ROOTS AND UNIMODALITY OF DOMINATION POLYNOMIAL OF GRAPHS

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Abstract. A dominating set of a graph G is a subset of vertices of G, say S, such that every vertex in $V(G) \setminus S$ is adjacent to at least one vertex of S. The domination polynomial of G is the polynomial $D(G,x) = \sum_{i=1}^{|V(G)|} d(G,i)x^i$, where $d(G,i)$ is the number of dominating sets of G of size i. In this talk we obtain some results about the roots of domination polynomial of graphs. More precisely we obtain that all roots of $D(G,x)$ lies in the set $\{z : |z+1| \leq \sqrt[\delta+1]{2^n - 1}\}$, where $\delta = \delta(G)$ is the minimum degree of vertices of G. We prove that $D(G,x)$ has at least $\delta - 1$ non-real roots. In addition, we show that if all roots of $D(G,x)$ are real, then $\delta = 1$. We construct an infinite family of graphs such that all roots of their polynomials are real. Finally we consider the unimodality of the average of the domination polynomial of all labeled graphs. For every $n \geq 1$, let $\Phi_n(x)$ be the average of the domination polynomial of all labeled graphs on n vertices. We prove that $\Phi_n(x)$ is log-concave and unimodal.

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