RANDOM CLUSTER MODEL ON REGULAR GRAPHS

IN COLLABORATION WITH FERENC BENCS AND MÁRTON BORBÉNYI

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Abstract. For a graph $G = (V, E)$ with $v(G)$ vertices the partition function of the random cluster model is defined by

$$Z_G(q, w) = \sum_{A \subseteq E(G)} q^{k(A)} w^{|A|},$$

where $k(A)$ denotes the number of connected components of the graph (V, A). In this talk we sketch the proof of the following result. If $(G_n)_n$ is a sequence of d-regular graphs such that the girth $g(G_n) \to \infty$, the length of the shortest cycle, then the limit

$$\lim_{n\to\infty} \frac{1}{v(G_n)} \ln Z_{G_n}(q, w) = \ln \Phi_{d,q,w}$$

exists if $q \geq 2$ and $w \geq 0$, and is equal to

$$\Phi_{d,q,w} := \max_{t\in[-\pi,\pi]} \Phi_{d,q,w}(t),$$

where

$$\Phi_{d,q,w}(t) := \left(\sqrt{1+\frac{w}{q}}\cos(t) + \sqrt{\frac{(q-1)w}{q}}\sin(t)\right)^d + (q-1)\left(\sqrt{1+\frac{w}{q}}\cos(t) - \sqrt{\frac{w}{q(q-1)}}\sin(t)\right)^d.$$

The same conclusion holds true for a sequence of random d-regular graphs with probability 1.

For integer q our work extends the results of Dembo, Montanari, Sly and Sun, and we prove a conjecture of Helmuth, Jenssen and Perkins. This is joint work with Ferenc Bencs and Márton Borbényi.

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