A new 3T-1R parallel robot

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Abstract

The paper presents a new family of fully-parallel robots providing 3 translations and 1 rotation about a given axis. Such machines are aimed to use in robotics or machining. The ability of the proposed structure to provide the needed dof[1] is first proved; two possible constructions are analyzed: symmetrical and asymmetrical. Finally, constructive designs are presented that show the effectiveness of the solution.

1 Introduction

From the first ideas of parallel mechanisms proposed by Gough [1] or Stewart [2], a lot of interesting mechanical devices or design methods have been extensively studied. In the late 80's, a new field of research and applications has been opened by Clavel who proposed the famous Delta structure [3] as a base for a "family" of parallel machines dedicated to high-speed applications. More recently, the machine-tool industry discovered the potential advantages of parallel mechanisms and major machine-tool companies are evaluating their own parallel machines.
Among the large number of different structures proposed by Academic researchers in the last 20 years, some can be regarded (see [5] for statistical figures) as more popular as far as industrial use is concerned: the Delta robot is definitely a success, as well as the so-called "hexapod" with 6 *U-P-S* chains in parallel (*U-P-S*: Universal-Prismatic-Spherical). This may be a result of, either the exceptional simplicity of the Delta 3-dof solution, or the enormous research effort dedicated to "hexapod" (see [4] for an extensive coverage of this issue). The machine proposed by Toyoda, HexaM [6], is also a promising solution, thanks to its simple design (this solution is an evolution of the Hexa robot [7]).
It is clear today that most efforts are dedicated to 6 dof machines which are now well known (see [8] for an exhaustive enumeration) or 3 dof machines (see [9] and [10] for good examples of such devices).
However, we strongly believe that there is a need for equipment providing more than 3 dof arranged in parallel *and* based on simpler arrangements than 6-dof structures. As a matter of fact, for most pick-and-place applications, at least four dof are required (3 translations and 1 rotation to arrange the object in its final location). On the Delta robot, this is achieved thanks to an additional *U-P-U* link between the base and the gripper. In some cases, this solution is smart and elegant. In those other cases (namely, for Delta's with huge workspace, or even more, for linear Delta's), this does not seem as efficient as a fully parallel arrangement. On the other hand, 6-dof fully-parallel machines currently in use in machining suffer from their complexity (they need at least 6 motors while the cutting process requires only 5 controlled axis –plus the spindle rotation–) and from their limited tilting angle. Solutions to these drawbacks have been proposed such as the smart -but complex- Eclipse machine [11], or the hybrid serial-parallel Tricept. We believe that another approach could be valid too: one can provide the spindle with 4 axis and let the work piece be moved by an additional 5th axis, more or less following the left-hand/right-hand robotics paradigm.
The aim of this paper is then clear: we propose a new family of 4-dof parallel robots that could be useful for high-speed handling in robotics and milling in machine-tool industry. Considering this important goal, it is amazing to remark that only few efforts have been devoted in the past to 4 dof parallel mechanisms. Apart from Koevermans flight simulator [12] and Reboulet 4-dof wrist [13], which both provide 3 rotations and 1 translation, we can mention few hybrid (that is to say, non fully-parallel) mechanisms as in [14] or [15].
In the following sections we first present the general arrangement we propose. Then we give design conditions that lead to a machine with 3 dof in translation and 1 dof in rotation about a given axis. Finally examples of possible practical constructions are given.

2 General concept

The general concept we propose is to build a fully-parallel mechanism with no passive chain, able to provide high performance in terms of speed and acceleration. Those considerations lead to three important consequences: the mechanism is based on 4 independent chains between the base and the nacelle; each chain is actuated; each actuator is fixed on the base. Such ideas have already proven their efficiency on high-speed equipment like the Delta robot, the Hexa robot and the HexaM machine-tool.
Knowing the advantages of this family of mechanisms, it is interesting to recall few important design features. Figure 1 represents a prototype of linear Delta, based on three actuated linear joints, and three pairs of rods equipped with ball joints at each end. This is the "classical" way to build a Delta, but this choice is driven more by technological issues than by pure kinematics.

[1] dof: degree of freedom

As a matter of fact, two ball joints on each rod introduce an internal dof for the rod which can rotate about its own axis. The arrangement depicted in Figure 3 suppresses these internal dof and keeps the same global behavior (letter P in a gray box represents an actuated P joint, letter U in a white box represents a passive U joint and letter S in a white box a passive spherical joint).

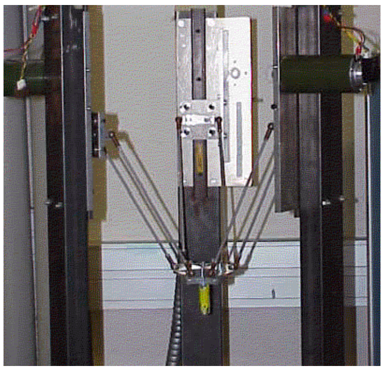

Figure 1. A linear Delta.

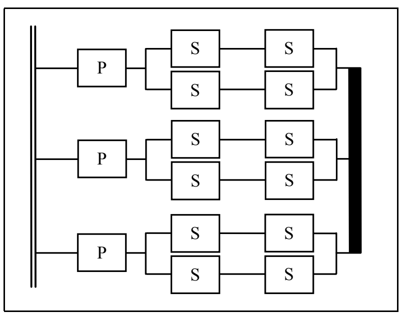

Figure 2. Practical construction of a linear Delta.

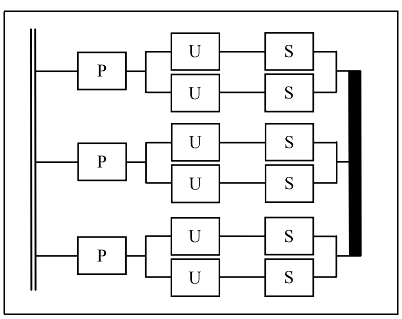

Figure 3. A linear Delta with no internal dof.

To go a little further, it is possible to consider each pair of rods (that is: two *U-S* chains parallel one to the other) as equivalent to a unique rod equipped with Universal joints at each extremity. A "minimal" Delta can be built following the scheme of Figure 4. The admissible motions for both chains are not exactly the same, but they can be regarded as "equivalent" in terms of number and type of degrees of motion: this will be discussed in the next sub-section. The reason for choosing such chains is simple: the motions needed for the nacelle must be admissible for every independent chains. For example, in the case of the Delta robot, the nacelle has 3 possible translations, and each chain provide at least those 3 translations. Here we need 3 translation and 1 rotation: thus, those chains are good candidates.

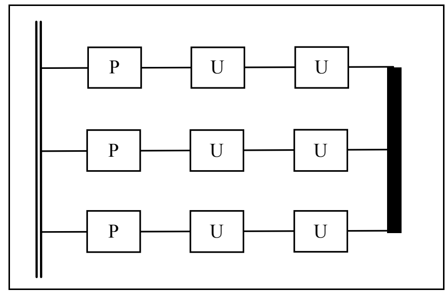

Figure 4. Arrangement for a "minimal" Linear Delta.

2.1 "Quasi-equivalence" between P-U-U and P-(U-S)$_2$

A "quasi-equivalence" exists between U-U and (U-S)$_2$ chains, when one considers (U-S)$_2$ chains with two parallel rods. This implies the following conditions:

- For a P-(U-S)$_2$ chain $\boldsymbol{A_{11}B_{11} = A_{12}B_{12}}$

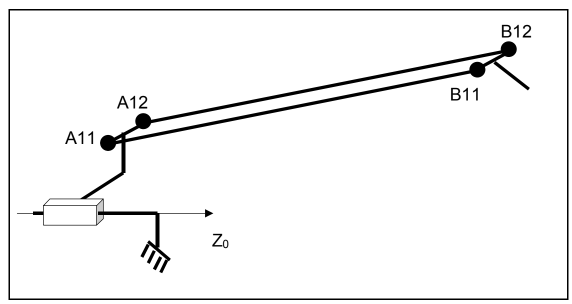

Figure 5. P-(U-S)$_2$ chain

- For a P-U-U chain $z_2 = z_3$
  $\theta_2 = -\theta_3$ (thus: $z_1 = z_4$)
  Moreover, for practical reasons (easy machining), we add the following conditions:
  $z_1 \bullet z_2 = 0 \quad z_3 \bullet z_4 = 0 \quad z_2 \bullet \boldsymbol{A_1B_1} = 0$

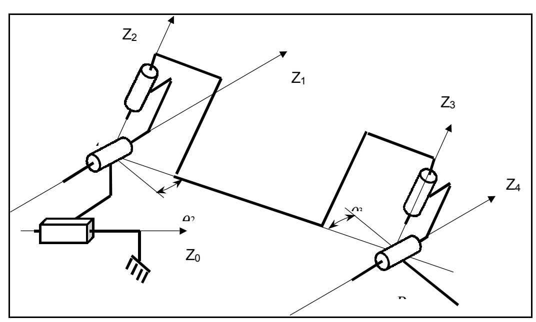

Figure 6. A P-U-U chain.

Both chains are 5-dof chains, and provide the same type of dof: 3 translations (T) and 2 rotations (R). They simply differ when the suppressed motion is considered:

- $P\text{-}(U\text{-}S)_2$ chain

Since both rods are considered as plain solids, the impossible motion is the rotation about the following vector:

$$\boldsymbol{A_{11}B_{11}} \times \boldsymbol{B_{11}B_{12}}$$

- P-U-U chain

The impossible motion is a rotation about the vector normal to axes z_3 and z_4 , that is:

$$z_3 \times z_4 \quad \text{or} \quad (\boldsymbol{A_1B_1} \times z_4) \times z_4$$

Note: the same “quasi-equivalence” exists for chains where the first (actuated) joint is a rotational joint.

With those remarks, it is possible to use P-U-U’s at conceptual design stage (for example, for deriving a mobility analysis), and to consider $P\text{-}(U\text{-}S)_2$’s at practical design stage.

2.2 Necessary dof

We resort to classical kinematics formulas to define the basic principle of our 4 dof mechanism. Indeed, a mechanism’s mobility is given by :

$$m = \sum_{i=1}^{N_l} dof_i - 6L \qquad (1)$$

where : dof_i is the number of dof of the i[th] link, L the number of loops, N_l the number of links. For a fully-parallel 4-dof mechanism with no passive chain, $L = 3$ and $m = 4$. Thus:

$$\sum_{i=1}^{N_l} dof_i = 4 + (6 \times 3) = 22 \qquad (2)$$

2.3 Symmetrical design

Our symmetrical design is based on four identical P-U-U chains which provide: $4 \times 5 = 20$ dof. Thus, two additional 1-dof joints are needed. We have decided to use two rotational joints because they are easy to manufacture with good accuracy at low cost. Additionally, we consider the arrangement of Figure 7 where two pairs of chains are created, each pair being connected to the nacelle by a R joint (R: revolute).

Note that the U-U chains can be replaced by $(U\text{-}S)_2$ chains (see Figure 7-b). Moreover, the actuated prismatic joints could be replaced by actuated rotational joints as well.

2.4 Asymmetrical design

Keeping the idea of creating two pairs of chains, we can reach the required total of 22 dof with 2 P-U-U’s and 2 P-U-S’s, as in Figure 8-a. Again, the P-U-U’s can be replaced by $P\text{-}(U\text{-}S)_2$’s, as in Figure 8-b.

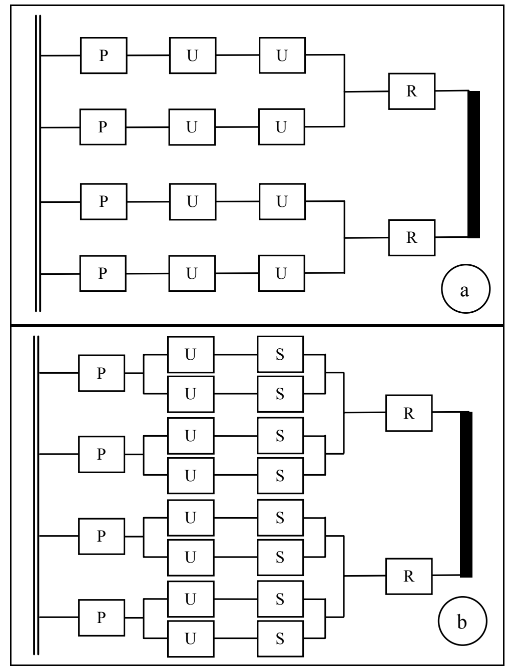

Figure 7. Basic principle of a symmetrical H4.

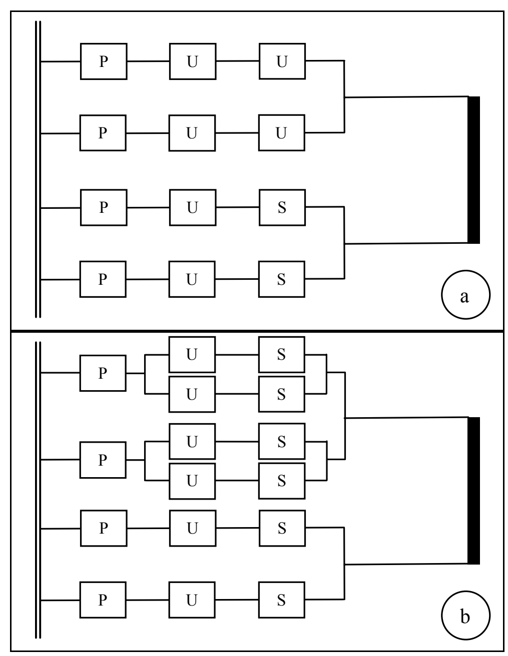

Figure 8. An asymmetrical design.

3 Symmetrical design: A necessary condition for a rotation about given axis

This section is dedicated to the derivation of a necessary condition for a 4-dof mechanism, based on 4 P-(U-S) $_2$ chains to provide 3 translations and 1 rotation **about a given axis**. We first recall mathematical properties that will be extensively used in the derivations. Then, the kinematic conditions will be studied in two phases: firstly, we consider the mechanism in its "central" position to establish a condition; then we verify that the kinematic properties remain the same when the mechanism leaves this position.

3.1 Preliminary remark

The two following properties will be used to provide a mathematical condition :

- If two independent kinematic chains, $\aleph_A$ and $\aleph_b$, connect the ground to a nacelle, $\Im$, the possible motion of $\Im$ is the motion admissible for both chains:

$$dof(\Im) = dof(\aleph_A) \cap dof(\aleph_B) \qquad (3)$$

- Under the same conditions, the motion that $\Im$ cannot make is the sum of the impossible motions for each chain:

$$\overline{dof(\Im)} = \overline{dof(\aleph_A)} \oplus \overline{dof(\aleph_B)} \qquad (4)$$

3.2 Possible motion in "central" position

We first consider a "central" position where the (U-S)$_2$ chains are in nominal positions, that is, such as the two rods are parallel to each other. A simple scheme of a possible H4 structure is depicted in

Figure 9 **(chain #3 is not plotted for simplicity)**. In order to simplify the equations in next sections, we introduce points A_i and B_i as the centers of segments $A_{i1}A_{i2}$ and $B_{i1}B_{i2}$, respectively. Thus:

$$\boldsymbol{A_{i1}B_{i1}} = \boldsymbol{A_{i2}B_{i2}} = \boldsymbol{A_iB_i}$$

Moreover, we denote:

$$\boldsymbol{u_i} = \boldsymbol{B_{i1}B_{i2}}$$

With such notations, the impossible motion for the tip of each P-(U-S)$_2$ chain is the rotation about: $\boldsymbol{A_iB_i} \times \boldsymbol{u_i}$.

3.2.1 Possible motions of link B_1B_2 and link B_3B_4

The link B_1B_2 is connected to the ground by P-(U-S)$_2$ chains. Each chain provides 3 translations; thus the link B_1B_2 can undergo the same translations.
Moreover, B_1B_2 cannot rotate about $\boldsymbol{A_1B_1} \times \boldsymbol{u_1}$, neither about $\boldsymbol{A_2B_2} \times \boldsymbol{u_2}$ (property (4)).

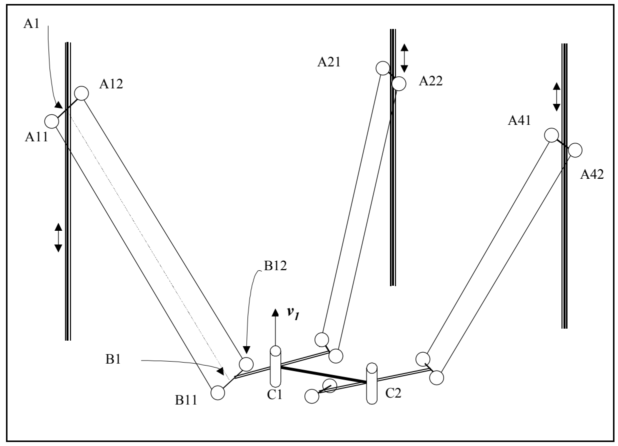

Figure 9. A scheme of a symmetrical H4.

Consequently, it can only rotate about the vector normal to the two previous vectors, that is:

$$(\boldsymbol{A_1B_1} \times \boldsymbol{u_1}) \times (\boldsymbol{A_2B_2} \times \boldsymbol{u_2}) \qquad (5)$$

For the same reasons, link B_3B_4 has 3 dof in translation, and can only rotates about:

$$(\boldsymbol{A_3B_3} \times \boldsymbol{u_3}) \times (\boldsymbol{A_4B_4} \times \boldsymbol{u_4}) \qquad (6)$$

3.2.2 Possible motion for link C_1C_2

Link C_1C_2 is connected, on one side, to B_1B_2 by a revolute joint whose direction is represented by vector $\boldsymbol{v_1}$, on the other side, to B_3B_4 by a revolute joint whose direction is represented by vector $\boldsymbol{v}_2$ Thus, we can consider two 5-dof "**meta-chains**" on each side of the nacelle. Those "meta-chains" have the following properties:

- First "meta-chain": 3 translations are possible, as well as 1 rotation about $\boldsymbol{v_1}$, and 1 rotation about $(\boldsymbol{A_1B_1} \times \boldsymbol{u_1}) \times (\boldsymbol{A_2B_2} \times \boldsymbol{u_2})$; thus the rotation about

  $$[(\boldsymbol{A_1B_1} \times \boldsymbol{u_1}) \times (\boldsymbol{A_2B_2} \times \boldsymbol{u_2})] \times \boldsymbol{v_1}$$

  is impossible.
- Second "meta-chain": 3 translations are possible, as well as 1 rotation about $\boldsymbol{v}_2$, and 1 rotation about $(\boldsymbol{A_3B_3} \times \boldsymbol{u_3}) \times (\boldsymbol{A_4B_4} \times \boldsymbol{u_4})$. Thus the rotation about

  $$[(\boldsymbol{A_3B_3} \times \boldsymbol{u_3}) \times (\boldsymbol{A_4B_4} \times \boldsymbol{u_4})] \times \boldsymbol{v_2}$$

  is impossible

Finally, the possible motions of the nacelle are: 3 translations, and the rotation about

$$([(\boldsymbol{A_1B_1} \times \boldsymbol{u_1}) \times (\boldsymbol{A_2B_2} \times \boldsymbol{u_2})] \times \boldsymbol{v_1}) \times ([(\boldsymbol{A_3B_3} \times \boldsymbol{u_3}) \times (\boldsymbol{A_4B_4} \times \boldsymbol{u_4})] \times \boldsymbol{v}_2)$$

3.2.3 Necessary condition for a rotation about a given axis

We make here the assumption that the two revolute joints placed on the nacelle have the same direction than the the vector of the desired rotation, represented by vector $\boldsymbol{v}$. This choice makes sense but it may be not the only one (note that placing two revolute joints parallel to each other on the nacelle is a good choice for practical matters too). The existence of a rotational motion about $\boldsymbol{v}$ can be written as follows:

$$\exists\alpha, \alpha \neq 0 \ / \ ([\boldsymbol{w_1}\times\boldsymbol{w_2}]\times\boldsymbol{v})\times([\boldsymbol{w_3}\times\boldsymbol{w_4}]\times\boldsymbol{v}) = \alpha\,\boldsymbol{v} \qquad (7)$$

where: $\boldsymbol{w_i} = \boldsymbol{A_i B_i}\times\boldsymbol{u_i}$

Equation (7) is equivalent to:

$$[([\boldsymbol{w_1}\times\boldsymbol{w_2}]\times\boldsymbol{v_1})\bullet\boldsymbol{v}](\boldsymbol{w_3}\times\boldsymbol{w_4}) - ([\boldsymbol{w_1}\times\boldsymbol{w_2}]\times\boldsymbol{v})\bullet(\boldsymbol{w_3}\times\boldsymbol{w_4})\,\boldsymbol{v} = \alpha\boldsymbol{v}$$

Thus:

$$-([\boldsymbol{w_1}\times\boldsymbol{w_2}]\times\boldsymbol{v})\bullet(\boldsymbol{w_3}\times\boldsymbol{w_4})\,\boldsymbol{v} = \alpha\,\boldsymbol{v}$$

And:

$$-([\boldsymbol{w_1}\times\boldsymbol{w_2}]\times\boldsymbol{v})\bullet(\boldsymbol{w_3}\times\boldsymbol{w_4}) \neq 0$$

So, we have:

$$([\boldsymbol{w_1}\times\boldsymbol{w_2}]\times(\boldsymbol{w_3}\times\boldsymbol{w_4}))\bullet\boldsymbol{v} \neq 0$$

Finally, the necessary condition is:

$$\Big(\ [(\boldsymbol{A_1B_1}\times\boldsymbol{u_1})\times(\boldsymbol{A_2B_2}\times\boldsymbol{u_2})]\times [(\boldsymbol{A_3B_3}\times\boldsymbol{u_3})\times(\boldsymbol{A_4B_4}\times\boldsymbol{u_4})]\ \Big)\bullet\boldsymbol{v} \neq 0 \qquad (8)$$

Remark:
The previous condition is only a *necessary* condition. We do not take into account any singular configuration in this derivation.

3.3 Stability of the hypothesis

The previous derivations have been done for a "central" position where two rods of a pair are parallel to each other. We will now derive conditions to keep this true when the nacelle moves. We already know that if equation (8) is satisfied, the nacelle has 3 dof in translation, and 1 dof in rotation about vector $\boldsymbol{v}$, as long as the rods in a pair stay parallel. The rods in a pair will stay parallel to each other if both links B_1B_2 and B_3B_4 move only in translation with respect to the ground. As a matter of fact, the possible motions of the nacelle imply that the two pivots fixed on the nacelle along the direction of vector $\boldsymbol{v}$ will keep this direction. Thus the only possible rotation for B_1B_2 (and B_3B_4) is about vector $\boldsymbol{v}$. But we previously stated (section 3.2.1) that B_1B_2 can only rotates about $(\boldsymbol{A_1B_1}\times\boldsymbol{u_1})\times(\boldsymbol{A_2B_2}\times\boldsymbol{u_2})$.
So, as long as $(\boldsymbol{A_1B_1}\times\boldsymbol{u_1})\times(\boldsymbol{A_2B_2}\times\boldsymbol{u_2})$ and $\boldsymbol{v}$ are not co-linear, the link B_1B_2 has no possible rotation. A similar remark can be made for B_3B_4 , leading to the two following conditions:

$$\begin{aligned}(\boldsymbol{A_1B_1}\times\boldsymbol{u_1})\times(\boldsymbol{A_2B_2}\times\boldsymbol{u_2}) &\neq \boldsymbol{v}\\ (\boldsymbol{A_3B_3}\times\boldsymbol{u_3})\times(\boldsymbol{A_4B_4}\times\boldsymbol{u_4}) &\neq \boldsymbol{v}\end{aligned} \qquad (9)$$

In brief, if conditions (9) are fulfilled, links B_1B_2 and B_3B_4 will keep the direction they had in "central" position, and the rods in a pair will stay parallel to each other. Basically, each "meta-chain" behaves as it belongs to a Delta structure since their "end-effector" (link B_1B_2 or link B_3B_4) moves only in translation

4 Asymmetrical design

An asymmetrical design can be set up if we chose $\boldsymbol{u_1} = \boldsymbol{u_2}$, in the expression (5) giving the possible rotation of the link B_1B_2 . This expression becomes:

$$(\boldsymbol{A_1B_1}\times\boldsymbol{u_1})\times(\boldsymbol{A_2B_2}\times\boldsymbol{u_1}) = \beta\,\boldsymbol{u_1}$$

which means that the only possible rotation of the link is about vector $\boldsymbol{u_1}$ (as long as $\boldsymbol{A_1B_1} \neq \boldsymbol{A_2B_2}$). Moreover, this link has always the 3 translations. Thus, the "meta-chain" already fulfilled our requirements: 3 translations and 1 rotation about a given axis. The two additional P-U-S chains are 6-dof chains and are then obviously compatible with the motions offered by the first "meta-chain" (*cf.* Figure 10).

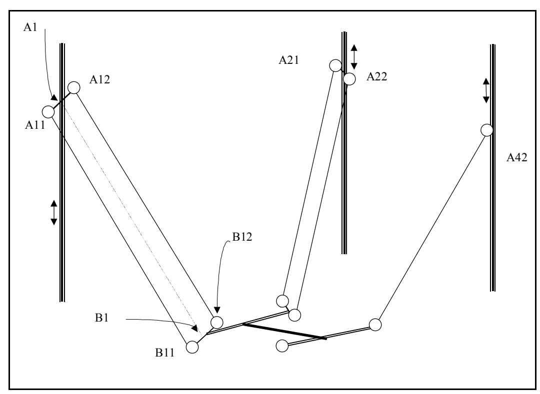

Figure 10. An asymmetrical H4.

As for the symmetrical H4, it is worth discussing the stability of the hypothesis: "the 2 rods of a P-(U-S)$_2$ chain are parallel". It is here obvious: the link B_1B_2 can rotate only about $\boldsymbol{u_1}$; thus the direction of $\boldsymbol{u_1}$ will remain constant, and the rods stay parallel.

5 Examples of implementation

This section is dedicated to the presentation of two possible implementations of the H4 concept: one symmetrical implementation; one asymmetrical implementation.
The arrangement of the symmetrical nacelle is given by Figure 11. The numerical data are the following: $\boldsymbol{v} = [0 \quad 0 \quad 1]^t$

$$\boldsymbol{u} = \begin{bmatrix} -1 & 1 & 1 & 0 \\ 1 & 1 & 0 & -1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \quad \boldsymbol{AB} = \begin{bmatrix} 1 & 1 & 0 & -1 \\ 1 & -1 & -1 & 0 \\ -1 & -1 & -1 & -1 \end{bmatrix}$$

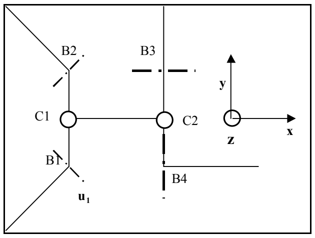

Figure 11. Example compatible with the condition.

This gives : $(\boldsymbol{w_1} \times \boldsymbol{w_2}) \times (\boldsymbol{w_3} \times \boldsymbol{w_4}) = [-2 \quad 6 \quad -4]^t$

Thus $(\boldsymbol{w_1} \times \boldsymbol{w_2}) \times (\boldsymbol{w_3} \times \boldsymbol{w_4}) \bullet \boldsymbol{z} \neq 0$. This arrangement is then a good candidate for a H4 robot. Such a solution is depicted in Figure 12. In Figure 13 an asymmetrical design is presented. Four linear actuators are arranged in an "horizontal" plane along a unique direction (say: x axis); two actuators are connected to P-$(U\text{-}S)_2$ chains with vector $\boldsymbol{u_1}$ parallel to the y axis; finally, the two last actuators are connected to P-U-S chains.

6 Conclusion

We have proposed in this paper a new family of 4-dof parallel robots which could be useful for high-speed handling or machining. One key point is the derivation of a necessary condition to obtain a rotation about a given axis. Practical implementations are currently under consideration and patents are pending for H4 family of robots.

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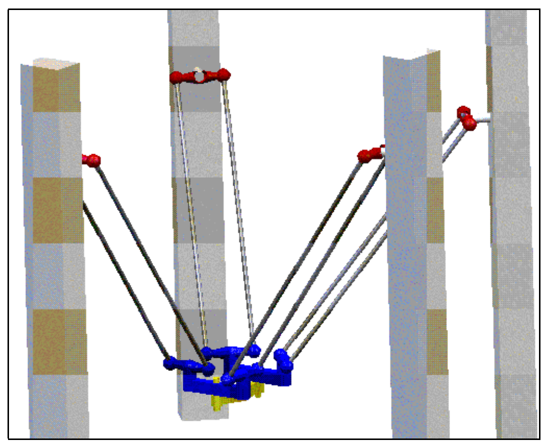

Figure 12. A P-$(U\text{-}S)_2$ H4 robot.

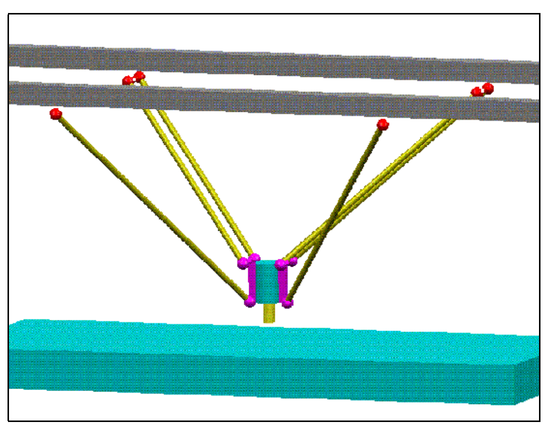

Figure 13. A P-$(U\text{-}S)_2$ / P-U-S H4 robot.