Applying CHC Models to Reasoning in Fictions

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Abstract. In figuring out the complete content of a fictional story, all kinds of consequences are drawn from the explicitly given material. It may seem natural to assume a closure deductive principle for those consequences. Notwithstanding, the classical closure principle has notorious problems because of the possibility of inconsistencies. This paper aims to explore an alternative approach to reasoning with the content of fictional works, based on the application of a mathematical model for conjectures, hypotheses and consequences (abbr. CHCs), extensively developed during the last years by Enric Trillas and some collaborators, with which deduction in this setting becomes more comprehensive.

Keywords: Soft-Computing, CHC-Models, Reasoning, Fiction, Philosophy.

1 Introduction

Issues concerning fiction has increasingly attracted attention of logicians, philosophers and computer scientists during the last years. Particularly, several formal systems have been proposed and applied in order to represent the way in which a cognitive agent reasons about a work of fiction, [13], [14], [16], [15].

Talking about fictions is often restricted to conversations about literature, movies or TV-shows. Nevertheless, appealing to fiction in many areas of formal and empirical sciences has been also very fruitful. A clear example of this is the great interest in relating fiction with scientific models [11], [12]. Fiction has been applied not only to explain how a scientist builds a model but also to determine what kind of ontological entities models are. In this way, models have been understood as fictional entities; and the work scientists do while modelling different phenomena has been compared to the work of authors who create fiction. This relationship between models and fiction can also work in the opposite direction. If that were the case, it would be possible to define fiction, in its turn, as a sort of model.

During the last decade, Enric Trillas with some collaborators have worked out in [3], and more recently in [2], [7], [8], [9], a mathematical model for conjectures,

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hypotheses and consequences (abbr. CHCs), in order to execute with this model certain mathematical and informal reasoning. These interesting mathematical models for CHCs have been established algebraically, and the statements and propositions of human thinking are represented as those elements in an orthocomplemented lattice. Additionally, several meaningful operators are defined, which act on each given set of premises, intuitively standing for the conjectures and hypotheses as well as the consequences of that set of premises. The election of Orthocomplemented lattices is justified because these are quite general algebraic structures, in order to establish a sufficiently extensive reasoning model in which CHCs can be mathematically described. Alternatively, other algebraic settings have been also studied [4], [5].

This paper will be concerned with formal reasoning applied to ordinary experience with works of fictions. It aims to explore an approach to reasoning with the content of fictional works, which especially deals with deduction in this setting, based on the application of CHC-Models. The article is organized as follows. First a short reference about current philosophical work on fiction is given. In second place, recent work on CHC-Models is addressed and it is shown how it can bear on reasoning in fictions. Finally, solutions to some problems concerning deduction in this setting are considered. The conclusion will point out some further research.

2 Philosophy and Logic on Fiction

Problems related to fiction has raised several troubles for classical conceptions in the field of philosophy of language and logic. Already starting from the work of Frege [17], the semantic role of fictional names, i.e., names of characters, creatures and places that belong to fictional works, has been far from clear. Moreover, the semantic value of fictional sentences turned out to be controversial. Notably, it is hard to establish whether a sentence like "Sherlock Holmes is a detective" is true, false or truth-valueless, but it is also embarrassing to accept that fictional sentences are never true.

From the standpoint of philosophy and logic, inference in fiction involves reasoning with incomplete information. This is due to the fact that fictional stories describe their characters, places, and events only in an incomplete way. It is not possible, for instance, to determine if Sherlock Holmes is 1.80 meters tall. Additionally, inference in fiction also involves reasoning with inconsistent information that can emerge from two sources. On the one hand, information belonging to a fiction contradicts reality in many aspects. For example, while according to Doyle's stories Sherlock Holmes used to live in London in 221B Baker Street, in the real London, there was no Sherlock Holmes who used to live there. On the other hand, some stories are based on a contradiction or contain inconsistent information. For instance, this would be the case of a story where it is said that a character x has and does not have certain property P. Specially, cases of this last type will be addressed later in this paper.

Consequently, it has turned out to be quite difficult to provide a formal account of reasoning in fiction based on classical semantics. Firstly, as it was shown, the standard approach to interpret classical languages, is objectual. According to this interpretation, the domain of discourse assumes the existence of a non-empty set of real objects. Therefore, sentences like "Sherlock Holmes is a detective" or "Sherlock Holmes is a fictional character", in this classical formal setting, must be evaluated as false. Secondly, the notion of logical consequence is defined in terms of necessary truth preservation. Clearly, it can be seen that this definition forces to take bare truth as the only semantic value acceptable in a consequence relation. In this sense, a conclusion would follow from the premises of a fictional story just in case that conclusion is as barely true as the premises. Admittedly, sentences that contain fictional names never hold in a classical interpretation. Hence, it is not possible to give a compelling formal account of reasoning in fiction inside classical semantics.

Anyway, it could be argued that it is possible to give a classical formal account of reasoning in fiction confined to propositional classical logic. In this way, it is possible to avoid speaking about objects and try to deal with fictional discourse at a propositional level. However, problems arise also in a classical propositional formal setting. On the one hand, classical propositional semantics is bivalent. The principle of bivalence states that every sentence expressing a proposition has exactly one truth value: true or false (one or zero). As a consequence, the proposition *"p or not p"* equals one. Nonetheless, as fictional works are essentially incomplete, it is impossible for some sentences to establish whether they are true or false. On the other hand, classical propositional logic obeys the principle *Ex falso quodlibet*. According to this principle, anything follows from an inconsistent set of premises. It turns out that the consequences of the proposition *"p and not p"* equals L –the entire language. But a work of fiction can contain contradictions. Hence, in a classical formal framework, dealing with inconsistent information will overgenerate propositions derived from a story. And even worse, any proposition could be drawn from a fictional work. Thence, classical propositional logic does not provide an adequate formal system for dealing with reasoning in fiction. The following sections will reaffirm this diagnosis on the basis of the relationship between classical propositional calculus and Boolean algebra.

3 Introducing CHC Models

CHCs Models have been conceived as mathematical tools for studying commonsense, everyday or ordinary reasoning. Clearly, a model of this kind is not coincidental with the reality that is modelled by it, but a simplification of the reality. Anyway, these mathematical models bring a good mean of applying formal deductive reasoning for trying to understand reality and to do more accurate and clearer philosophical reflections on it. Moreover mathematical models can be viewed as useful devices to construct new realities through computational methods.

From the perspective of CHC–Models, common-sense reasoning can be decomposed in the pair consisting of 'conjecturing+refuting', two terms that embed two different types of deducing. On the one hand there is a type of deducing, which corresponds with the informal type of deduction that people carry out in Common-sense reasoning. On the other hand, there is a more restrictive kind, which has to do with the formal or mathematical concept of deductive consequence. Admittedly this last one is the concept that Alfred Tarski [10] formulated into the well-known definition of a consequence operator.

Thus, in common-sense reasoning, deduction is an informal and weaker concept than in formal, mathematical reasoning. To better characterize what people do with common-sense deduction, a moderate dose of formalization will be introduced in the next section.

4 A Model for Reasoning in Fiction

Assuming that reasoning impose the existence of some previous information about the subject under consideration, in any reasoning task there exist, from the beginning, a body of information already available on the subject. This is usually expressed through a finite number of statements or premises in natural language, which also can include other type of expressions like numbers or functions. Thus, one can assume to start with that the information given by a fiction F is somehow stored under this form of representing knowledge. Certain other constraints will be imposed later.

As it was just said, this paper mainly aims to apply CHC Models to the formal treatment of reasoning in fiction, hence it is convenient to introduce firstly some concepts concerning CHC-Models from previous work of Enric Trillas and his collaborators. Specially paying attention to [7], [9] and to [8], which seem to be better suited to the present work.

In the setting of CHC Models knowledge is represented in an adequate algebraic structure in a set $\mathsf{L}, (\mathsf{L}, \leqslant, \cdot, +, ')$, containing a pre-order $\leqslant$ representing *if/then*, a unary operation $'$ representing *not*, and two binary operations representing the linguistic *and* $(\cdot)$ and the linguistic *or* $(+)$. Clearly the set of premises of any type of reasoning cannot be trivially inconsistent. If they were, reasoning would be absurd or utter nonsense. Thus it makes sense to assume that the set of premises must satisfy at least a somehow minimal requirement of consistency. Let be $\mathsf{P} = \{p_1, \ldots, p_n\}$ the subset of L with these statements, which are taken as premises. It will be assumed that the element (not necessarily in P) $p_\wedge = p_1 \cdot \ldots \cdot p_n$ is not self-contradictory, i.e., $p_\wedge \nleqslant p_\wedge^{'}$. Let $\mathcal{F}$ be such family of subsets in L. Thus, this lack of self-contradiction means that P does not contain absurd premises.

The following example –loosely adapted from [7]– illustrate these concepts and it will help to clarify their application when reasoning with information retrieved from a fiction. The knowledge representation part revolves around the following statements, which are true about the novel "Farenheit 451" by Ray Bradbury:

Guy Montag is a fireman
A fireman burns books
Guy Montag loves books

Moreover a very clear fact also is: "Neither does Guy Montag love nor burn books".

Example 1. Let L be an ortholattice with the elements **f** for *Guy Montag is a fireman*, **b** for *A fireman burns books*, and **l** for *Guy Montag loves books*, and its corresponding negations, conjunctions and disjunctions. It is also the case that *Guy Montag is a fireman*, *Guy Montag is a fireman and does not burn books*, and *neither does Guy Montag love nor burn books.* Representing *and*, *or* and *not* as stated before, the set of premises is $\mathsf{P} = \{f, f \cdot b', (l \cdot b)'\}$. Thus the *core-value* of this information can be identified with $p_\wedge = f \cdot f \cdot b' \cdot (l \cdot b)' = f \cdot f \cdot b' \cdot (b' + l') = f \cdot b'$.

Some reasoning can be done in the lattice. Notably some inferences can be drawn, on the basis of the information of a fiction F, once conceded that $a \leqslant b$ means that b is a logical consequence of a. All these inferences conform the class of "conjectures" in the setting of CHC-Models. Among conjectures, consequences, hypothesis and speculations are distinguished. However to keep things as simple as possible, these classification between different types of conjectures, can be put aside and an overall distinction can be made only between "conjectures" and "consequences" for the sake of convenience. Thus, from the example it follows,

- *Guy Montag does not burn books, b'*, is a consequence of P, since $p_\wedge = f \cdot b' \leqslant b'$.
- The statement *"Guy Montag is a fireman and He does not burn books and He loves books"*, $f \cdot b' \cdot l$ and *"He loves books"*, l, are conjectures of P
- The statements *"he burns books"* and *"he is not a fireman"* are refutations of P, since $f \cdot b' \leqslant b'$, and $f \cdot b' \leqslant f = (f')'$

Understandably for representing reasoning in natural language a richer and more complex framework would be highly desirable. Specially, aspects of tense and other subtleties of common language are most difficult to interpret in these more rigid algebraic structures. However, within this limited and closed framework, this example may still count as a formalization of a piece of human reasoning.

5 The Problem of Consistency

At first glance, in the previous example, a contradiction looms over the conclusions. On the one hand it holds on the story that "Guy Montag is a fireman" and, also that "a fireman burns books". On the other hand one knows that "Guy Montag does not burn books". Accordingly, it can be inferred then that "Guy Montag is not a fireman'". This is a very simple inference supported by the inference rule known as "Modus Tollens", from $a \to b$ and b', it follows a', where $\to$ is the material conditional, interpreted as $a \to b = a' + b$. Nevertheless, in spite of its obviousness, this inference presupposes some particular structural features. As argued in [1] "deduction" and "inference", must be clearly distinguished when formalizing reasoning. And one has to be aware also that the validity of formulas belonging to the language, depends on the particular deductive system at stage. In the example, for the inference to be valid, L must be endowed with the algebraic structure of a Boolean algebra, and the consequence operator should be the greatest one for such a framework. The verification of the inequality expressing Modus Tollens , i.e., $b' \cdot (a \to b) = b' \cdot (a' + b) = a' \cdot b' \leqslant a'$ in ortholattices as much as in De Morgan algebras, will cause the validity of the laws of Boolean algebras. Conversely, if the consequence operator is changed, the validity of the inference scheme can no longer be guaranteed. Arguably, the problem in this case does not have to do with the formalization of a conditional proposition as material conditional. Actually, there is no conditional statement to be formalized at all. In spite of this, according to certain naïve reading of the story, it is right to think that "either Guy Montag is not a fireman or he does burn books". A logic-minded person who sticks to classical logic, would feel that things could not be otherwise. But they are, because in the story, Guy Montag manages to do both, he remains a fireman and refrains from burning books. Unquestionably, the problem can be ascribed to the consequence operator, which is the greatest one for a Boolean algebra, and contributes to validate mentally this inference scheme.

Thus, it seems impossible to confine oneself to the premises and to ensure an overall consistency, while keeping this kind of propositional reasoning in the setting of classical logic or Boolean algebras. These are the kind of problems, which are frequent when reasoning in fictions. The closure of classical deductive consequence, together with the meaning attributed to classical connectives, implies that some undesirable conclusions must be accepted in spite of contradicting explicit information. Hence, the type of consistency concerning reasoning in fiction has to comply somehow with standards of inference and consequence other than those of classical logic. Additionally, the example about Guy Montag, helps to see that "consequences" are not the only type of deductions gained from the premises. There are also "conjectures", which are in fact a lot more useful.

The following definition from [7] specifies what is meant by a conjecture, relative to a given problem on which some information conveyed by a set $\mathsf{P} = \{p_1, p_2, \ldots, p_n\}$ of n premises is known.

Definition 1. *q is a conjecture from P, provided q is not incompatible with the information on the given problem once it is conveyed throughout all p_i in P.*

The most important things in this definition are, on the one side, how to state that P is consistent; and on the other side, how to interpret the requirement that q is not incompatible with the information given by P.

Accordingly, to apply CHC-Models to reasoning in fictions, an appropriate notion of incompatibility will be needed in each case. This notion of incompatibility should be conveniently dissociated from the closure property of classical deduction.

In the development of CHC-Models during the last decade, the construction of conjecture's operators has relied persistently on standard consequences' operators. Recently, in [1] it has been shown that to keep the most typical properties of the concept of conjecture, it suffices to only consider operators that are extensive and monotonic, but without enjoying the closure property. It is worth to get a glimpse at this. Given a non empty set of sentences L let $\mathcal{F}$ be a family of subsets in L. Then a standard consequence operator, is a mapping $\mathbf{C}$: $\mathcal{F} \to \mathcal{F}$, such that,

- $\mathsf{P} \subset \mathbf{C}(\mathsf{P})$, $\mathbf{C}$ is *extensive*
- If $\mathsf{P} \subset \mathsf{Q}$, then $\mathbf{C}(\mathsf{P}) \subset \mathbf{C}(\mathsf{Q})$, $\mathbf{C}$ is *monotonic*,
- $\mathbf{C}(\mathbf{C}(\mathsf{P})) = \mathbf{C}(\mathsf{P})$, or $\mathbf{C}^2 = \mathbf{C}$, $\mathbf{C}$ is a *closure.*

for all P, Q in $\mathcal{F}$. In addition, *consistent* operators of consequence verify

- If $q \in \mathbf{C}(\mathsf{P})$, then $q' \notin \mathbf{C}(\mathsf{P})$

In [7] several consequence operators lacking in closure are distinguished. Especially there are three operators of consequence, which are significant to the purpose of formalizing reasoning in fictions,

- $\mathbf{C}_1(\mathsf{P}) = \{q \in \mathsf{L} : r(P) \cdot q' = 0\}$
- $\mathbf{C}_2(\mathsf{P}) = \{q \in \mathsf{L} : p_\wedge \cdot q' \leqslant (p_\wedge \cdot q')'\}$
- $\mathbf{C}_3(\mathsf{P}) = \{q \in \mathsf{L} : p_\wedge \leqslant q\} = \mathbf{C}_\wedge(\mathsf{P})$.

where $r(P)$ refers to the *core-value* of the information gathered in the premises, which could verify for example, $r(P) \leqslant p_\wedge$.

Concerning these consequence operators, for $i = \{1, 2\}$ it is $\mathsf{P} \subset \mathbf{C}_i(\mathsf{P})$, and if $\mathsf{P} \subset \mathsf{Q}$, then $\mathbf{C}_i(\mathsf{P}) \subset \mathbf{C}_i(\mathsf{Q})$. Nevertheless, $\mathbf{C}_i$ cannot be always applicable to $\mathbf{C}_i(\mathsf{P})$ since it easily can be $r\mathbf{C}_i(P) = 0$, due to the lack of consistency of $\mathbf{C}_i$. Furthermore, $\mathbf{C}_\wedge(\mathsf{P})$ is also a consistent operator of consequence.

Hence, the corresponding operator of conjectures $Conj_i$ does not come from an operator of consequences, but only from an extensive and monotonic one, for which the closure property has no sense, since $\mathbf{C}_i(P)$ cannot be taken as a body of information in the sense of [7], i.e., guaranteed free from incompatibility.

Remark 1. To have $\mathbf{C}(\mathsf{P}) \subset Conj_C(P)$, it suffices for $\mathbf{C}$ to be a consistent operator of consequences. Hence, the consistency of $\mathbf{C}$ is what characterizes the inclusion of $\mathbf{C}(\mathsf{P})$ into $Conj_C(P)$, that consequences are a special type of conjectures. Thus, when the premises harbour inconsistencies, which cannot be taken away, $\mathbf{C}_1$ and $\mathbf{C}_2$ seem to be the only alternatives to take. Apparently, it is the case while reasoning in fictional stories.

6 Designing the Appropriate Framework

The problem of knowledge representation is one of the most important aspects of any formalization process. Notably, for representing ordinary reasoning, which usually involves natural language, more flexible constructions are needed. Concerning algebraic structure, it turns out that algebras of fuzzy sets enjoy such a flexibility. In [7] and [6] an abstract definition of a *Basic Flexible Algebra* is given. Latices with negations and, in particular, ortholatices and De Morgan algebras are instances of BFAs. Also the standard algebras of fuzzy sets $\left([0,1]^X, \mathsf{T}, \mathsf{S}, \mathsf{N}\right)$ are particular BFAs.

Previous to any specific application of CHC-Models, some questions must be addressed concerning the representational framework. First of all, where do the objects ('represented' statements) belong to. That is, which is L, such that $\mathsf{P} \subset \mathsf{L}$ and $q \in \mathsf{L}$? Secondly, with which algebraic structure is endowed L?. And lastly, how to state that P is consistent, and how to translate that *q is not incompatible*?

To assume that P is free of incompatible elements is to concede that there are not elements p_i, p_j in P, such that $p_i \leqslant {p'}_j$, or $p_i \cdot p_j = 0$. To avoid the odd case $p_i \cdot \ldots \cdot p_n = 0$ it is convinient to assume that $r(P) \neq 0$.

To keep things simpler in a first attempt, it will be convenient to pay attention only to the cases in which the BFA is an ortholatice. As a matter of fact, a De Morgan algebra offers a basic suitable structure for reasoning with the content of a fiction. Moreover, some clarification is in order concerning the formulation of the key principles of Non-contradiction and Excluded Middle. For that goal, the following distinction between the incompatibility concept of contradictory and self-contradictory elements in a BFA may be introduced.

– Two elements a, b in a BFA are said to be *contradictory* with respect to the negation $'$, if $a \leqslant b'$.
– An element a in BFA is said to be *self-contradictory* with respect to the negation $'$, if $a \leqslant a'$.

In dealing with De Morgan algebras, these principles, formulated in the way that is typical of standard modern logic ($a \cdot a' = 0; a + a' = 1$), do not hold. Nevertheless, with an alternative formulation, De Morgan algebras also verify those principles, that is,

– **NC**: $a \cdot a' \leqslant (a \cdot a')'$
– **EM**: $(a + a')' \leqslant \left((a + a')'\right)'$

7 More on Consequence Operators and Conjectures

Arguably, it is worth to submit reasoning in fictions to the supposition that L is endowed with an ortholatice structure $\mathcal{L} = (\mathsf{L}, \cdot, +, ', 0, 1)$. Among the alternatives to express the non incompatibility between the premises and a conjecture q, there are two, which seem to be adequate in the present case: $r(P) \cdot q \neq 0$ and $r(P) \cdot q \nleq (r(P) \cdot q)'$. It means that either the proposition added to the core is admissible or that the set formed this way is not auto-contradictory or impossible. The applications will make sense as much as $r(P) = p_\wedge \neq 0$.

Manifestly, it was readily seen that there is a close connection between conjectures and consequence operators. Moreover it has been also shown that there are conjecture operators $Conj_i$, which do not come from an operator of consequence. Anyway, even in the case of fictions, it seems that logical consequences are always counted as a particular case of conjectures. Then, it is natural that one wonders whether there is also a consequence operator $\mathbf{C}(P) \subset Conj_i$. Notably, it turns out that $\mathbf{C}_\wedge$ is such an operator and it is $\mathbf{C}_\wedge \subset Conj_i(P)$. Hence, $\mathbf{C}_\wedge$ turns out to be in this case the safer conjectures. The set of conjectures, which no longer include $\mathbf{C}_\wedge$, is the set

$$Conj_i(P) - \mathbf{C}_\wedge(P) = \{q \in Conj_i(P) : q < p_\wedge\} \cup \{q \in Conj_i(P) : qNCp_\wedge\} \quad (1)$$

where *NC* stands for non *order comparable*. Each set in (1) that remains when $\mathbf{C}_\wedge$ is taken away, has more risky deductions obtained by reasoning from the premises.

Furthermore, any operator of conjectures verify some of the following properties [7] ,

1. $Conj(P) \neq \emptyset$
2. $0 \notin Conj(P)$
3. There exist an operator $\mathbf{C}$ such that $Conj(P) = \{q \in \mathsf{L} : q' \notin \mathbf{C}(P)\}$
4. *Conj* is expansive: $P \subset Conj(P)$
5. *Conj* is anti-monotonic: If $P \subset Q$, then $Conj(Q) \subset Conj(P)$

Concerning this intended application to reasoning in fictions, all these features of conjecture operators are attractive. Especially, anti-monotonicity is very appealing. As noted also in [7], $Conj_{\mathbf{C}}$ is anti-monotonic *if and only if* $\mathbf{C}$ is monotonic. That is, conjectures and consequences are particularly linked with respect to monotony. When reasoning in fictions, it is also important to observe that a more encompassing set of premises can make showing up conjectures that one could not see before. This is an admissible interpretation of anti-monotonicity in this setting.

8 Conclusion

By using CHC-Models, deductions are treated differently in connection with reasoning. In particular, several types of deduction are distinguished. Moreover, by separating consequence and inference, a new perspective on the use of logic for reasoning is gained. It is possible now to apply more complex algebraic settings to account for different types of deductions, which taken together give a more promissory approach to the variety of human reasoning. Specifically, the application sketched in this paper, helps to show how a complex type of reasoning concerning fictional content, can be approached from the methodological perspective of CHC-Models. Deductions are no longer submitted to a classical closure principle. An alternative model, based on consequences and conjectures, can control reasoning instead. The advantages of this approach deliver promising results. Arguably, more complex information, such as inexact or fuzzy knowledge, coming from fictions, can be also represented. Even in this case, by changing the algebraic setting, this type of content shall be also accommodated. Future work on the subject can be also addressed to consistently combine suitable contextual principles into CHCs models, in order to search for conjectures and consequences thereby generated. These models can be achieved by mimicking simple fictional scenarios, on which subjects can perform specific inference tasks, whose results are somehow circumscribed by determinate constraints.

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