EVALUATING GRAPH POLYNOMIALS ON COMPLETE GRAPHS

IN COLLABORATION WITH TOMER KOTEK AND VSEVOLOD RAKITA

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Abstract. Let $T(G; X, Y)$ be the Tutte polynomial for graphs. We start with the study the sequence $t_{a,b}(n) = T(K_n; a, b)$ where a, b are integers, and show that for every $\mu \in \mathbb{N}$ the sequence $t_{a,b}(n)$ is ultimately periodic modulo μ provided $a \neq 1 \mod \mu$ and $b \neq 1 \mod \mu$. This result is related to a conjecture by A. Mani and R. Stones from 2016. The theorem is a consequence of a more general theorem which holds for a wide class of graph polynomials definable in Monadic Second Order Logic augmented with modular counting quantifiers. This gives also similar results for the various substitution instances of the bivariate matching polynomial and the trivariate edge elimination polynomial $\xi(G; X, Y, Z)$ introduced by I. Averbouch, B. Godlin and the second author in 2008. Finally, we also discuss Harary polynomials on complete graphs. Given a graph property P, the Harary polynomial $H_P(G; k)$ counts the number of P-colorings of G with at most k colors. All our results depend on the Specker-Blatter Theorem from 1981, or its extension due to the author and E. Fischer, 2003, which studies modular recurrence relations of combinatorial sequences which count the number of labeled graphs.

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