Semi-Automatic Modeling by Constraint Acquisition ⋆

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Abstract. Constraint programming is a technology which is now widely used to solve combinatorial problems in industrial applications. However, using it requires considerable knowledge and expertise in the field of constraint reasoning. This paper introduces a framework for automatically learning constraint networks from sets of instances that are either acceptable solutions or non-desirable assignments of the problem we would like to express. Such an approach has the potential to be of assistance to a novice who is trying to articulate her constraints. By restricting the language of constraints used to build the network, this could also assist an expert to develop an efficient model of a given problem. This paper provides a theoretical framework for a research agenda in the area of interactive constraint acquisition, automated modelling and automated constraint programming.

1 Introduction

Over the last 30 years, considerable progress has been made in the field of Constraint Programming (CP), providing a powerful paradigm for solving complex problems. Applications in many areas such as resource allocation, scheduling, planning and design have been reported in the literature [16]. However, the use of CP still remains limited to specialists in the field. Modelling a problem in the constraint formalism requires significant expertise in constraint programming. This precludes novices from being able to use CP on complex problems without the help of an expert. This has a negative effect on the uptake of constraint technology in the real-world by non-experts [5].

In addition, in many practical applications humans find it difficult to articulate their constraints. While the human user can recognize examples of where their constraints should be satisfied or violated, they cannot articulate the constraints themselves. However, by presenting examples of what is acceptable, the human user can be assisted in developing a model of the set of constraints she is trying to articulate. This can be regarded as an instance of constraint acquisition. One of the goals of our work is to assist the, possibly novice, human user by providing semi-automatic methods for acquiring the user's constraints.

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Furthermore, even if the user has sufficient experience in CP to encode her problem, a poor model can negate the utility of a good solver based on state-of-the-art filtering techniques. For example, in order to provide support for modelling, some solvers provide facilities for defining constraints extensionally (i.e., by enumerating the set of allowed tuples). Such facilities considerably extend the expressiveness and ease-of-use of the constraints language, thus facilitating the definition of complex relationships between variables. However, a disadvantage of modelling constraints extensionally is that the constraints lose any useful semantics they may have which can have a negative impact on the inference and propagation capabilities of a solver. As a result, the resolution performance of the solver can be significantly deteriorated in the parts of the problem where such constraints are used. Therefore, another goal of our work is to facilitate the expert user who wishes to reformulate her problem (or a part of it that is suspected of slowing down the resolution). Given sets of accepted/forbidden instantiations of the (sub)problem (that can be generated automatically from the initial formulation), the expert will be able, for instance, to test whether an optimised constraint library associated with her solver is able to model the (sub)problem in a way which lends itself to being efficiently solved.

However, constraint acquisition is not only important in an interactive situation involving a human user. Often we may wish to acquire a constraint model from a large set of data. For example, given a large database of tuples defining buyer behaviour in a variety of markets, for a variety of buyer profiles, for a variety of products, we may wish to acquire a constraint network which describes the data in this database. While the nature of the interaction with the source of training data is different, the constraint acquisition problem is fundamentally the same.

This paper makes the following contributions:

1. We present an approach to semi-automatically acquiring models of constraint satisfaction problems from examples;
2. We present an algorithm for acquiring such models, we prove its correctness and report on some empirical results which demonstrate some properties of our approach;
3. Finally, we give some insights into our research agenda in this area.

The remainder of this paper is organised as follows. Section 2 presents an overview of the related work in this area, Section 3 provides some preliminary definitions on constraint networks. Section 4 briefly presents the machine learning techniques that can be used for our problem. In Section 5, we formulate our problem as a learning problem. Section 6 presents the technique in detail, and proves some properties of the approach that are guaranteed. In Section 7, some of the issues that the approach raises are presented, and their possible effects on the learning process are illustrated by some preliminary experiments. A brief overview of the main topics in our research agenda are outlined in Section 8. Some concluding remarks are made in Section 9.

2 Related Work

Recently, researchers have become more interested in techniques for solving problems where users have difficulties articulating constraints. Padmanabhuni *et al.* have proposed a framework for learning constraints [12]. In [14], the goal of Rossi and Sperduti is not exactly to help the user learning a constraint network, but to help her learning the valuations of the tuples in a semi-ring constraint network where the constraint structures are already given. Freuder and Wallace have considered suggestion strategies for applications where a user cannot articulate all constraints in advance, but can articulate additional constraints when confronted with something which is unacceptable [3]. Freuder and O'Sullivan have focused on generating constraints which model tradeoffs between variables in problems which have become over-constrained during an interactive configuration session [2]. Version spaces have been reported by O'Connell *et al* for acquiring single constraints, with a focus on acquisition from humans where dialog length is a critical factor [9, 10]. The focus of their work was interactive acquisition of constraints from users of differing abilities. The motivation for their work comes from the field of design [11]. Page and Frisch have studied the issue of generalising constraints and PAC-learning of constrained atoms in an inductive logic programming context [13].

Furthermore, a considerable amount of work has been done in constraint reasoning to identify polynomial classes of constraint networks [1, 15]. Unfortunately, these polynomial classes are weak in terms of expressivity and are capable of encoding only small number of interesting real problems. These polynomial classes are based on static properties, and do not adapt themselves to the kind of problem the user wants to solve. We, therefore, have the problem of finding a problem which fits a polynomial class, and not the reverse. The approach proposed in this paper intends to be more flexible. It does not require that we define *a priori* classes in which the expressivity of a problem is to be tested, but lets the expert define it by herself. In this context we can imagine an interactive scenario in which the user tries more and more expressive libraries of constraints until her problem can be adequately expressed. While the manner in which the constraint language (library) is chosen is critical, it is outside the scope of this paper. However, we could use a case-based reasoning approach selecting libraries depending on the type of problem being tackled [5].

3 Preliminaries

Definition 1 (Constraint Network) *A constraint network is defined as a triplet* $(\mathcal{X}, \mathcal{D}, \mathcal{C})$ *where:*

- $\mathcal{X} = \{X_1, \ldots, X_n\}$ *is a set of variables.*
- $\mathcal{D} = \{D_{X_1}, \ldots, D_{X_n}\}$ *is the set of their domains: each variable* X_i *takes its values in the domain* D_{X_i}.
- $\mathcal{C} = (C_1, \ldots, C_m)$ *is a sequence of constraints on* $\mathcal{X}$ *and* $\mathcal{D}$*, where a constraint* C_i *is defined by the sequence* $var(C_i)$ *of variables it involves, and the relation* $rel(C_i)$ *specifying the allowed tuples on* $var(C_i)$.

We regard the constraints as a sequence to simplify the forthcoming notations.

Definition 2 (Instance) *Let* $Y = \{Y_1, \cdots, Y_k\}$ *be a subset of* $\mathcal{X}$*. An* instance e_Y *on* Y *is a tuple* $(v_1, \ldots, v_k) \in D_{Y_1} \times \cdots \times D_{Y_k}$*. This instance is partial if* $Y \neq \mathcal{X}$*, complete otherwise (noted* e*). An instance* e_Y *on* Y violates *the constraint* C_i *iff* $var(C_i) \subseteq Y$ *and* $e_Y[var(C_i)] \notin rel(C_i)$.

Definition 3 (Solution) *A complete instance on the set $\mathcal{X}$ of variables is a* solution *of the constraint network $N = (\mathcal{X}, \mathcal{D}, \mathcal{C})$ iff it does not violate any constraint. Otherwise it is a* non solution. *$Sol(N)$ denotes the set of solutions of N.*

4 The Fundamental Problem

As a starting point, we assume that the user knows the set of variables of her problem and their domains of possible values. She is also assumed to be able to classify an instance as positive (a solution) or negative (non-solution). Therefore, the available data are the set $\mathcal{X}$ of the variables of the problem, their domains $\mathcal{D}$, a subset E^+ of the solutions of the problem, and a set E^- of non-solutions.

In addition to the "assisting the expert" perspective, the aim is to code the problem efficiently, using only efficient constraint relations between these variables; i.e. a library of constraints with efficient propagation features is assumed to be given. Indications can also be given revealing the possible location of the constraints, by defining variables between which constraints must be found (learned), or by restricting ourselves to binary constraints only. These semantic and structural limitations define the inductive bias:

Definition 4 (Bias) *Given a set $\mathcal{X}$ of variables and the set $\mathcal{D}$ of their domains, a* bias $\mathcal{B}$ *on $(\mathcal{X}, \mathcal{D})$ is a sequence $(B_1, \ldots, B_m)$ of local biases, where a local bias B_i is defined by a sequence $var(B_i) \subseteq \mathcal{X}$ of variables, and a set $L(B_i)$ of possible relations on $var(B_i)$.*

The set $L(B_i)$ of relations allowed on a set of variables $var(B_i)$ can be any library of constraints of arity $|var(B_i)|$.

Definition 5 (Membership of a Bias) *Given a set $\mathcal{X}$ of variables and the set $\mathcal{D}$ of their domains, a sequence of constraints $\mathcal{C} = (C_1, \ldots, C_m)$* belongs *to the bias $\mathcal{B} = (B_1, \ldots, B_m)$ on $(\mathcal{X}, \mathcal{D})$ if $\forall C_i \in \mathcal{C}, var(C_i) = var(B_i)$ and $rel(C_i) \in L(B_i)$. We note $\mathcal{C} \in \mathcal{B}$.*

The problem consists in looking for a sequence of constraints $\mathcal{C}$ belonging to a given bias $\mathcal{B}$, and whose solution set is a superset of E^+ containing no element of E^-.

Definition 6 (Constraint Acquisition Problem) *Given a set of variables $\mathcal{X}$, their domains $\mathcal{D}$, two sets E^+ and E^- of instances on $\mathcal{X}$, and a bias $\mathcal{B}$ on $(\mathcal{X}, \mathcal{D})$, the* constraint acquisition problem *consists in finding a sequence of constraints $\mathcal{C}$ such that:*

$\mathcal{C} \in \mathcal{B}$,
$\forall e^- \in E^-$, e^- is a non solution of $(\mathcal{X}, \mathcal{D}, \mathcal{C})$, and,
$\forall e^+ \in E^+$, e^+ is a solution of $(\mathcal{X}, \mathcal{D}, \mathcal{C})$.

If the sets E^+ and E^-, called the *training data*, are provided by an interaction with the user, then the acquisition problem can be regarded as the modelling phase for the user's problem. Otherwise, it can be regarded as an assistance to the expert for an automatic reformulation of her problem.

We can point out that if $E^+ \cup E^- = D_{X_1} \times \cdots \times D_{X_n}$, and $\mathcal{B}$ is a bias on $(\mathcal{X}, \mathcal{D})$ containing $n(n-1)/2$ local biases such that for each pair of variables (X_i, X_j), $\exists B_i \in \mathcal{B}$

with $var(B_i) = (X_i, X_j)$, and $L(B_i) = \mathcal{P}(D_{X_i} \times D_{X_j})$,[1] then the constraint acquisition problem answers the representability problem of a relation $\rho = E^+$ with a binary constraint network [8].

5 Constraint Acquisition as Concept Learning

Concept induction is a well known paradigm in Machine Learning. The underlying problem can be described the following way: given a set H of hypotheses, two training data sets (E^+ of positive and E^- of negative instances), find an hypothesis h consistent with this training data, i.e., which rejects all the negative instances and accepts all the positive instances. The concept providing the training data is called the target concept.

In our context, this concept is the unknown network that we are looking for that consistently captures all the information given by the user in the training set. So, in our vocabulary:

- An hypothesis h is a sequence of constraints,
- H is the set of possible sequences of constraints belonging to $\mathcal{B}$,
- The target concept is the sequence of constraints $\mathcal{C}$ we are looking for,
- A positive instance is a solution of $(\mathcal{X}, \mathcal{D}, \mathcal{C})$, a negative one is a non-solution of $(\mathcal{X}, \mathcal{D}, \mathcal{C})$.

There are many techniques from the field of Machine Learning, from decision trees to neural networks or genetic algorithms. We propose here a method based on version spaces [7], which has several nice properties amongst which the most interesting from our perspective are: they provide two approximations of the target concept, an upper bound and a lower bound; their computation is incremental with respect to the training data; and the result does not depend on the order of the instances in the training set (commutativity). This last property is essential in an interactive acquisition process.

We briefly present version spaces, which rely on the partial-order based on inclusion in the set H of hypotheses.

Definition 7 (Generalisation relation $\leq_G$) *Given $(\mathcal{X}, \mathcal{D})$ a set of variables and their domains, an hypothesis h_1 is less general than or equal to an hypothesis h_2 (noted $h_1 \leq_G h_2$) iff the set of solutions of $(\mathcal{X}, \mathcal{D}, h_1)$ is a subset of this of $(\mathcal{X}, \mathcal{D}, h_2)$.*

A version space does not only provide one consistent hypothesis, but the whole subset of H consistent with the training data:

Definition 8 (Version Space) *Given $(\mathcal{X}, \mathcal{D})$ a set of variables and their domains, E^+ and E^- two training data sets , and H a set of hypotheses, the* version space *is the set:*

$$V = \{h \in H / E^+ \subseteq Sol(\mathcal{X}, \mathcal{D}, h), E^- \cap Sol(\mathcal{X}, \mathcal{D}, h) = \emptyset\}$$

Because of its nice property of incrementality with respect to the training data, a version space is learned by incrementally processing the training instances of E^+ and E^-. In addition, due to the $\leq_G$ partial order, a version space V is completely characterised by two boundaries: the specific boundary S of maximally specific (minimal) elements of V (according to $\leq_G$), and the general boundary G of maximally general (maximal) elements.

[1] E being a set, $\mathcal{P}(E)$ is the set of subsets of E.

Property 1 *Given a version space V, and its boundaries S and G, $\forall h \in V, \exists s \in S$ and $\exists g \in G / s \leq_G h \leq_G g$.*

In the general case, V is exponential in the size of the data. So, thanks to Property 1, the constraint acquisition problem is restricted to computing the bounds S and G of the version space consistent with (E^+, E^-).

Given a set of hypotheses H on $(\mathcal{X}, \mathcal{D})$, and the training data (E^+, E^-), if there does not exist any $h \in H$ consistent with (E^+, E^-), then the version space acquisition will finish in a state where there exists $s \in S$ and $g \in G$ such that $s \neq g$ and $g \leq_G s$. This is called the *collapsing* state of the version space.

6 Learning the Constraint Version Space

In this section, we describe the process of learning the version space corresponding to the constraint acquisition problem on $(\mathcal{X}, \mathcal{D})$ with the two training sets (E^+, E^-) of solutions and non-solutions, and bias $\mathcal{B}$ on $(\mathcal{X}, \mathcal{D})$.

Let us first define the concepts that will be used at the single constraint level. We can project the generalisation relation $\leq_G$ at the constraint level. To be completely consistent with version space theory, we define $L^H(B_i) = L(B_i) \cup \{\perp, \top\}$, where $\perp$ is the empty relation $\perp$, and $\top$ the universal relation. Note that without loss of generality, the universal relation can be stated as belonging to any library of constraints. Thus, $\leq_g$ is a partial order on $L^H(B_i)$ such that $\forall r_1, r_2 \in L^H(B_i), r_1 \leq_g r_2 \Leftrightarrow r_1 \subseteq r_2$.

Given $L_1 \subseteq L^H(B_i)$ and $L_2 \subseteq L^H(B_i)$, we note that $L_1 \leq_g L_2$ iff $\forall r_1 \in L_1, \forall r_2 \in L_2, r_1 \leq_g r_2$.

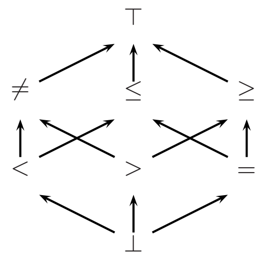


Fig. 1: $L^H(B_i)$

Example 1 Let $L(B_i) = \{<, \leq, =, \geq, >, \neq\}$ be a given local bias. Fig. 1 shows the set $(L^H(B_i), \leq_g)$, which in this case is a lattice.

Restricting ourselves to each constraint individually, we introduce a local version space for each local bias. Because $\leq_g$ is, like $\leq_G$, a partial order, each local version space inherits Property 1. Thus, each local version space is completely characterized by its own local specific and general boundaries.

Definition 9 (Local boundaries) *$L(S_i)$ (resp. $L(G_i)$) is the set of relations of $L^H(B_i)$ which appear in an element of S (resp. G):*

$$L(S_i) = \{r \in L^H(B_i) / \exists s \in S : (var(Bi), r) \in s\}$$

$$L(G_i) = \{r \in L^H(B_i) / \exists g \in G : (var(Bi), r) \in g\}$$

S_i and G_i are the corresponding sets of constraints:

$$S_i = \{(var(B_i), r)\},\ \textit{where}\ r \in L(S_i); \qquad G_i = \{(var(B_i), r)\},\ \textit{where}\ r \in L(G_i)$$

6.1 The learning process

We are now ready to describe the CONACQ algorithm (Algorithm 1), which takes as input two training sets E^+, E^-, and returns the corresponding version space V on the bias $\mathcal{B}$. We present step by step the different scenarios that can occur when a training instance is processed.

Instances from $\boldsymbol{E^+}$ A positive instance e^+ must be a solution of all the networks $(\mathcal{X}, \mathcal{D}, h)$ for which $h \in V$. So,

$$\forall h \in V, \forall C_i \in h, e^+[var(C_i)] \in rel(C_i)$$

Projecting onto the local version spaces of each local bias B_i, we obtain the following property:

Property 2 (Projection property of S_i's) *Each local specific boundary S_i must accept all the positives instances. $L(S_i)$ is thus the set of maximally specific relations (minimal w.r.t. $\leq_g$) of $L(B_i)$ that accept all E^+:*

$$L(S_i) = min_{\leq_g}\{r \in L^H(B_i) / \forall e^+ \in E^+, e^+[var(B_i)] \in r\}$$

Corollary 1 *The specific boundary S is the Cartesian product of the local specific boundaries S_i's, i.e., the set of hypotheses, where each constraint takes its relation from $L(Si)$:*

$$S = \bigtimes_{i \in 1..m} S_i$$

From Property 2, when a positive instance e^+ is presented, each local bias B_i can be processed individually (line 2 of Algorithm CONACQ). If the specific boundary of a constraint already accepts this positive instance, it is skipped (line 3), else the boundary goes up to the most specific relations of the local version space (i.e., the relations of $L^H(B_i)$ between $L(S_i)$ and $L(G_i)$) that accept e^+ (line 4). If no such relation exists, this means that no hypothesis can accept this positive instance. Then, the algorithm terminates since a collapsing state has been encountered (line 5).

Instances from $\boldsymbol{E^-}$ A negative instance e^- must be a non solution for all the networks $(\mathcal{X}, \mathcal{D}, h)$ where $h \in V$. So,

$$\forall h \in V, \exists C_i \in h / e^-[var(C_i)] \notin rel(C_i)$$

Since at least one violated constraint is sufficient for an instance to be regarded as negative, instead of all satisfied constraints necessary in the case of a positive instance, G does not have the projection property exhibited by S:[2]

[2] If this was not the case the constraint defined on $rel(B_i)$ would be sufficient to reject all negative instance of E^-.

$$L(G_i) \neq max_{\leq_g}\{r \in L^H(B_i)/\forall e^- \in E^-, e^-[var(B_i)] \notin r\}$$

We can only say that $\forall e^- \in E^-, \exists i/\forall r \in L(G_i), e^-[var(B_i)] \notin r$. However, the cause of the rejection (which constraint(s) has been violated) may not be obvious. Furthermore, storing only the local general boundaries G_i's is not sufficient to express this uncertainty.

The traditional approach to version space learning involves storing the set of all possible global boundaries G. However, this can require exponential space and time [4]. In order to ensure our algorithm remains polynomial, we do not store this set, but encode each negative instance e^- as a clause, Cl. Each constraint that could possibly be involved in the rejection of a negative example e^- will be encoded as a meta-variable in the clause Cl. Semantically, the clause Cl represents the disjunction of the possible explanations for the rejection of this negative instance. In other words, it encodes a disjunction of the constraints that could have been responsible for the inconsistency in the instance.

When a negative instance e^- is presented, a new clause, initially empty, is built by processing each local bias B_i one-by-one (lines 12-14). Those biases whose specific boundary, $L(S_i)$, already accepts the negative instance, e^-, are skipped (line 15). The reason being that S_i is the maximally specific boundary of the local version space for B_i and, by definition, we know that at each step of the learning process the constraint defined on $rel(B_i)$ cannot be involved in the rejection of e^-, since this has already been deemed acceptable by at least one positive example.

For all the other constraints, a subset of which is responsible for the rejection e^-, we compute A_i, the subset of maximally specific relations (w.r.t. $\leq_g$) between $L(S_i)$ and $L(G_i)$ which accept $e^-[var(B_i)]$, i.e. the least upper bound that B_i must not take if it is proven to be a contributor to the rejection of e^- (line 16). Depending on this set of relations we have two alternative courses of action to consider. Firstly, if the set A_i is empty it means that all possible relations for the constraint defined on $rel(B_i)$ already reject e^-. Therefore, every hypothesis in the version space is consistent with e^- so there is nothing to do for e^-; we are then ready to process the next instance (line 17). Secondly, if A_i is not empty, we add the meta-variable $(L(G_i) <_g A_i)$ to the clause Cl (line 18). The semantic of this meta-variable is *"if B_i is involved in the rejection of e^-, then G_i must be made more specific than A_i"*.

To reject e^-, a sequence of constraints h must satisfy at least one meta-variable in the clause Cl encoding e^-. We will denote this as $h \models Cl$. The set of clauses, characterizing all the set E^-, is denoted as $\mathcal{K}$. Below we will summarise the maintenance of these clauses for clarity.

Maintenance of the set of clauses

i Maintenance of clauses when positive instances are added.
If a clause Cl contains the meta-variable $(L(G_i) <_g A_i)$ and if after the processing of positive instances, $L(S_i) \cap A_i$ becomes not empty, then the relations in $L(S_i) \cap A_i$ are no longer a possible explanation for the rejection of the negative instance denoted by Cl. Let A'_i be the set of relations of A_i that are not in $L(S_i)$. If A'_i is not empty, the new explanation becomes $(L(G_i) <_g A'_i)$. Otherwise the meta-variable $(L(G_i) <_g A_i)$ is erased from Cl (lines 6-9).

Algorithm 1: The CONACQ Algorithm

$\mathcal{K} \leftarrow \emptyset$
foreach B_i **do** $L(S_i) \leftarrow \{\bot\};\ L(G_i) \leftarrow \{\top\}$
foreach *training instance e* **do**
1 **if** $e \in E^+$ **then**
2 **foreach** B_i **do**
3 **if** $\exists r \in L(S_i)/e[var(B_i)] \notin r$ **then**
4 $L(S_i) \leftarrow min_{\leq_g}\{r/S_i \leq_g r \leq_g G_i$ and $e[var(B_i)] \in r\}$
5 **if** $L(S_i) = \emptyset$ **then** "collapsing"
6 **foreach** $Cl/(L(G_i) <_g A_i) \in Cl$ *and* $A_i \cap L(S_i) \neq \emptyset$ **do**
7 $Cl \leftarrow Cl \setminus (L(G_i) <_g A_i)$
8 $A'_i \leftarrow A_i \setminus L(S_i)$
9 **if** $A'_i \neq \emptyset$ **then** $Cl \leftarrow Cl \cup (L(G_i) <_g A'_i)$
10 **if** $Cl = \emptyset$ **then** "collapsing"
11 **if** $Cl = \{(L(G_i) <_g A_i)\}$ **then**
$G_i \leftarrow max_{\leq_g}\{r/S_i \leq_g r \leq_g G_i$ and $r <_g A_i\}$; $\mathcal{K} \leftarrow \mathcal{K} \setminus Cl$

12 **if** $e \in E^-$ **then**
13 $Cl \leftarrow \emptyset$; $reject \leftarrow false$
14 **foreach** B_i ***while*** $\neg reject$ **do**
15 **if** $\exists r \in L(S_i)/e[var(B_i)] \notin r$ **then**
16 $A_i \leftarrow min_{\leq_g}\{r/S_i \leq_g r \leq_g G_i$ and $e[var(B_i)] \in r\}$
17 **if** $A_i = \emptyset$ **then** $reject \leftarrow true$
18 **else** $Cl \leftarrow Cl \cup (L(G_i) <_g A_i)$

if $\neg reject$ **then**
19 **if** $Cl = \emptyset$ **then** "collapsing"
20 **if** $Cl = \{(L(G_i) <_g A_i)\}$ **then**
$G_i \leftarrow max_{\leq_g}\{r/S_i \leq_g r \leq_g G_i$ and $r <_g A_i\}$; $\mathcal{K} \leftarrow \mathcal{K} \setminus Cl$
21 **else if** $\nexists Cl'/Cl' \subseteq Cl$ **then** $\mathcal{K} \leftarrow \mathcal{K} \cup Cl$
22 **foreach** $Cl''/Cl \subset Cl''$ **do** $\mathcal{K} \leftarrow \mathcal{K} \setminus Cl''$

ii Empty clause implies that the version space has collapsed.

When a clause Cl, encoding a negative instance e^-, is empty, either during its construction (line 19), or following the addition of some positive instances (line 10), it implies that there does not exist a possible explanation for rejecting the negative instance e^-, then the algorithm collapses.

iii The one-meta-variable clauses.

The one-meta-variable clauses represent the only possible explanation for the rejection of the coded negative instances. They must be reported on the local version space then removed from $\mathcal{K}$. Let $(L(G_i) <_g A_i)$ be a such clause: $L(G_i)$ must be made more specific than A_i.(lines 11 and 20).

iv Subsumption of negative instances.

A negative instance e_1^- (encoded by Cl_1) subsumes e_2^- (encoded by Cl_2) iff $Cl_1 \subseteq Cl_2$. Indeed, if Cl_1 is satisfied, then Cl_2 too. It is thus useless to store Cl_2. Lines 21-22 of Algorithm CONACQ maintain this minimality of the base of clauses $\mathcal{K}$.

Example 2 In Figure 2, we describe the learning process on a single constraint C_{12} and where B_{12} is the bias of Figure 1. Initially $L(S_{12}) = \{\bot\}$, and $L(G_{12}) = \{\top\}$. When the positive instance $e_1^+ = (1,1)$ is received, $L(S_{12})$ goes up in $L^H(B_{12})$ to the most specific relations accepting this tuple (a), namely $\{=\}$. We know now that the final relation of constraint C_{12} will be more general than or equal to "$=$". When the negative instance $e_2^- = (2,1)$ is received, it is necessary to restrict $L(G_{12})$ such that it will reject this tuple (b). At this step: $L(S_{12}) = \{=\}$, and $L(G_{12}) = \{\leq\}$. The negative instance $e_3^- = (0,3)$ forbids the relation "$\leq$" (c). Then $L(S_{12}) = L(G_{12}) = \{=\}$, that is a local convergence.

E	X_1	X_2
e_1^+	1	1
e_2^-	2	1
e_3^-	0	3

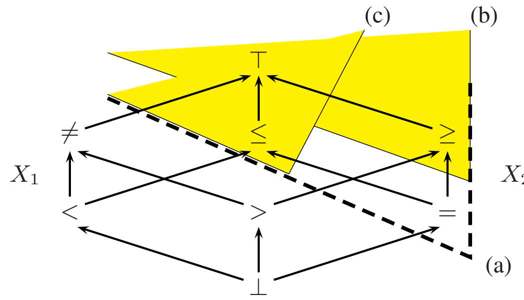
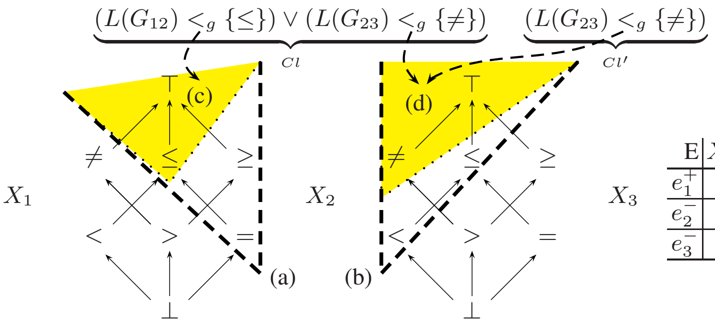


Fig. 2: A local example

Example 3 In Figure 3, we present an example of our algorithm using a constraint network involving three variables X_1, X_2 and X_3. The biases used are $\{B_{12}, B_{23}\}$, where $var(B_{12}) = (X_1, X_2)$, $var(B_{23}) = (X_2, X_3)$ and $L^H(B_{12}) = L^H(B_{23})$, which for the purposes of this example we will assume to be the same as that presented as $L^H(B_i)$ in Figure 1. When processing the positive example $e_1^+ = (2,2,5)$, due to the projection property of S, S_{12} **and** S_{23} go up (i.e., $L(S_{12}) \leftarrow \{=\}$ and $L(S_{23}) \leftarrow \{<\}$) (a) and (b).

However, since G does not have this projection property: when $e_2^- = (1,3,2)$ is received, either C_{12} must reject the tuple $(1,3)$ **or** C_{23} must reject $(3,2)$. Therefore, we build the clause $Cl = (rel(C_{12}) <_g^L \{\leq\}) \vee (rel(C_{23}) <_g^L \{\neq,\})$ to store these alternatives (c) or (d). When the negative instance $e_3^- = (1,1,0)$ is received, we know that the constraint C_{12} is not involved in its rejection, because $e_3^-[var(C_{12})] = (1,1)$ is an allowed tuple of $L(S_{12}) = \{=\}$. The only explanation for the rejection of e_3^- is $Cl' = (rel(C_{23}) <_g^L \{\neq,\})$ (d). Note than Cl' subsumes Cl (i.e., $Cl' \subseteq Cl$). The explanation of the rejection of e_3^- (d) is also a valid one for e_2^-: Cl becomes subsumed by Cl', and thus is discarded (the explanation (c) is discarded too). After these three instances, the CONACQ result is $L(S_{12}) = \{=\}$, $L(G_{12}) = \{\top\}$, C_{23}: $L(S_{23}) = \{<\}$ and $L(G_{23}) = \{\leq\}$.



E	X_1	X_2	X_3
e_1^+	2	2	5
e_2^-	1	3	2
e_3^-	1	1	0

Fig. 3: A global example

6.2 Correctness of Algorithm CONACQ

The theorem of representability [7] says that if cn the target network belongs to H (the hypothesis space) and if S and G are the minimal and maximal boundaries of the version space wrt the generalisation order $\leq_G$, then at every step of training, cn is such that $\exists s \in S, \exists g \in G / s \leq_G cn \leq_G g$.

To ensure the correctness of Algorithm CONACQ, we have only to prove that S and G are the minimal and maximal boundaries of V. We just give a sketch of the proof. For S, this is easy to prove because of the projection property of S. The global S is the union of the local S_i's. And by construction, each local S_i is the minimal boundary of V on $var(S_i)$. For G, this is more complex because G is not equivalent to the union of its local projections. Remember that because of its space complexity, we do not store G in extension but only its clause characterisation $\mathcal{K}$. To be in G, an hypothesis h must reject all negative instances, i.e., $h \models \mathcal{K}$. It is an implicant[3] of $\mathcal{K}$. In addition, h must be as general as possible wrt $\leq_G$, which means that it satisfies as few meta-variables as possible. It is thus a prime implicant of $\mathcal{K}$:

$$G = \{g \in \bigtimes_{i \in 1..m} G_i / g \models^\star \mathcal{K}\}$$

Now, an hypothesis $h \in G$ must be such that $E^- \cap Sol(\mathcal{X}, \mathcal{D}, h) = \emptyset$, and such that $\nexists h' \neq h / h \leq_G h'$ and $E^- \cap Sol(\mathcal{X}, \mathcal{D}, h') = \emptyset$. Suppose there exists such a h'. Since it rejects all the negative instances, we have $h' \models \mathcal{K}$ by construction of $\mathcal{K}$. And then, h would not be a prime implicant, which contradicts the assumption.

Since our S and G have the required properties, the following corollary applies:

Corollary 2 ([7]) *Given a target network cn and a space H of hypotheses, the version space collapses iff $cn \notin H$, which means that the selected bias is not sufficient.*

7 Experiments and Observations

We report here on some preliminary experiments to evaluate the learning capabilities of our approach. Rather than focusing on techniques for minimising the number of inter-

[3] A term i is an implicant of a set of clauses B iff $i \models B$. It is a prime implicant, noted $i \models^\star B$, iff for each other implicant i', $i' \not\subset i$.

actions, our focus here is on studying a number of properties of the CONACQ algorithm which provide motivation for our research agenda.

We performed experiments with a simulated teacher, which plays the role of the user, and a simulated learner that uses the algorithm presented earlier. The teacher has the knowledge of a randomly generated (target) network, represented by the triple $< 50, 8, C >$, defining a problem involving 50 variables with domains $\{1, ..8\}$, and a number C of constraints. Each constraint is randomly chosen from the bias $\{<, =, >, \leq, \neq, \geq\}$. The teacher provides the learner with solutions and non solutions. The learner acquires a version space for the problem using CONACQ algorithm.

7.1 Experiment 1: Effect of the order of the instances

In this following experiment, we assess aspects of the runtime characteristics of the CONACQ algorithm. In particular, we study computing time and the size of the version space, while varying the order in which examples are presented. Instances from a set E of size 100 are given by the teacher to the learner based on a $< 50, 8, 50 >$ network. The set E contains 10 positive and 90 negative instances.

Table 1 presents the time needed by the learner to acquire the version space, V, for the example set while varying the arrival time of the 10 positive instances. The positive instances were presented at the beginning (a), middle (b), and end (c) of the interaction between teacher and learner.

Table 1: Effect of the timing of the introduction of positive instances

Introduction time for positives	0 (a)	50 (b)	90 (c)
Computing time (in sec.)	3.3	5.1	8.6
$log(\|V\|)$	2,234	2,234	2,234

We observe that *"the sooner, the better"* seems to be the good strategy for the introduction of positive instances. Indeed, the specific bound S rises quickly in the space of hypotheses with positive instances, reducing the size of the version space. Because of that, the CPU time needed is also reduced when positive instances arrive at the beginning. But we can see that the final size of the version space is not affected by the order of the instances. This is due to the commutativity property of version spaces, discussed in Section 5.

7.2 Experiment 2: Utility of partial instances

In some cases, the user can reject an instance while justifying it by a negative sub-instance. For example, in a real-estate setting the customer (teacher) might reject an apartment citing the reason that *"this living-room is too small for me"*. The estate agent (learner) knows that the violation is due to the variables defining the living room, which can being very helpful for handling negative examples. The utility of such justified rejections can be measured by providing our learner with partial instances. In the following experiment (Table 2), the teacher provides the learner with 90 partial negative instances (after 10 complete positive ones) in the training data. We consider partial instances involving 2, 5, 10 variables, and report the size of the version space and of the set of clauses after 100 instances have been given.

Table 2: Effect of the partial instances

Nb of variables involved in instances of E^-	50	10	5	2
$log(\|V\|)$	2,234	2,233	2,225	2,144
$\|K\|$ (10^4 meta-variables)	7.6	6.1	3.2	0

We observe that partial instances speed up the process of convergence of the version space. The smaller these partial instances are, the more helpful they are. This opens a promising way of helping the learning process: asking the user to justify why she rejects some instances can assist in reducing the length of the dialog with the teacher. This is a critical issue if we are learning in an interactive setting from a human user.

7.3 Experiment 3: Non-representable constraints and version space collapse

We have seen that our algorithm learns a network expressed using a given bias. An overly restrictive bias leads to version space collapse since it is unlikely to be capable of expressing the target network. This happens because some of the constraints of the target network are non-representable in the given bias. In the following experiment, (Table 3), we analyse the effect on the speed of version space collapse by varying the proportion of non-representable constraints in the target network. For each proportion of non-representable constraints we learned 100 different $< 50, 8, 50 >$ target networks. Non-representable constraints, generated randomly, were not members of our bias $\{<, =, >, \leq, \neq, \geq\}$. The table presents the number of times the version space collapsed after 1,000 instances where presented. Note that the focus of this experiment is related to automatic reformulation rather than interactive modelling.

Table 3: Effect of non-representable constraints

Ratio of non representable constraints	10%	25%	50%
% of collapsing	47	78	100

We observe that the more non-representable constraints in the target network, the faster the version space collapses. This is to be expected since version spaces are sensitive to noise (errors) in the training data. However, an interesting observation was made during this analysis. On a number of constraint networks containing very few non-representable constraints we observed that the version space did not collapse, even after millions of instances were presented. This seems to violate Corollary 2. However, what occured in these situations was that the non-representable constraints in the target *become* representable by propagation of other constraints. As an example, consider three variables X_1, X_2, and X_3 linked by the following constraints in the target network: $X_1 = X_2$, $X_2 = X_3$, and $(X_1 = X_3) \vee (X_1 = X_3 + 5)$. Obviously, the constraint between X_1 and X_3 is not representable in the bias $\{<, =, >, \leq, \neq, \geq\}$, and every network containing it is expected to cause the version space to collapse. However, this is not the case in this example, because we can restrict the constraint between X_1 and X_3 to $X_1 = X_3$ by transitivity. This transitive closure constraint is representable.

It is worth noting that if we had used a technique that learned each constraint in the network separately, the version space for our problem would have collapsed had we attempted to acquire the constraint between X_1 and X_3.

7.4 Observation: Implicit constraints

The general phenomenon of constraints that can be inferred by other constraints can have another effect, which is to prevent the version space from converging to the smallest possible on each constraint taken separately. Consider again an example with three variables X_1, X_2 and X_3, linked in the target network by the constraints $X_1 = X_2$, $X_2 = X_3$, and $X_1 = X_3$. Furthermore, consider what occurs at some point in the learning process when the version space local to the constraint defined on (X_1, X_3) contains both $X_1 = X_3$ and $X_1 \leq X_3$. If the training data contains only complete instances, it is impossible to converge to the constraint $X_1 = X_3$ because every negative instance that would permit us to discard $X_1 \leq X_3$ from the version space (e.g., $((X_1, 1), (X_2, 2), (X_3, 2)))$ will also be rejected by $X_1 = X_2$ or by $X_2 = X_3$. Thus, the boundary G will never determine that culpability lies with $X_1 \leq X_3$.

Applying some levels of local consistency seems to be a promising approach to improving the reduction of the version space, by adding implicit constraints to the learned network. In the previous example, path-consistency would be enough to deduce that the only candidate between X_1 and X_3 is the constraint $X_1 = X_3$. This phenomenon can cause the VS to keep a size bigger than it should have.

8 Aspects of our Research Agenda

In this paper we have presented an approach to acquiring models of constraint satisfaction problems from examples. There is significant scope for research in this area. Here we give some insights into some aspects of our research in this area.

Standard version space learning algorithms are sensitive to noise in the training data and, as a consequence, are brittle to false positives and negatives provided to the algorithm. However, some recent work from the machine learning community gives us a basis for making our approach more robust to such errors [6].

Another issue, and one which is particularly interesting, is that of implicit constraints and redundancy which has already been discussed in Section 7. When we deal with partial instances, this will have some interesting implications, such as the effect that the order in which examples are provided has on the representability of a particular problem in the given constraint language.

Finally, we consider the effect that various models of interaction can have on the speed with which we can learn the target problem, particularly from the perspective of minimising the number of interactions with the user. Some preliminary results have already been reported on this issue [10, 11].

9 Conclusion

We have proposed an original method to learn constraint networks from instances that should or should not be solutions. The technique used is based on version spaces, a machine learning paradigm that has good properties (e.g., incrementality, commutativity, wrt the training data) that will be essential in a process interacting with a user. Even if this paper is mainly a description of the general process of learning a constraint network from instances, we can easily foresee the many applications it could have, in assisting a novice in modelling her problem, or helping an expert to test whether a given library of constraints with good computational properties can encode her problem.

We have presented preliminary experiments that show that our approach raises several important issues, such as the speed of the learning process, or the question of the implicit constraints. Based on these experiments, and the framework in general, we have given an insight into some of the very interesting research issues which are raised by our work.

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