ON COUNTING ORIENTATIONS FOR GRAPH HOMOMORPHISMS AND FOR DUALLY EMBEDDED GRAPHS USING THE TUTTE POLYNOMIAL OF MATROID PERSPECTIVES

EMERIC GIOAN

ABSTRACT. An (oriented) matroid perspective (or strong map, or quotient, or morphism) is an ordered pair of (oriented) matroids satisfying some structural relationship. In the particular case of graphs, two notable types of perspectives can be considered: graph homomorphisms, and dually embedded graphs on a surface. Considering an orientation of a graph in the pair naturally yields a consistent orientation for the second graph in the pair. The Tutte polynomial of such a perspective is a classical 3-variable polynomial (also called Las Vergnas polynomial in the case of dually embedded graphs), whose coefficients and evaluations allow to count pairs of orientations of certain types (with respect to directed cycles and cuts). For instance, the evaluation at $(0, 0, 1)$ is classically known to count orientations giving an acyclic first graph and a strongly connected second graph (in the homomorphism situation), or giving two strongly connected dually embedded graphs (in the embedding situation). In this presentation, we show how coefficients and evaluations of the Tutte polynomial can be interpreted in terms of counting equivalence classes of pairs of orientations, named activity classes, as well as pairs of orientations where some edge orientations are fixed. These properties appear when the edge set is linearly ordered. They are available in general for oriented matroid perspectives (generalizing a joint work with Michel Las Vergnas).

LIRMM, CNRS, UNIVERSITÉ DE MONTPELLIER, FRANCE
E-mail address: emeric.gioan@lirmm.fr
URL: https://www.lirmm.fr/~gioan/