ALMOST UNIMODAL AND REAL-ROOTED GRAPH POLYNOMIALS

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Abstract. It is well known that the coefficients of the matching polynomial are unimodal. Unimodality of the coefficients (or their absolute values) of other graph polynomials has been studied as well. One way to prove unimodality is to prove real-rootedness.

Recently I. Beaton and J. Brown (2020) proved the for almost all graphs the coefficients of the domination polynomial form a unimodal sequence, and C. Barton, J. Brown and D. Pike (2020) proved that the forest polynomial (aka acyclic polynomial) is real-rooted iff G is a forest.

Let $\mathcal{A}$ be a graph property, and let $a_i(G)$ be the number of induced subgraphs of order i of a graph G which are in $\mathcal{A}$. Inspired by their results we prove:

Theorem: If $\mathcal{A}$ is the complement of a hereditary property, then for almost all graphs in $G(n, p)$ the sequence $a_i(G)$ is unimodal.

Theorem: If $\mathcal{A}$ is a hereditary property which contains a graph which is not a clique or the complement of a clique, then the graph polynomial $P_{\mathcal{A}}(G; x) = \sum_i a_i(G)x^i$ is real-rooted iff $G \in \mathcal{A}$.

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