A SHORT(ER) PROOF OF BRYLAWSKI'S RELATIONS FOR TUTTE POLYNOMIAL COEFFICIENTS

GARY GORDON

ABSTRACT. The Tutte polynomial $f(M; x, y) = \sum b_{ij}x^i y^j$ can be defined for any set with a rank function r. Under mild assumptions about the rank function, one can extend Brylawski's relations for the coefficients: For all $0 \le k < n, \sum_{i=0}^{k} \sum_{j=0}^{k-i} (-1)^j {k-i \choose j} b_{i,j} = 0$. Brylawski discovered these identities for matroids in 1972, and he published two different proofs. We were able to generalize one of those proofs to greedoids (a generalization of matroids) in "Linear relations for a generalized Tutte polynomial" in 2014. The inductive proofs required several lemmas.

Recently, Beke, Csáji, Csikvári, and Pituk found a shorter, non-inductive proof of these identities. I'll discuss their proof, and some consequences of the identities.

LAFAYETTE COLLEGE *E-mail address*: gordong@lafayette.edu