H4: a new family of 4-dof parallel robots.

François Pierrot[♦], Olivier Company

Abstract—The goal of this paper is to propose a new family of fully-parallel robots providing 3 translations and 1 rotation about a given axis. Such machines are aimed to use in robotics or machining. The ability of the proposed structure to provide the needed dof is first proved. Position and velocity relationships are then given. Finally, constructive designs are presented that show the effectiveness of the solution with either linear or rotational actuators.

Index Terms—parallel robots, kinematics

I. Introduction

Parallel robots have induced numerous research activities in the Academic community for many years now. Indeed, from the first ideas proposed by Gough [1] or Stewart [2], a lot of interesting mechanical devices or design methods have been extensively studied. In the late 80's, a new field of both research and applications has been opened by Clavel who proposed the famous Delta structure [3] as a base for a "family" of parallel machines dedicated to high-speed applications. More recently, the machine-tool industry discovered the potential advantages of parallel mechanisms and most major machine-tool companies are today in the process of extensive tests and evaluation of their parallel machines for 5-axis milling (Gidding&Lewis, Ingersoll, Hexel, Toyoda, Neos …), for drilling (Hitachi, …), for 3-axis milling (Honda, …).

Even if an incredibly large number of different structures have been proposed by Academic researchers in the last 20 years, some of them can be regarded (see [5] for statistical figures) as more popular as far as industrial use is concerned: the Delta robot is definitely a success, as well as the so-called "hexapod" with 6 *U-P-S* chains in parallel (*U-P-S*: Universal-Prismatic-Spherical). This can be seen as a result of, either the exceptional simplicity of the Delta 3-dof solution, or the enormous research effort dedicated to "hexapod" (see [4] for an extensive coverage of this issue). The machine proposed by Toyoda, HexaM [6], can be a promising solution as well, thanks to its simple design (this solution is an evolution of the Hexa robot [7]).

But it is clear today that most efforts are dedicated to 6 dof (dof: degrees of freedom) machines which are now well known (see [8] for an exhaustive enumeration) or 3 dof machines (see [9] and [10] for good examples of such devices).

However, we strongly believe that there is a need for equipment providing more than 3 dof arranged in parallel *and* based on simpler arrangements than 6-dof structures. As a matter of fact, for most pick-and-place applications, at least four dof are required (3 translations to move the carried object from one point to the other, 1 rotation to arrange the object in its final location). On the Delta robot, this is achieved thanks to an additional *U-P-U* link between the base and the gripper. In some cases, this solution is smart and elegant. In those other cases (namely, for a Delta with a huge workspace, or even more, for linear Delta's), this does not seem as efficient as a fully paralel arrangement. On the other hand, 6-dof fully-parallel machines currently in use in machining suffer from their complexity (they need at least 6 motors while the cutting process requires only 5 controlled axis –plus the spindle rotation–) and from their limited tilting angle. Solutions to these drawbacks have been proposed such as the smart but complex Eclipse machine [11], or the hybrid serial-parallel Tricept built by Neos Company. We believe that another approach could be valid too: one can provide the spindle with 4 axis and let the work piece be moved by an additional 5th axis, more or less following the left-hand/right-hand robotics paradigm.

The aim of this paper is then clear: we propose a new family of 4-dof parallel robots that could be useful for high-speed handling in robotics and milling in machine-tool industry. Considering this important goal, it is amazing to remark that only few efforts have been devoted in the past to 4 dof parallel mechanisms. Apart from Koevermans flight simulator [12] and Reboulet 4-dof wrist [13], which both provide 3 rotations and 1 translation, we can mention few hybrid (that is to say, non fully-parallel) mechanisms as in [14] or [15].

In the following sections we first present the general arrangement we propose. Then we give design conditions that leads to a machine with 3 dof in translation and 1 dof in rotation about a given axis. Then, basics models are derived. Finally examples of possible practical constructions are given.

II. General concept.

The general concept we propose is to build a fully-parallel mechanism with no passive chain, and able to provide high performance in terms of speed and acceleration. Those considerations lead immediately to three important consequences: the mechanism is based on 4 independent kinematic chains between the base and the nacelle, each chain is actuated, each actuator is fixed on the base. Such technological and conceptual ideas have already proven their efficiency on high-speed equipment for the Delta robot, the Hexa robot and the HexaM machine-tool.

So, knowing the advantages of this family of mechanisms, it is interesting to recall few important design features. Figure 1 represents a prototype of linear Delta, based on three actuated linear joints, and three pairs of rods equipped

♦ LIRMM – UMR 5506 CNRS / U.M. 2
161, rue Ada 34392 Montpellier Cedex 5, France

with ball joints at each end. This is the “classical” way to build a Delta, but this choice is driven more by technological issues than by pure kinematics.

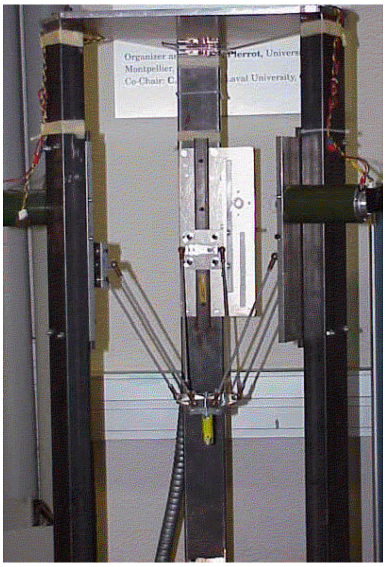

Figure 1. A linear Delta.

As a matter of fact, two ball joints on each rod introduce an internal dof for the rod which can rotate about its own axis. An arrangement such as the one depicted in Figure 2 suppresses these internal dof and keeps the same global behavior (letter P in a gray box represents an actuated P joint, while letter U in a white box represents a passive U joint and letter S in a white box a passive spherical joint).

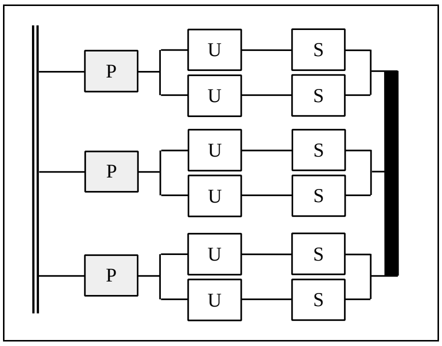

Figure 2. A linear Delta with no internal dof.

To go a little further, it is possible to consider each pair of rods (that is: two *U-S* chains parallel one to the other) as equivalent to a unique rod equipped with Universal joints at each extremity.
The admissible motions for both chains are not exactly the same (see section III), but hey can be regarded as “equivalent” in terms of number and type of degrees of motion.

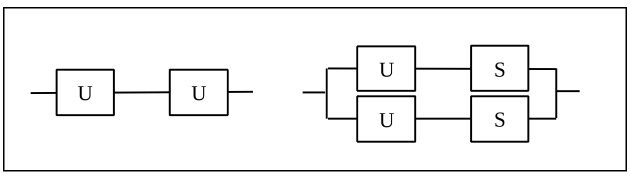

Figure 3. “Equivalence” between U-U and $(U\text{-}S)_2$ chains.

Consequently a “minimal” Delta can be built following the scheme of Figure 4.

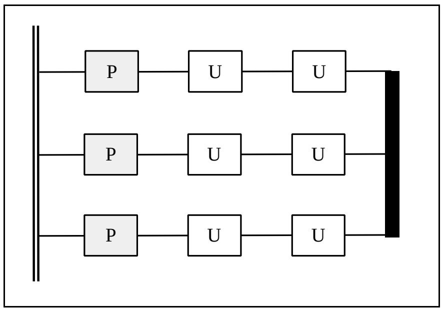

Figure 4. Kinematic arrangement for a “minimal” Delta.

Starting from that point we can now resort to classical kinematics formulas to define the basic principle of a 4 dof mechanism.

According Grübler’s formula, a mechanism’s mobility is given by :

$$m = 6N_P - 6N_l + \sum_{i=1}^{N_l} dof_i \qquad (1)$$

where :

- N_p is the number of independent solids,
- N_l is the number of links,
- dof_i is the number of dof of the i[th] link.

For sake of simplicity we would rather write equation (1) in a slightly different form. Indeed, considering that a “kinematic closed loop” (that is: a chain going from the base to the nacelle, and then back to the base) is composed of n solids and $n+1$ links, equation (1) leads to:

$$m = \sum_{i=1}^{N_l} dof_i - 6L \qquad (2)$$

where : L is the number of loops.

For a fully-parallel 4-dof mechanisms involving no passive chain, $L = 3$ and $m = 4$. Thus:

$$\sum_{i=1}^{N_l} dof_i = 4 + (6 \times 3) = 22 \qquad (3)$$

The four P-U-U chains provide: $4 \times 5 = 20$ dof. Thus each additional joint must be only 1 dof joint. We have decided to use two rotational joints because such joints are extremely easy to manufacture with good accuracy at low cost. Additionally, we consider first the arrangement of Figure 5 where two pairs of chains are created, each pair being connected to the nacelle by a R joint (R: revolute).

Note that the actuated prismatic joints can be replaced by actuated rotational joints, and that the U-U chains can be replaced by $(U\text{-}S)_2$ chains as well (see Figure 5-b). To date, we have only set up conditions for a 4-dof machine. The next section will provide the reader with conditions to be fulfilled for obtaining 3 translations and 1 rotation about a given axis.

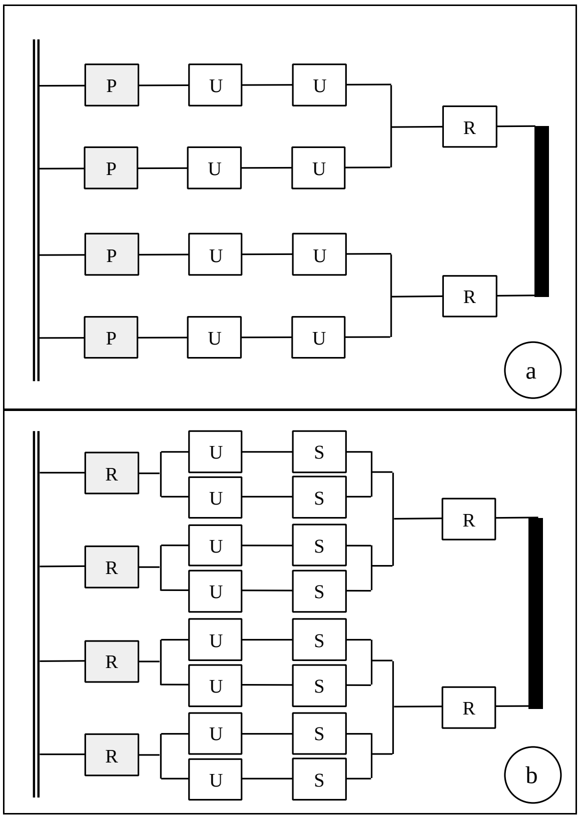

Figure 5. H4 basic principle.

III. Rotation about given axis: necessary condition

The current section is dedicated to the derivation of a necessary condition for a 4-dof mechanism, based on P-(U-S)2 chains, to provide 3 translations and 1 rotation **about a given axis**.

We first recall mathematical properties that will be extensively used in the derivations. Then, the kinematic conditions will be studied in two phases: firstly, we consider the mechanism in its "central" position to establish a condition; then we verify that the kinematic properties remain the same when the mechanism leaves this position.

A. *Preliminary remark*

The two following properties will be used to provide a mathematical condition :

- If two independent kinematic chains, $\aleph_A$ and $\aleph_b$, connect the ground to a nacelle, $\Im$, the possible motion of $\Im$ is the motion admissible for both chains:

$$dof(\Im) = dof(\aleph_A) \cap dof(\aleph_B) \qquad (4)$$

- Under the same conditions, the motion that $\Im$ cannot make is the sum of the impossible motions for each chain:

$$\overline{dof(\Im)} = \overline{dof(\aleph_A)} \oplus \overline{dof(\aleph_B)} \qquad (5)$$

B. *Duality P-U-U P-(U-S)2*

1) *Geometrical hypothesis*

We do not consider here completely general chains. Indeed the motions needed for the nacelle must be admissible for every independent chains. For example, in the case of the Delta robot, the nacelle has 3 possible translations, and thus each of the three chains provide at least the same 3 translations.

This implies the following geometrical conditions:

- For a P-(U-S)$_2$ chain
  - $\boldsymbol{A_{11}B_{11}} = \boldsymbol{A_{12}B_{12}}$

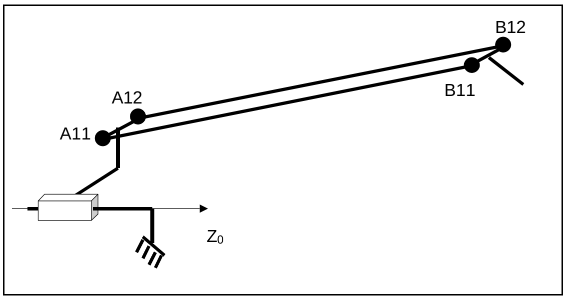

Figure 6. Liaison P-(U-S)$_2$

- For a P-U-U chain
  - $\boldsymbol{z_2} = \boldsymbol{z_3}$
  - $q_2 = -q_3$ (thus: $\boldsymbol{z_1} = \boldsymbol{z_4}$)

  Moreover, for practical reasons (mostly related to easiness of machining), we add the following conditions:
  - $\boldsymbol{z_1} \bullet \boldsymbol{z_2} = 0$
  - $\boldsymbol{z_3} \bullet \boldsymbol{z_4} = 0$
  - $\boldsymbol{z_2} \bullet \boldsymbol{A_1B_1} = 0$

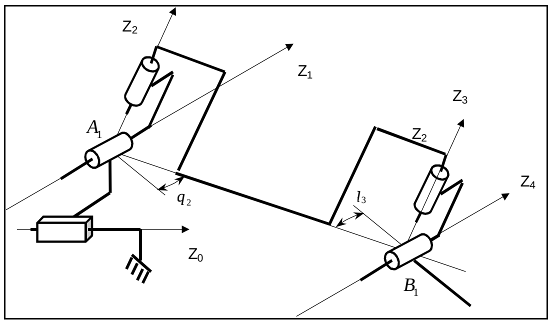

Figure 7.Chaîne P-U-U.

2) *Impossible motions*

- P-(U-S)$_2$ chain

A P-(U-S)$_2$ chain provides 5 degrees of freedom: 3 translations (T) and 2 rotations (R). Since both rods are considered as plain solids, the impossible motion is the rotation about the following vector:

$$\boldsymbol{A_{11}B_{11}} \times \boldsymbol{B_{11}B_{12}}$$

- P-U-U chain

This chain has the same motion capabilities (3T and 2 R); however, the impossible motion is a rotation about the vector normal to axes $\boldsymbol{z_3}$ and $\boldsymbol{z_4}$, that is:

$$\boldsymbol{z_3} \times \boldsymbol{z_4}$$

or

$$(\boldsymbol{A_1 B_1} \times \boldsymbol{z_4}) \times \boldsymbol{z_4}$$

3) Duality

In the next sections, we consider only P-(U-S)$_2$ chains to derive mathematical conditions. If P-U-U chains are considered, one can simply replace $\boldsymbol{A_{11} B_{11}} \times \boldsymbol{B_{11} B_{12}}$ by $(\boldsymbol{A_1 B_1} \times \boldsymbol{z_4}) \times \boldsymbol{z_4}$ in every expression. Of course, the same duality exists for chains where the first (actuated) joint is a rotational joint.

C. Possible motion in "central" position

We first consider a "central" position where the (U-S)$_2$ chains are in nominal positions, that is, such as the two rods are parallel to each other.

1) Notations

A simple scheme of a possible H4 structure is depicted in Figure 8 (chain #3 is not plotted for sake of simplicity). In order to simplify the equations in next sections, we introduce points A_i and B_i as the centers of segments $A_{i1}A_{i2}$ and $B_{i1}B_{i2}$, respectively. Thus:

$$\boldsymbol{A_{i1} B_{i1}} = \boldsymbol{A_{i2} B_{i2}} = \boldsymbol{A_i B_i}$$

Moreover, we will note:

$$\boldsymbol{u_i} = \boldsymbol{B_{i1} B_{i2}}$$

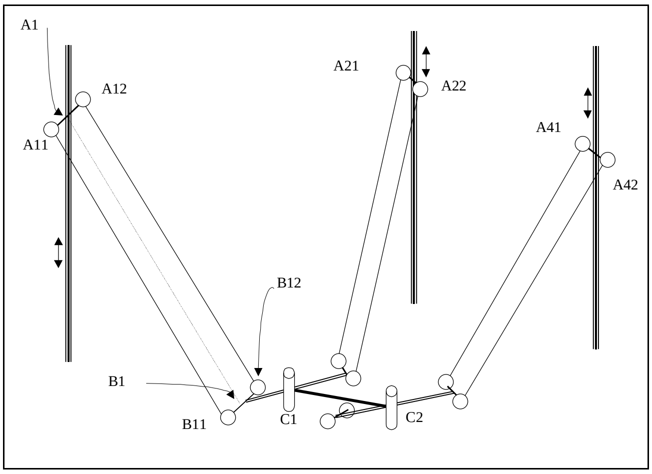

Figure 8. A scheme of H4.

With such notations, the impossible motion for the tip of each P-(U-S)$_2$ chain is the rotation about: $\boldsymbol{A_i B_i} \times \boldsymbol{u_i}$.

2) Possible motions of link B_1B_2 and link B_3B_4

The link B_1B_2 is connected to the ground by P-(U-S)$_2$ chains. Each chain provides 3 translations; thus the link B_1B_2 can undergo the same translations.

Moreover B_1B_2 cannot rotate about $\boldsymbol{A_1 B_1} \times \boldsymbol{u_1}$, neither about $\boldsymbol{A_2 B_2} \times \boldsymbol{u_2}$ (property given by equation (5).

Consequently, it can only rotate about the vector normal to the two previous vectors, that is:

$$(\boldsymbol{A_1 B_1} \times \boldsymbol{u_1}) \times (\boldsymbol{A_2 B_2} \times \boldsymbol{u_2}) \qquad (6)$$

For the same reasons, the link B_3B_4 has 3 dof in translation, and can only rotates about:

$$(\boldsymbol{A_3 B_3} \times \boldsymbol{u_3}) \times (\boldsymbol{A_4 B_4} \times \boldsymbol{u_4}) \qquad (7)$$

3) Possible motion for the link C_1C_2

The link C_1C_2 is connected, on one side, to B_1B_2 by a revolute joint whose direction is represented by vector $\boldsymbol{v_1}$, on the other side, to B_3B_4 by a revolute joint whose direction is represented by vector $\boldsymbol{v}_2$

Thus, we can consider two "meta-chains", with 5 dof, on each side of the nacelle. Those "meta-chains" have the following properties:

- First "meta-chain": 3 translations are possible, as well as 1 rotation about $\boldsymbol{v_1}$, and 1 rotation about $(\boldsymbol{A_1 B_1} \times \boldsymbol{u_1}) \times (\boldsymbol{A_2 B_2} \times \boldsymbol{u_2})$; thus the rotation about
$$[(\boldsymbol{A_1 B_1} \times \boldsymbol{u_1}) \times (\boldsymbol{A_2 B_2} \times \boldsymbol{u_2})] \times \boldsymbol{v_1}$$
is impossible.
- Second "meta-chain": 3 translations are possible, as well as 1 rotation about $\boldsymbol{v}_2$, and 1 rotation about $(\boldsymbol{A_3 B_3} \times \boldsymbol{u_3}) \times (\boldsymbol{A_4 B_4} \times \boldsymbol{u_4})$. Thus the rotation about
$$[(\boldsymbol{A_3 B_3} \times \boldsymbol{u_3}) \times (\boldsymbol{A_4 B_4} \times \boldsymbol{u_4})] \times \boldsymbol{v_2}$$
is impossible

Finally, the possible motions of the nacelle are

- The 3 translations,
- The rotation about

$$([(\boldsymbol{A_1 B_1} \times \boldsymbol{u_1}) \times (\boldsymbol{A_2 B_2} \times \boldsymbol{u_2})] \times \boldsymbol{v_1}) \times ([(\boldsymbol{A_3 B_3} \times \boldsymbol{u_3}) \times (\boldsymbol{A_4 B_4} \times \boldsymbol{u_4})] \times \boldsymbol{v}_2)$$

4) A necessary condition for a rotation about a given axis

We make here the assumption that the two revolute joints placed on the nacelle have the same direction than the the vector of the desired rotation, represented by vector $\boldsymbol{v}$. This choice makes sense but it may be not the only one (note that placing two revolute joints parallel to each other on the nacelle is a good choice for practical matters too)

The existence of a rotational motion about $\boldsymbol{v}$ can be written as follows:

$$\exists a,\, a \neq 0 \;/\; ([\boldsymbol{w_1} \times \boldsymbol{w_2}] \times \boldsymbol{v}) \times ([\boldsymbol{w_3} \times \boldsymbol{w_4}] \times \boldsymbol{v}) = a\ \boldsymbol{v} \quad (8)$$

where: $\boldsymbol{w_i} = \boldsymbol{A_i B_i} \times \boldsymbol{u_i}$

Equation (8) is equivalent to:

$$[([\boldsymbol{w_1} \times \boldsymbol{w_2}] \times \boldsymbol{v_1}) \bullet \boldsymbol{v}](\boldsymbol{w_3} \times \boldsymbol{w_4}) - ([\boldsymbol{w_1} \times \boldsymbol{w_2}] \times \boldsymbol{v}) \bullet (\boldsymbol{w_3} \times \boldsymbol{w_4}) \boldsymbol{v} = a\boldsymbol{v}$$

Or :

$$-([\boldsymbol{w_1} \times \boldsymbol{w_2}] \times \boldsymbol{v}) \bullet (\boldsymbol{w_3} \times \boldsymbol{w_4}) \boldsymbol{v} = a\ \boldsymbol{v}$$

Thus:

$$-([\boldsymbol{w_1} \times \boldsymbol{w_2}] \times \boldsymbol{v}) \bullet (\boldsymbol{w_3} \times \boldsymbol{w_4}) \neq 0$$

So, we have:

$$([\boldsymbol{w_1} \times \boldsymbol{w_2}] \times (\boldsymbol{w_3} \times \boldsymbol{w_4})) \bullet \boldsymbol{v} \neq 0$$

Finally, the necessary condition is:

$$\left([(\boldsymbol{A_1B_1} \times \boldsymbol{u_1}) \times (\boldsymbol{A_2B_2} \times \boldsymbol{u_2})] \times [(\boldsymbol{A_3B_3} \times \boldsymbol{u_3}) \times (\boldsymbol{A_4B_4} \times \boldsymbol{u_4})] \right) \bullet \boldsymbol{v} \neq 0 \quad (9)$$

Remark:
The previous condition is only a *necessary* condition. We do not take into account any singular configuration in this derivation.

D. Stability of our hypothesis

The previous derivations have been done for a "central" position where two rods of a pair are parallel to each other. We will now derive conditions to keep this true when the nacelle moves.
We already know that if equation (9) is satisfied, the nacelle has 3 dof in translation, and 1 dof in rotation about vector $\boldsymbol{v}$, as long as the rods in a pair stay parallel.
The rods in a pair will stay parallel to each other if both links B_1B_2 and B_3B_4 move only in translation with respect to the ground.
As a matter of fact, the possible motions of the nacelle imply that the two pivots fixed on the nacelle along the direction of vector $\boldsymbol{v}$ will keep this direction. Thus the only possible rotation for B_1B_2 (and B_3B_4) is about vector $\boldsymbol{v}$.
But we previously stated (equation (6)) that B_1B_2 can only rotates about $(\boldsymbol{A_1B_1} \times \boldsymbol{u_1}) \times (\boldsymbol{A_2B_2} \times \boldsymbol{u_2})$.
So, as long as $(\boldsymbol{A_1B_1} \times \boldsymbol{u_1}) \times (\boldsymbol{A_2B_2} \times \boldsymbol{u_2})$ and $\boldsymbol{v}$ are not co-linear, the link B_1B_2 has no possible rotation. A similar remark can be made for B_3B_4, leading to the two following conditions:

$$\begin{aligned} (\boldsymbol{A_1B_1} \times \boldsymbol{u_1}) \times (\boldsymbol{A_2B_2} \times \boldsymbol{u_2}) &\neq \boldsymbol{v} \\ (\boldsymbol{A_3B_3} \times \boldsymbol{u_3}) \times (\boldsymbol{A_4B_4} \times \boldsymbol{u_4}) &\neq \boldsymbol{v} \end{aligned} \quad (10)$$

In brief, if conditions (10) are fulfilled, the links B_1B_2 and B_3B_4 will keep the direction they had in "central" position, and the rods in a pair will stay parallel to each other. Basically, each "meta-chain" behaves as it belongs to a Delta structure since their "end-effector" (the link B_1B_2 or the link B_3B_4) moves only in translation

We can now derive easily the classical relationships for position and velocity

IV. Position Relationship

In order to write the position relationship, we define the following parameters and variables:

- The (linear) actuators slide along guide-ways oriented along a unitary vector, $\boldsymbol{z_i}$, and the origin is point P_i;
- The rods length is L_i
- The position of the end effector, namely point D, is defined by a position vector, $\boldsymbol{D} = [x \quad y \quad z]^t$, and a scalar, q, representing the orientation angle about $\boldsymbol{v}$.

Consequently, the position of each point A_i is given by:

$$\boldsymbol{A_i} = \boldsymbol{P_i} + q_i \boldsymbol{z_i} \quad (11)$$

the position of points C_1 and C_2 is given by:

$$\begin{aligned} \boldsymbol{C_1} &= \boldsymbol{D} + \boldsymbol{Rot(v,q)}(\boldsymbol{DC_1}) \\ \boldsymbol{C_2} &= \boldsymbol{D} + \boldsymbol{Rot(v,q)}(\boldsymbol{DC_2}) \end{aligned} \quad (12)$$

and the position of each point B_i is given by:

$$\begin{aligned} \boldsymbol{B_1} &= \boldsymbol{C_1} + \boldsymbol{C_1B_1} \\ \boldsymbol{B_2} &= \boldsymbol{C_1} + \boldsymbol{C_1B_2} \\ \boldsymbol{B_3} &= \boldsymbol{C_2} + \boldsymbol{C_2B_3} \\ \boldsymbol{B_4} &= \boldsymbol{C_2} + \boldsymbol{C_2B_4} \end{aligned} \quad (13)$$

The position relationship can then be written in the following way:

$$\|\boldsymbol{A_iB_i}\|^2 = L_i^2 \quad (14)$$

As an example, here are the calculations for the first axis:

$$\boldsymbol{A_1B_1} = [\boldsymbol{D} + \boldsymbol{Rot(v,?)}(\boldsymbol{DC_1}) + \boldsymbol{C_1B_1} - \boldsymbol{P_1}] - q_1 \boldsymbol{z_1}$$

$$(\boldsymbol{A_1B_1})^2 = q_1^2 - 2\, q_1\, \boldsymbol{d_1} \bullet \boldsymbol{z_1} + \boldsymbol{d_1}^2 \quad (15)$$

where: $\boldsymbol{d_1} = [\boldsymbol{D} + \boldsymbol{Rot(v,?)}(\boldsymbol{DC_1}) + \boldsymbol{C_1B_1} - \boldsymbol{P_1}]$

Finally, the two solutions for the position relationship are given by:

$$q_1 = \boldsymbol{d_1} \bullet \boldsymbol{z_1} \pm \sqrt{(\boldsymbol{d_1} \bullet \boldsymbol{z_1})^2 - \boldsymbol{d_1}^2 + L_1^2} \quad (16)$$

Similar derivations give the solutions for q_2, q_3 and q_4.

V. VELOCITY RELATIONSHIP.

The velocity of the nacelle can be defined by resorting to a velocity vector for the translation, $v_D = [\dot{x} \quad \dot{y} \quad \dot{z}]^t$, and a scalar for the rotation about vector $\boldsymbol{v}$, $\dot{q}$.
Thus, the velocity of points C_1 and C_2 can be written as follows:

$$\boldsymbol{v}_{C_1} = \boldsymbol{v}_D + \dot{?}\,\boldsymbol{v} \times \boldsymbol{DC}_1$$
$$\boldsymbol{v}_{C_2} = \boldsymbol{v}_D + \dot{?}\,\boldsymbol{v} \times \boldsymbol{DC}_2$$

Moreover, since the links B_1B_2 and B_3B_4 move only in translation, the following relations hold:

$$\boldsymbol{v}_{B_1} = \boldsymbol{v}_{B_2} = \boldsymbol{v}_{C_1}$$
$$\boldsymbol{v}_{B_3} = \boldsymbol{v}_{B_4} = \boldsymbol{v}_{C_2}$$

On the other hand, velocity of points A_i is given by:

$$\boldsymbol{v}_{A_i} = \dot{q}_i \boldsymbol{z}_i$$

The velocity relationship can then be written thanks to the classical property:

$$\boldsymbol{v}_{A_i} \bullet \boldsymbol{A}_i\boldsymbol{B}_i = \boldsymbol{v}_{B_i} \bullet \boldsymbol{A}_i\boldsymbol{B}_i \qquad (17)$$

Equation (17) can be written for $i = 1,...,4$ and the results grouped in a matrix form, such as:

$$\boldsymbol{J}_q \dot{\boldsymbol{q}} = \boldsymbol{J}_x \dot{\boldsymbol{x}}$$

where

$$\boldsymbol{J}_q = \begin{bmatrix} (\boldsymbol{A_1B_1} \bullet \boldsymbol{z_1}) & 0 & 0 & 0 \\ 0 & (\boldsymbol{A_2B_2} \bullet \boldsymbol{z_2}) & 0 & 0 \\ 0 & 0 & (\boldsymbol{A_3B_3} \bullet \boldsymbol{z_3}) & 0 \\ 0 & 0 & 0 & (\boldsymbol{A_4B_4} \bullet \boldsymbol{z_4}) \end{bmatrix}$$

$$\dot{\boldsymbol{q}} = [\dot{q}_1 \quad \dot{q}_2 \quad \dot{q}_3 \quad \dot{q}_4]^t$$

$$\boldsymbol{J}_x = \begin{bmatrix} (\boldsymbol{A_1B_1})_x & (\boldsymbol{A_1B_1})_y & (\boldsymbol{A_1B_1})_z & (\boldsymbol{A_1B_1} \times \boldsymbol{DC_1}) \bullet \boldsymbol{v} \\ (\boldsymbol{A_2B_2})_x & (\boldsymbol{A_2B_2})_y & (\boldsymbol{A_2B_2})_z & (\boldsymbol{A_2B_2} \times \boldsymbol{DC_1}) \bullet \boldsymbol{v} \\ (\boldsymbol{A_3B_3})_x & (\boldsymbol{A_3B_3})_y & (\boldsymbol{A_3B_3})_z & (\boldsymbol{A_3B_3} \times \boldsymbol{DC_2}) \bullet \boldsymbol{v} \\ (\boldsymbol{A_4B_4})_x & (\boldsymbol{A_4B_4})_y & (\boldsymbol{A_4B_4})_z & (\boldsymbol{A_4B_4} \times \boldsymbol{DC_2}) \bullet \boldsymbol{v} \end{bmatrix}$$

$$\dot{\boldsymbol{x}} = [\dot{x} \quad \dot{y} \quad \dot{z} \quad \dot{q}]^t$$

In the next section, we finally propose practical implementation based our analysis.

VI. EXAMPLES OF IMPLEMENTATION.

$$\boldsymbol{v} = [0 \quad 0 \quad 1]^t$$

$$\boldsymbol{AB} = \begin{bmatrix} -1 & -1 & 1 & 1 \\ -1 & 1 & 1 & -1 \\ 1 & 1 & 1 & 1 \end{bmatrix} \quad \boldsymbol{u} = \begin{bmatrix} -1 & 1 & 1 & -1 \\ 1 & 1 & -1 & -1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

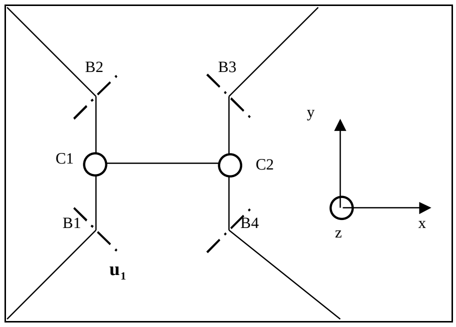

Figure 9. Example not compatible with the condition.

This gives : $(\boldsymbol{w_1} \times \boldsymbol{w_2}) \times (\boldsymbol{w_3} \times \boldsymbol{w_4}) = [0 \quad 16 \quad 0]^t$
Thus: $(\boldsymbol{w_1} \times \boldsymbol{w_2}) \times (\boldsymbol{w_3} \times \boldsymbol{w_4}) \bullet \boldsymbol{z} = 0$. This means that the nacelle cannot rotate about $\boldsymbol{v} = [0 \quad 0 \quad 1]^t$.

$$\boldsymbol{u} = \begin{bmatrix} -1 & 1 & 1 & 0 \\ 1 & 1 & 0 & -1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \quad \boldsymbol{AB} = \begin{bmatrix} 1 & 1 & 0 & -1 \\ 1 & -1 & -1 & 0 \\ -1 & -1 & -1 & -1 \end{bmatrix}$$

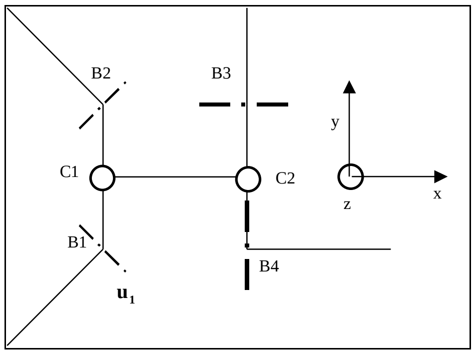

Figure 10. Example compatible with the condition.

This gives : $(\boldsymbol{w_1} \times \boldsymbol{w_2}) \times (\boldsymbol{w_3} \times \boldsymbol{w_4}) = [-2 \quad 6 \quad -4]^t$
Thus $(\boldsymbol{w_1} \times \boldsymbol{w_2}) \times (\boldsymbol{w_3} \times \boldsymbol{w_4}) \bullet \boldsymbol{z} \neq 0$. This arrangement is then a good candidate for a H4 robot. Such a solution is depicted in Figure 11. Note that it is possible to build an equivalent mechanism with rotational actuators, as shown in Figure 12.
From those basic implementations, it is possible to create more advanced mechanisms based on a particular case of equation (6) which gives the direction for the possible rotation of the link B_1B_2.
If we choose $\boldsymbol{u_1} = \boldsymbol{u_2}$, then equation (6) becomes:

$$(\boldsymbol{A_1B_1} \times \boldsymbol{u_1}) \times (\boldsymbol{A_2B_2} \times \boldsymbol{u_1}) = b\ \boldsymbol{u_1}$$

which means that the only possible rotation of this link is about vector $\boldsymbol{u_1}$ (as long as $\boldsymbol{A_1B_1} \neq \boldsymbol{A_2B_2}$). Moreover, this link has always the 3 translations. Thus, the "meta-chain" already fulfilled our requirements: 3 translations and 1 rotation about a given axis. However, this "meta-chain" is only equipped with 2 actuators; thus 2 other chains (with one actuator each) are needed in parallel to lead to a fully-parallel machine.

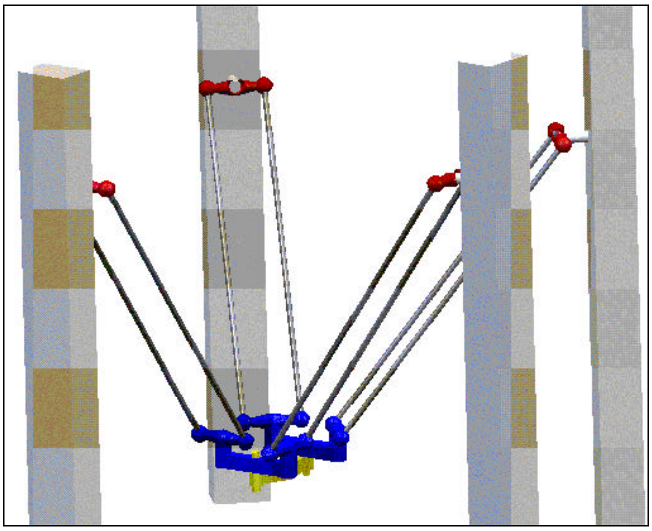

Figure 11. A P-(U-S)$_2$ H4 robot.

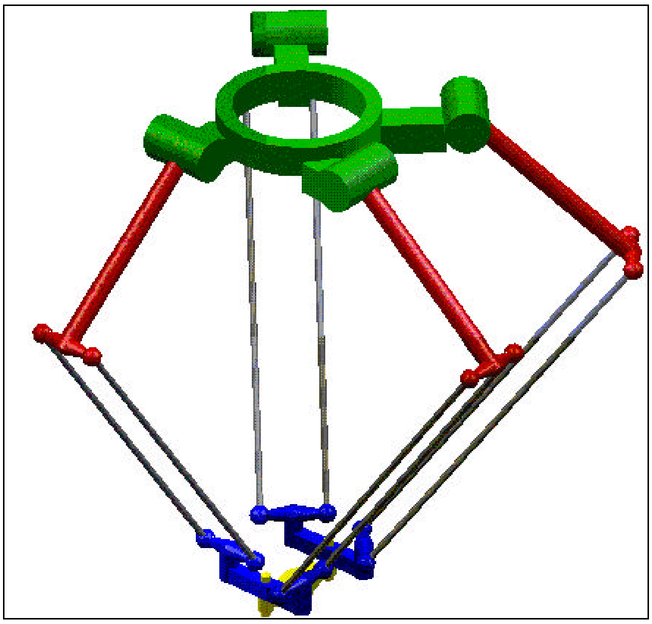

Figure 12. A R-(U-S)$_2$ H4 robot.

Those two additional chains must be 6-dof chains (according Grübler's formula) and are then obviously compatible with the motions offered by the first "meta-chain". We propose to use 2 P-U-S chains, as depicted in Figure 13.

VII. Conclusion

We have proposed in this paper a new family of 4-dof parallel robots which could be useful for high-speed handling or machining. One key point is the derivation of a necessary condition to obtain a rotation about a given axis. Practical implementations are currently under consideration and patents are pending for H4 family of robots.

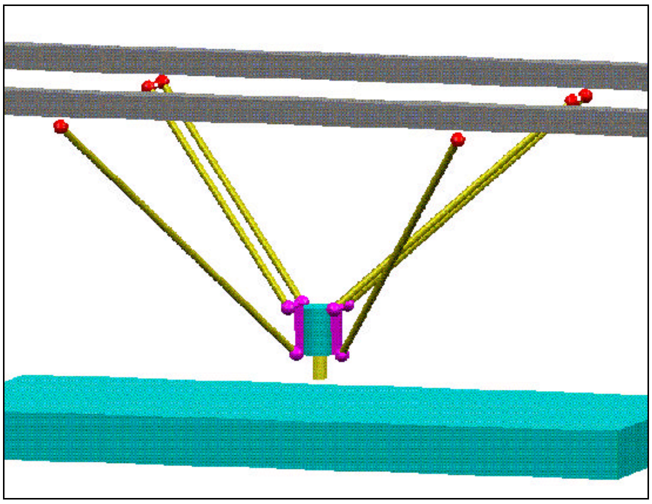

Figure 13. A P-(U-S)$_2$ / P-U-S H4 robot.

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