COUNTING IN GRAPHS AND BIPARTITION POLYNOMIALS

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Abstract. We consider counting problems in undirected finite graphs and present some old and new results on counting dominating sets, edge coverings, and Eulerian subgraphs. These counting problems are related to graph polynomials. We present some interesting new relations and representations of the domination polynomial, Ising polynomial, Euler polynomial, and edge cover polynomial.

The common theme of all these seemingly unrelated results is the way of proving the statements. We show that a trivariate graph polynomial, the *bipartition polynomial*, is a universal tool for proofs in enumerative graph theory. The usefulness of the bipartition polynomail is based on three facts:

- Many known graph polynomials are evaluations of the bipartition polynomial: domination , Euler, cut, Ising, matching, vertex cover, edge cover, and independence polynomial (the latter three only in case of regular graphs). Further graph polynomials, like the neighborhood polynomial, can be easily derived from the bipartition polynomial.
- The bipartition polynomial allows a list of interesting representations as sum ranging over vertex or edge subsets of the graph, the set of all bipartite subgraphs, the set of complete forests and as a sum of edge cover polynomials.
- The bipartition polynomial of four-colored graphs satisfies a recurrence relation with respect to local edge and vertex operations.

In addition, the bipartition polynomial has a surprising power in distinguishing non-isomorphic graphs:

- Despite intensive computer search, we could not find any pair of non-isomorphic trees with coinciding bipartition polynomial. We know that there is no such pair for trees of order less than 19. Could it be that the bipartition polynomial distinguishes all trees?
- The smallest pair of non-isomorphic graphs with the same bipartition polynomial is of order 10 and size 20. We could not find any pair of non-isomorphic graphs with coinciding bipartition but different Tutte polynomial. However, we are not able to prove this fact in general. Could it be that the Tutte polynomial is determined by the bipartition polynomial?

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