdf report) for more theoretical work

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- `diehard` uses dependent inputs when independence is required

dieharder documentation

speaks about "test failures"

> *Many dieharder tests, despite our best efforts, are numerically unstable or have only approximately known target statistics or are straight up asymptotic results, and will eventually return a failing result even for a gold-standard generator (such as AES), or for the hypercautious the XOR generator with AES, threefish, kiss, all loaded at once and xor'd together.* $\langle \ldots \rangle$ *Failure with numbers of psamples within an order of magnitude of the AES thresholds should probably be considered possible test failures, not generator failures. Failures at levels significantly less than the known gold standard generator failure thresholds are, of course, probably failures of the generator.*

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- almost as sensitive as the original test

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- equally sensitive if R_i are truly random

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- some preliminary results (M.Popov, master thesis under supervision of A.Romashchenko)

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- goal: add some “unpredictability” or “entropy” from physical sources
- “`/dev/random` typically blocks if there is less entropy available than requested”, “the generator keeps an estimate of the number of bits of noise in the entropy pool”, etc. (wikipedia)
- (naive) idea of “entropy” as some kind of liquid that can be measured, kept in a pool, etc. similar to caloric theory

RFC 4086

on extracting randomness from weak random source:

> *For an example of using a strong mixing function, reconsider the case of a string of 308 bits, each of which is biased 99% toward zero. The parity technique* $\langle \ldots \rangle$ *reduces this to one bit, with only a* 1/1000 *deviance from being equally likely a zero or one. But, applying the equation for information* $\langle \ldots \rangle$ *[Shannon entropy], this* 308*-bit skewed sequence contains over* 5 *bits of information. Thus, hashing it with SHA-1 and taking the bottom* 5 *bits of the result would yield* 5 *unbiased random bits and not the single bit given by calculating the parity of the string.*

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[Not justified: parity argument uses independence, and SHA-1 trick is not justified even in the independence case]

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- using $B[x][y]$ where B is a balanced matrix and x and y are independent weak random sources
- some preliminary experiments done

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- making experiments with existing physical randomness inputs (sound cards, physical devices) and analyzing their properties and ways to extract good random bits out of them
- last, but not least: theoretical work to understand properties of randomness (algorithmic information theory, computability theory approach to randomness, models of computation, randomness in game-theoretic approach to probability theory, etc.)

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- properties of randomness tests in algorithmic information theory (G.Novikov)
- detection of regularities and algorithmic statistics (A.Milovanov, N.Vereshchagin), presented at CCC 2017)

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- mutual information and its operational characterization (A.Romashchenko, with M.Zimand, Towson University)

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