ON THE NUMBER OF FORESTS AND TREES IN LARGE REGULAR GRAPHS

FERENC BENCS

ABSTRACT. For a graph $G=(V,E)$ let $T_G(x,y)$ be the Tutte-polynomial. Let $(G_n)_n$ be a sequence of d-regular graphs with girth $g(G_n)\to\infty$, the length of the shortest cycle, then the limit

$$\lim_{n\to\infty} T_{G_n}(x,y)^{1/v(G_n)} = \begin{cases} (d-1)\left(\frac{(d-1)^2}{(d-1)^2-x}\right)^{d/2-1} & \text{if } x\le d-1,\\ x\left(1+\frac{1}{x-1}\right)^{d/2-1} & \text{if } x> d-1.\end{cases}$$

for $x\ge 1$ and $0\le y\le 1$. If $(G_n)_n$ is a sequence of random d-regular graphs, then the statement holds true asymptotically almost surely.

This theorem generalizes results of McKay ($x=1, y=1$, spanning trees of random d-regular graphs) and Lyons ($x=1, y=1$, spanning trees of large-girth d-regular graphs). Interesting special cases are $T_G(2,1)$ counting the number of spanning forests, $T_G(2,0)$ counting the number of acyclic orientations.

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