Implementing the Core Chase for **Horn-$\mathcal{ALCH}$**

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2021

Entailment: a fundamental problem in knowledge representation

Knowledge base $K =$

Rules	**Facts**
$\alpha = Mother(x) \rightarrow Parent(x)$ $\beta = Parent(x) \rightarrow \exists y.IsTheParentOf(x, y)$ $\gamma = IsTheParentOf(x, y) \rightarrow Human(y)$	$Mother(\textbf{Marie})$

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Query $Q = \{IsTheParentOf(\textbf{Marie}, x), Human(x)\}$

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Conjunctive query entailment problem

Determine if the knowledge base K entails the query Q

Entailment: a fundamental problem in knowledge representation

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Definition

A factbase U is a universal model for a knowledge base K if for every query Q, K entails Q if and only if $U \vDash Q$.

One of the solution to resolve the conjunctive query entailment problem: The Chase

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Trigger $t_1 = (\alpha, \{x \mapsto \textbf{Marie}\})$

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Trigger $t_3 = (\gamma, \{x \mapsto \mathbf{Marie}, y \mapsto y_0\})$

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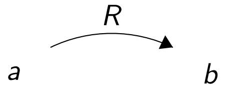
Query $Q = \{IsTheParentOf(\textbf{Marie}, x), Human(x)\}$

$$\bullet \xrightarrow{\text{IsTheParentOf}} y_0\text{: Human}$$

Marie: Mother, Parent

The oblivious chase

one rule: $\alpha = R(x, y) \rightarrow \exists z.R(y, z) \wedge R(z, y)$
one fact: $R(a, b)$



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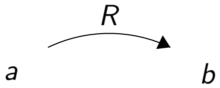

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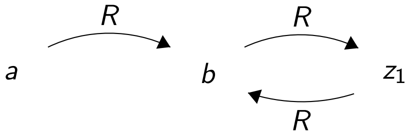

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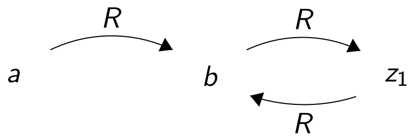

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$$\begin{tikzcd}
a \arrow[r, bend left, "R"] & b \arrow[r, bend left, "R"] & z_1 \arrow[l, bend left, "R"] \arrow[r, bend left, "R"] & z_2 \arrow[l, bend left, "R"]
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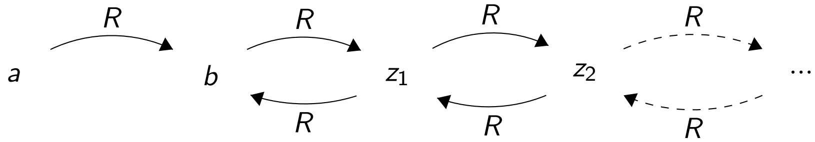

The core chase

Definition

A core of a factbase F is one of the minimal subset of F that is logically equivalent to F.

For example, if $F = \{B(x, y), R(y, z), B(x, w), R(w, z)\}$, then $Core(F) = \{B(x, y), R(y, z)\}$ because $\{w \mapsto y\}$ is a homomorphism from F to $Core(F)$.

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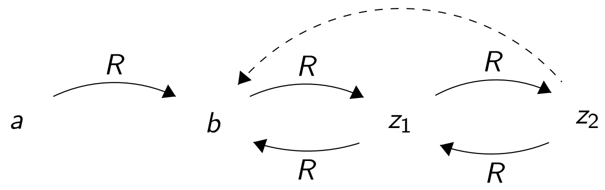

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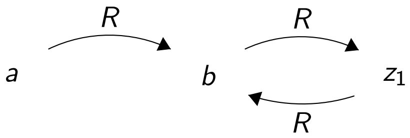

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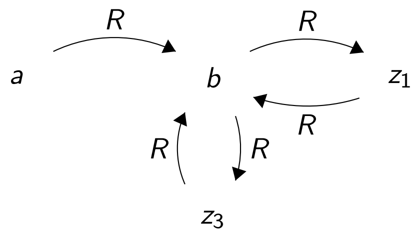

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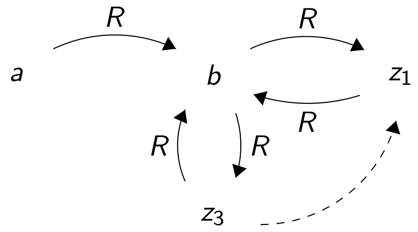

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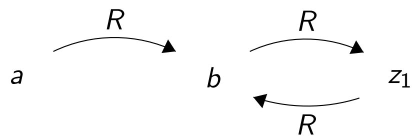


Some results

Theorem

For a knowledge base K, the result of the oblivious chase is an universal model of K.

Theorem

The knowledge base $K = (R, F)$ admits a finite universal model if and only if the core chase algorithm terminates on K.

Our restriction: **Horn-$\mathcal{ALCH}$**

The rules are of the form:

$$A(x) \land B(x) \to C(x)$$

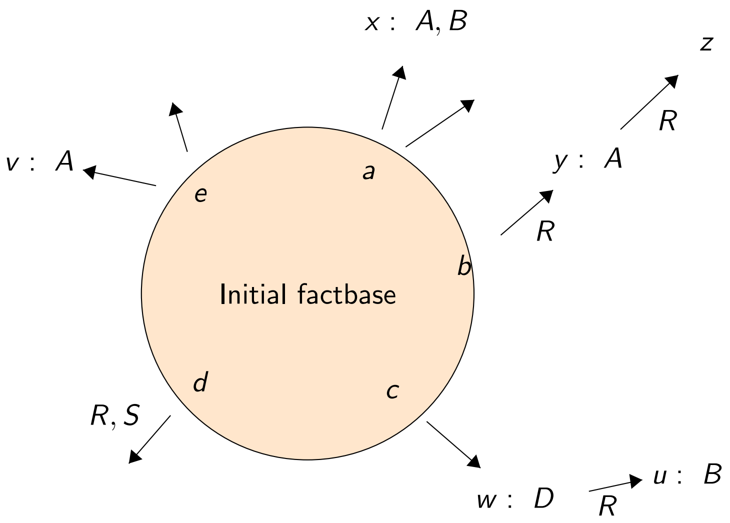
$$A(x) \land R(x,y) \to B(y)$$

$$R(x,y) \land B(y) \to A(x)$$

$$R(x,y) \land S(x,y) \to V(x,y)$$

$$A(x) \to \exists y.R(x,y) \land B(y)$$

Exemple



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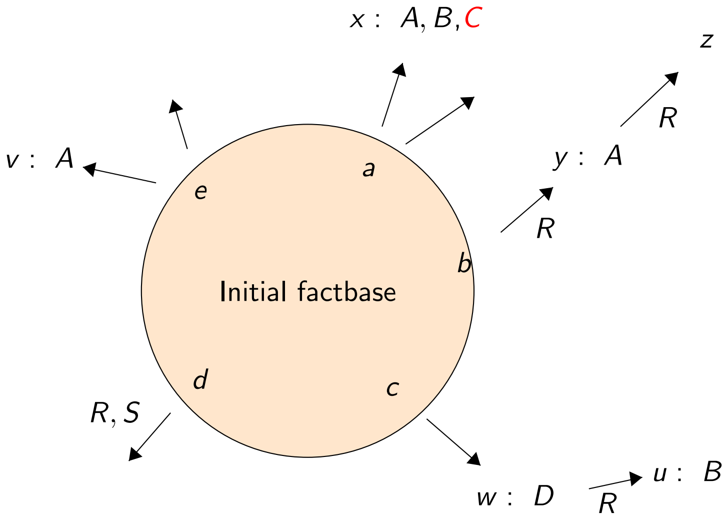
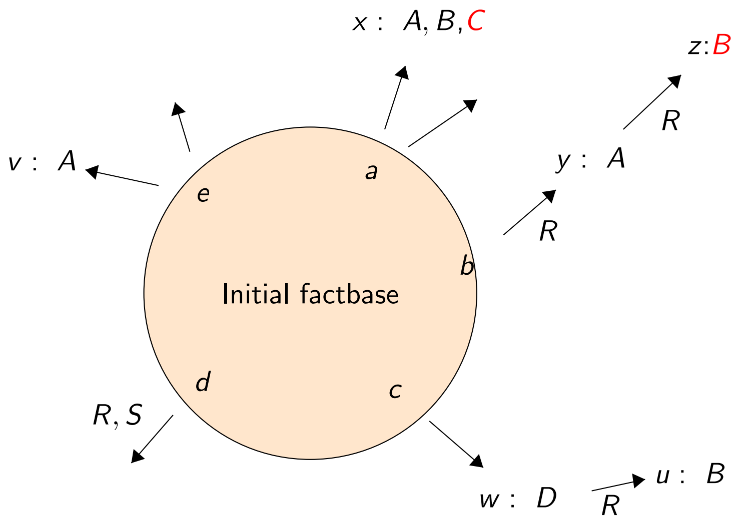


$$A(x) \wedge B(x) \rightarrow C(x)$$

Exemple



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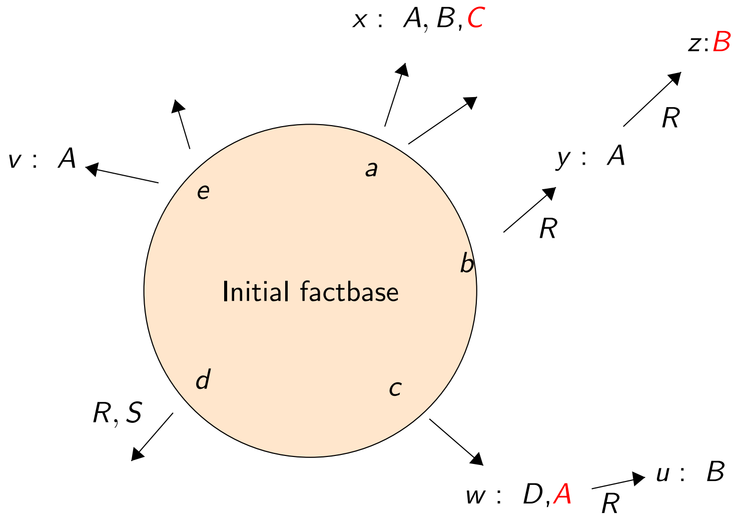
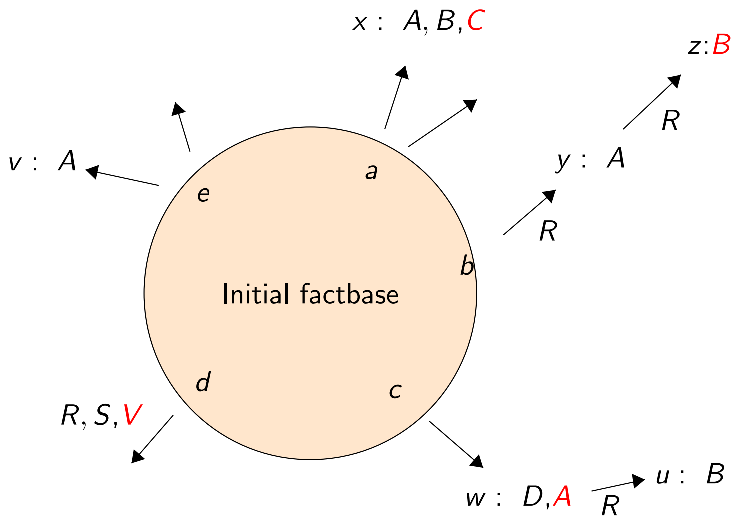


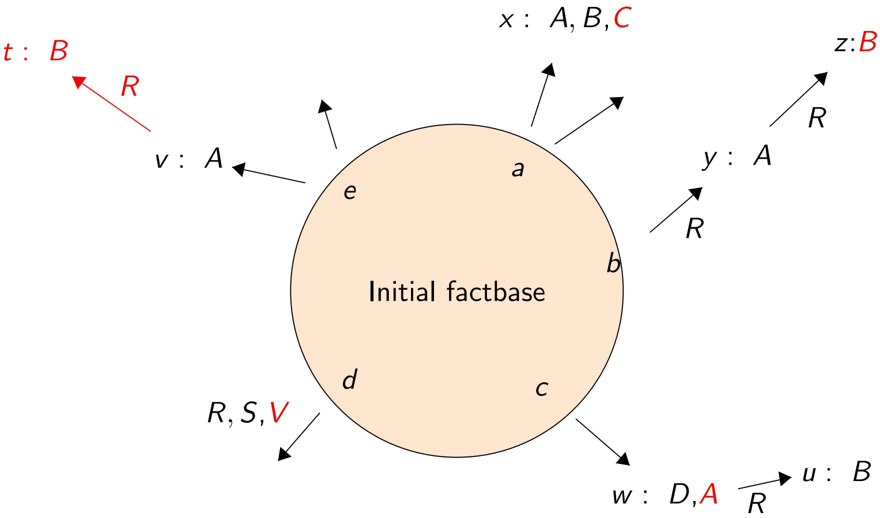
$$R(x,y) \wedge B(y) \rightarrow A(x)$$

Exemple



$$R(x,y) \wedge S(x,y) \rightarrow V(x,y)$$

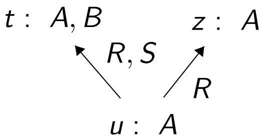
Exemple



$$A(x) \rightarrow \exists y.R(x,y) \wedge B(y)$$

A better algorithm in this restriction: The merge chase

The variable z is mergeable on the term t:



The variable z is mergeable on the term t:

$$\begin{tikzcd}
t : A, B & & z : A \\
& u : A \arrow[ul, "{R, S}"'] \arrow[ur, "R"'] &
\end{tikzcd}$$

Theorem

If the variable z is mergeable on the term t, the facts generated on z can be generated on t. (à reformuler)

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The variable z is mergeable on the term t:

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The variable z is mergeable on the term t:

$$R(x, y) \land B(y) \to A(x)$$

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If the variable z is mergeable on the term t, the facts generated on z can be generated on t. (à reformuler)

A better algorithm in this restriction: The merge chase

The variable z is mergeable on the term t:

$$R(x, y) \wedge S(x, y) \rightarrow V(x, y)$$

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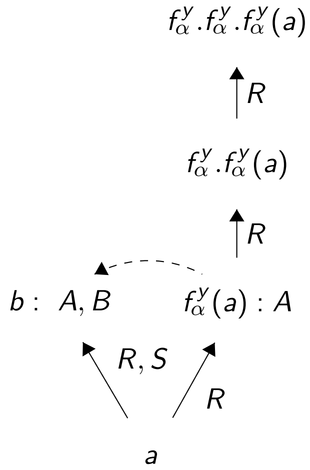
If the variable z is mergeable on the term t, the facts generated on z can be generated on t. (à reformuler)

A better algorithm in this restriction: The merge chase

The atomic merge



A better algorithm in this restriction: The merge chase

The atomic merge

$$
\begin{tikzcd}
f_\alpha^y.f_\alpha^y(b) & & f_\alpha^y.f_\alpha^y.f_\alpha^y(a) \\
f_\alpha^y(b) \arrow[u, dashed, "R"'] & & f_\alpha^y.f_\alpha^y(a) \arrow[u, "R"'] \\
b : \; A, B \arrow[u, dashed, "R"'] & & f_\alpha^y(a) : A \arrow[u, "R"'] \\
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\end{tikzcd}
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A better algorithm in this restriction: The merge chase

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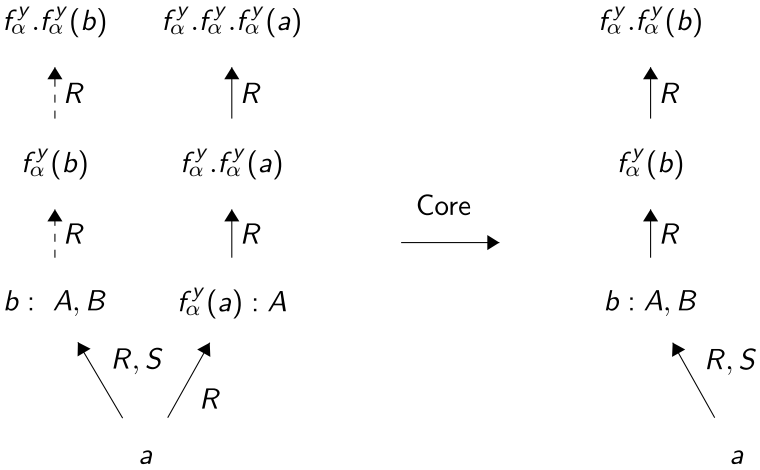

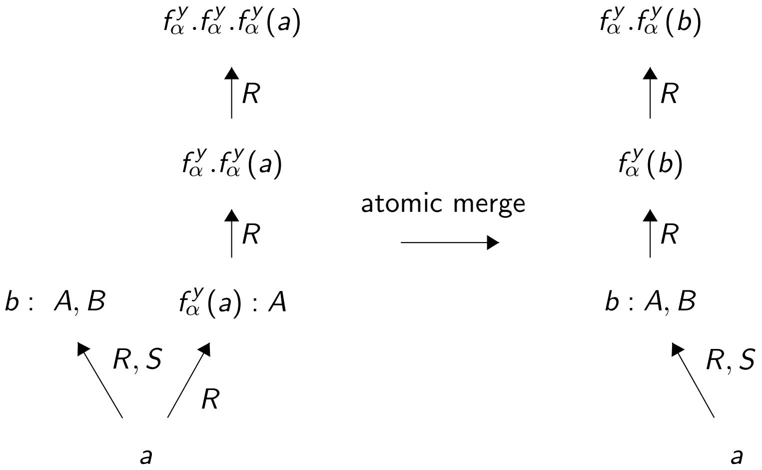

$f^y_\alpha.f^y_\alpha.f^y_\alpha(a)$
R
$f^y_\alpha.f^y_\alpha(a)$
R
$b:\ A, B$
$f^y_\alpha(a) : A$
R, S
R
a
atomic merge
$f^y_\alpha.f^y_\alpha(b)$
R
$f^y_\alpha(b)$
R
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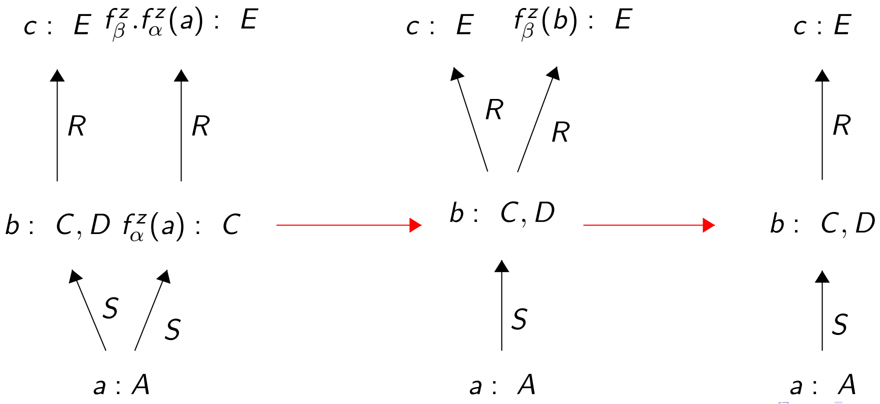
R, S
a

Definition

For a factbase F, $Merge(F)$ is obtained by applying all the possible atomic merging on F.



The merge chase

Theorem

For a factbase F, $Merge(F)$ is a core.

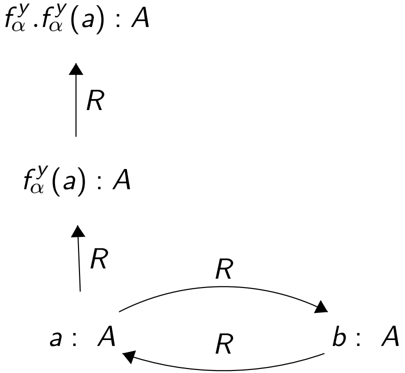
Definition

The merge chase for a **Horn-**$\mathcal{ALCH}$ knowledge base K is the core chase where we replace the computation of a core by the merge operation.

Corollary

The knowledge base $K = (R, F)$ admits a finite universal model if and only if the merge chase algorithm terminates on K.

the atomic merging operation has to be applied exhaustively



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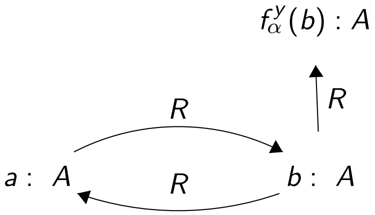

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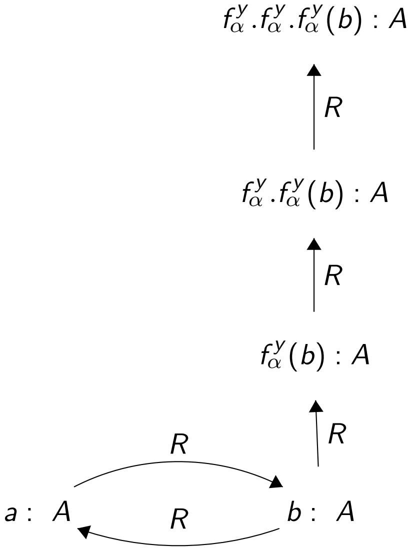

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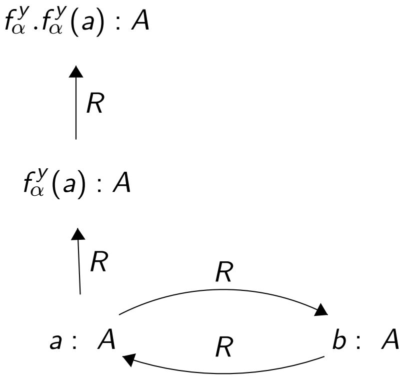

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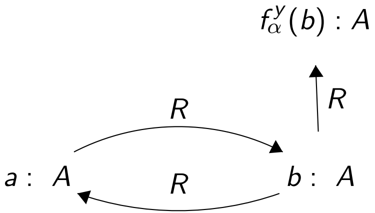

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$$\begin{tikzcd} a : A \arrow[r, bend left, "R"] & b : A \arrow[l, bend left, "R"] \end{tikzcd}$$

Conclusion

- Find **Horn-**$\mathcal{ALCH}$
- define the merge chase;
- prove that the merge chase works;
- Extend our work to **Horn-**$\mathcal{ALCHI}$: $R(x, y) \wedge S(x, y) \rightarrow T(y, x)$;
- Implement the merge chase to compare.